
Experiments on Oscillatory and Pulsatile Particle-Laden Flows: Resonance and Turbulence Transition

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Abstract

Fluid mechanics research often relies on the simplification of flows as statistically steady. However, in reality, many processes across biological, environmental, and engineering systems are governed by periodic forcing of the flow velocity. This can introduce complex nonlinear dynamics, such as *resonance* and *turbulence*, leading to an amplification of unsteady forces and structural loads. To explore these phenomena, novel experimental methodologies are applied to investigate how periodic forcing interacts with inherent fluid timescales, to identify critical observables for nonlinear interactions, and to establish mechanisms for flow prediction.

In the first part of this research, the transition to turbulence in a pulsatile particle-laden pipe flow is studied. This is primarily motivated by the human cardiovascular system, where plasma carries suspended particles like blood cells and emboli, and where turbulence is linked to the development of circulatory diseases. Despite this importance, the underlying turbulence transition mechanisms remain poorly understood due to the complexity of the system. To address this gap, a novel experimental setup was developed to investigate the dynamic interactions between particles and pulsatile flow in a straight rigid pipe. By implementing a Lagrangian Particle Tracking method using Shake-The-Box in combination with refractive index matched hydrogel particles, this study provides the first simultaneous, three-dimensional measurements of both the fluid velocity field and particle dynamics in a pipe flow. The findings reveal turbulence emerging as intermittent patches which grow and relaminarize during each period, triggered by vortices in the wakes of individual particles. This turbulence onset in a particle-laden flow follows the mechanisms of steady pipe flow, with a development primarily dominated by the advective timescale of the pulsation rather than phase-specific helical instabilities. Notably, this transition dynamic remains consistent across both complex physiological and harmonic waveforms, and was also observed for geometric perturbations of a pipe bend. This suggests a universal underlying regime of turbulence onset driven by large perturbation mechanisms.

The second flow examined in the thesis is liquid sloshing resonance, a phenomenon critical to aerospace engineering where large amplitude fluid oscillations in propellant tanks can compromise structural integrity and control. Predicting these resonant states is notoriously difficult due to the inherently nonlinear nature of the fluid dynamics. This research explores sloshing in a rectangular container subjected to harmonic forcing, employing newly developed optical measurements and comparing experimental data with different resonance modeling approaches. A state-of-the-art multimodal analytical sloshing model is shown to systematically overpredict resonance peaks, even for relatively small amplitude waves. In contrast, a nonlinear Duffing oscillator provides an excellent fit for the low amplitude dynamics. Critically, both models fail at larger sloshing amplitudes, as they do not account for the nonlinear dissipation arising with more complex wave motion. The research identifies the phase lag between the forcing and the fluid response as a critical observable and indicator for the onset of these dissipative effects. The most successful modeling results were achieved using the spectral submanifold theory. This data-driven approach captures nonlinear effects in both frequency and dissipation without requiring prior knowledge of the underlying fluid mechanics.

Kurzfassung

In der Fluidmechanik werden Strömungen häufig als statistisch stationär vereinfacht. Viele Prozesse in biologischen, ökologischen und technischen Systemen sind real jedoch durch periodische Anregungen bestimmt. Dies kann komplexe nichtlineare Dynamiken wie *Resonanz* und *Turbulenz* hervorrufen, was zu starken instationären Kräften führt. Um diese Phänomene zu untersuchen, werden neuartige experimentelle Methoden angewandt, zur Untersuchung von: Interaktionen zwischen periodischer Anregung und inhärenten Fluid-Zeitskalen, kritische Messgrößen für nichtlineare Interaktionen und Mechanismen zur Strömungsvorhersage.

Im ersten Teil dieser Arbeit wird der Übergang zur Turbulenz in einer pulsierenden, partikelbeladenen Rohrströmung untersucht. Dies ist primär durch den menschlichen Blutkreislauf motiviert, in welchem Turbulenzen mit der Entstehung von Herz-Kreislauf-Erkrankungen in Verbindung gebracht werden. Trotz dieser Relevanz sind die zugrunde liegenden Mechanismen des Turbulenzübergangs nur unzureichend verstanden. Um diese Lücke zu füllen, wurde ein neuartiger Versuchsaufbau entwickelt, um die dynamischen Interaktionen zwischen Partikeln und pulsierender Strömung in einem geraden, starren Rohr zu untersuchen. Mittels der Shake-The-Box Methodik in Kombination mit Brechungsindex angepassten Hydrogel-Partikeln liefert diese Studie die ersten simultanen, 3D Messungen von Geschwindigkeitsfeld und der Partikeldynamik. Die Ergebnisse zeigen, dass Turbulenz in Form von intermittierenden Strukturen in jeder Periode auftritt, ausgelöst durch Wirbel im Nachlauf einzelner Partikel. Dieser Turbulenzeinsatz folgt den Mechanismen der stationären Rohrströmung, wobei die Entwicklung primär durch die advective Zeitskala der Pulsation und nicht durch phasenspezifische helixförmige Instabilitäten dominiert wird. Bemerkenswerterweise bleibt diese Übergangsdynamik sowohl bei komplexen physiologischen als auch bei harmonischen Wellenformen konsistent und wurde auch bei geometrischen Störungen durch einen Rohrbogen beobachtet. Dies deutet auf ein universelles, zugrunde liegendes Regime des Turbulenzeinsatzes hin, das durch starke Störungsmechanismen getrieben wird.

Die zweite in der Arbeit untersuchte Strömung ist die Resonanz von Flüssigkeits-schwappen, ein für die Raumfahrt kritisches Phänomen, bei dem starke Wellenbewegung in Treibstofftanks die strukturelle Integrität und Steuerung gefährden können. Die Vorhersage dieser Resonanzzustände ist aufgrund der nichtlinearen Fluidodynamik äußerst schwierig. Diese Arbeit untersucht das Schwappen in einem rechteckigen Behälter unter harmonischer Anregung, wobei neu entwickelte optische Messverfahren eingesetzt und verschiedene Modellierungsansätze verglichen werden. Es wird gezeigt, dass ein modernes multimodales analytisches Schwappmodell Resonanzspitzen systematisch überschätzt, selbst bei relativ kleiner Wellenamplitude. Im Gegensatz dazu liefert ein nichtlinearer Duffing-Oszillator eine exzellente Übereinstimmung für niedrige Amplituden. Entscheidend ist, dass beide Modelle bei größeren Schwappamplituden versagen, da sie die nichtlineare Dissipation in komplexen Wellen nicht berücksichtigen. Diese Forschungsarbeit identifiziert den Phasenversatz zwischen der Anregung und Fluidoszillation als kritische Messgröße und Indikator für den Einsatz dieser dissipativen Effekte. Die erfolgreichsten Modellierungsergebnisse wurden mit der Theorie des Spectral Submanifold erzielt. Dieser Daten-basierte Ansatz erfasst nichtlineare Effekte sowohl in der Frequenz als auch in der Dissipation, ohne das Vorkenntnisse der zugrunde liegenden Strömungsmechanik erforderlich sind.

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List of Abbreviations

DNS	Direct Numerical Simulation
LNG	Liquefied Natural Gas
LPT	Lagrangian Particle Tracking
NS	Navier-Stokes
PIV	Particle Imaging Velocimetry
PTV	Particle Tracking Velocimetry
SSM	Spectral Submanifold
STB	Shake-The-Box
SW	Sexl-Womersley

List of Symbols

A	pulsation amplitude or dimensionless excitation amplitude in sloshing	
a_n, b_n	Fourier coefficients of a pulsation waveform	
c_n	mass ratio of sloshing mass for mode n	
d_n	parameters of the multimodal model (B.3)	
D	pipe diameter	m
D_p	particle diameter	m
D_{bend}	diameter of geometric bend perturbation	m
E	eigenfunction of sloshing wave components	m ² /s
$\mathbf{E}$	spatial eigenfunction of a perturbation	m/s
E_n	parameters of the multimodal model (B.3)	
$\mathbf{f}$	vector of external forces	kg/(s ² m ²)
f	frequency	Hz
f_n	natural frequency of mode n	Hz
$\hat{f}_n$	dimensionless waveform of mode n in the multimodal model	
F	driving force in sloshing	m/s ²
F_{adv}	advective or inertial forces	N/m ³
F_{diff}	diffusive or viscous forces	N/m ³
G	energy growth from linear stability analysis	m/s ²
g	gravitational acceleration	m/s ²
h	filling level	m
h_{crit}	critical filling level for softening-hardening transition	m
h_0	parameter of the multimodal model (B.3)	
J_0	Bessel function of the first kind and order zero	
k	turbulent kinetic energy	m ² /s ²
$\bar{k}$	azimuthally averaged turbulent kinetic energy	m ² /s ²
l	container depth	m
L	pipe length	m
L_a	length of the pushing rod in the sloshing drive system	m
L_{bend}	length of bending perturbation	m
L_E	entrance length for a fully developed laminar pipe flow	m
L_f	characteristic length scale of a flow	m
L_m	pipe length for pressure measurements	m
L_o	offset of bending perturbation	m
L_{pulse}	pipe length passed during one pulsation period	m
m	mass	kg
m_n	sloshing mass of mode n	kg
m_0	resting mass in sloshing tank	kg
M_1	frequency shift parameter from multimodal model (B.3)	
n	mode number	
$\mathbf{n}$	normal vector	
$\tilde{n}$	refractive index	

$\tilde{n}_1, \tilde{N}_1, \tilde{N}_2$	parameters of the multimodal model (B.3)	
p	pressure	Pa
p_{diff}	normalized pressure difference between perturbed and laminar flow	
$\hat{p}_n$	pressure amplitude of mode n	Pa
P_n	parameters of the multimodal model (B.3)	
q_n	parameters of the multimodal model (B.3)	
r	radial position	m
R	pipe radius	m
R^2	coefficient of determination	m
t	time	s
t_{adv}	advective timescale of the flow	s
t_{diff}	diffusive timescale of the flow	s
t_{end}	ending time turbulence patches in a period	s
t_f	time scale of fluid velocity change	s
t_{onset}	onset time turbulence patches in a period	s
t_p	time scale of particle response	s
T	pulsation and oscillation period	s
T_{exp}	experiment duration	s
$\mathbf{u}$	velocity vector	m/s
u_p	particle velocity	m/s
$\mathbf{u}_0$	base state velocity vector	m/s
$\mathbf{u}'$	perturbation velocity vector	m/s
$\bar{u}$	azimuthally averaged velocity	m/s
$\hat{u}$	peak velocity during one period	m/s
U	mean velocity	m/s
V_f	volume of fluid	m ³
V_p	volume of particles	m ³
w	container width	m
w_0	parameter of the multimodal model (B.3)	
x	sloshing center of mass position	m
$\mathbf{x}$	position vector	m
x_a	excitation amplitude in sloshing	m
x_n	sloshing center of mass position of mode n	m
$\hat{x}$	sloshing center of mass amplitude	m
X	dimensionless sloshing center of mass position	
$\hat{X}$	dimensionless sloshing center of mass amplitude	
$\hat{X}_m$	dimensionless sloshing center of mass amplitude from resonance model	
$\hat{Y}$	dimensionless amplitude parameter of the multimodal model	
De	Dean number	
$\hat{De}$	peak Dean number during one period	
De_m	phase averaged Dean number	
Re	Reynolds number	
$\hat{Re}$	peak Reynolds number during one period	
Re_m	phase averaged Reynolds number	
Re_c	critical Reynolds number	
$Re_{c,p}$	critical Reynolds number of particle-laden flow	
St	Stokes number	
Wo	Womersley number	

α	incident angle of a refraction	
β_n	temporal evolution of mode n of the multimodal model	
γ_n	viscous damping ratio of mode n	
δ_n	viscous damping rate of mode n	1/s
ϵ_n	nonlinear Duffing parameter of mode n	1/(m ² s ²)
ε	dimensionless Duffing parameter	
ε_t	rate of dissipation of turbulent kinetic energy	m ² /(s ³)
ζ	surface elevation	m
θ	azimuth angle	
λ	friction factor	
λ_E	eigenvalue of a perturbation	1/s
μ	dynamic viscosity	Pa s
ν	kinematic viscosity	m ² /s
ξ	container width coordinate	m
ρ	density	kg/m ³
σ	standard deviation	
ϕ	phase lag	
ϕ_m	phase lag from resonance model	
ϕ_p	particle concentration	
Φ	velocity potential	m ² /s
ω	angular frequency	1/s
ω_p	angular momentum of a particle	1/s
ω_n	natural angular frequency of mode n	1/s
ω_z	streamwise vorticity	1/s
Ω	dimensionless excitation frequency	

Chapter 1

Introduction

Fluid flows exhibit a vast diversity in their temporal and spatial dynamics. In many studies, such flows are treated as statistically steady over time, to simplify analysis and modeling. However, even nominally steady flows can develop periodic flow patterns as a result of intrinsic instabilities. Two examples of this are shown in Fig. 1.1, where the flow around a cylinder forms a wake of periodic vortices (Fig. 1.1a) or an atmospheric shear flow becomes unstable and develops periodic waves (Fig. 1.1b). In contrast, fluid flows subject to an external period forcing remain comparatively less explored, despite their prevalence in many natural or engineering systems. These flows are characterized by an imposed periodic acceleration and deceleration to the bulk fluid motion, which will strongly influence their dynamics compared to steady driving. Within this context, it is important to differentiate between oscillating flows, where the bulk flow fully reverses direction during each period, resulting in a phase-averaged mass transport of zero, and pulsatile flows, which exhibit an oscillating component superimposed on a nonzero mean flow (see Fig. 1.2).

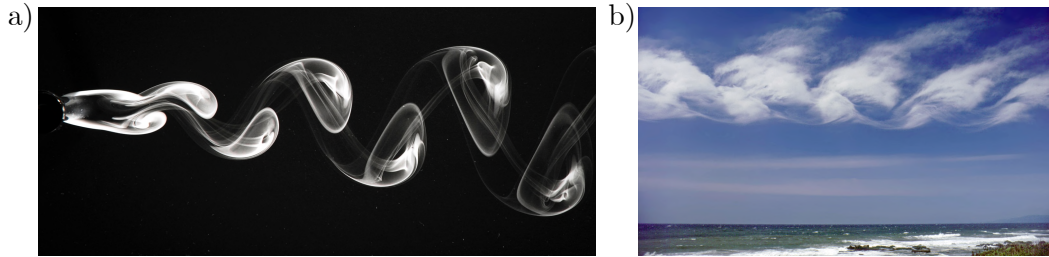


FIGURE 1.1: Examples of periodic flows developing in different systems. a) A Karman vortex street, caused by periodic vortex shedding in the wake of a rigid body. b) Periodic waves in the Kelvin-Helmholtz instability, occurring due to shear velocity between two different layers in the flow. Image credit for the pictures through Wikimedia Commons¹.

Fluid flows with oscillating or pulsatile driving play a crucial role in various fields, including biology, environmental science, and engineering. These periodic flows can span over many magnitudes in length, from the cardiovascular flow in the smallest blood vessels ($\mathcal{O}(10^{-6} \text{ m})$) (Sebastian and Dittrich, 2018), to the motion of ocean waves ($\mathcal{O}(1 \text{ m})$) (Deike, 2022) and even oscillations within accretion disks of orbital motions ($\mathcal{O}(> 10^{11} \text{ m})$) (Balbus and Hawley, 1998). The characteristic frequencies of periodic motions are similarly diverse, ranging from very slow tidal flows ($\mathcal{O}(10^{-5} \text{ Hz})$) (Hendershott and Munk, 1970), to the human heart rate ($\mathcal{O}(1 \text{ Hz})$) (Ku, 1997), up to high frequency acoustics ($\mathcal{O}(> 10^3 \text{ Hz})$) (Zhang et al., 2020). When trying to understand periodically forced flow systems, two ubiquitous phenomena have been the focus of researchers and engineers for centuries: *resonance* and *turbulence*. Both mechanisms are associated with large unsteady forces, unwanted vibrations, and increased structural loads if not properly accounted for (Bodnár, Galdi, and Necasová,

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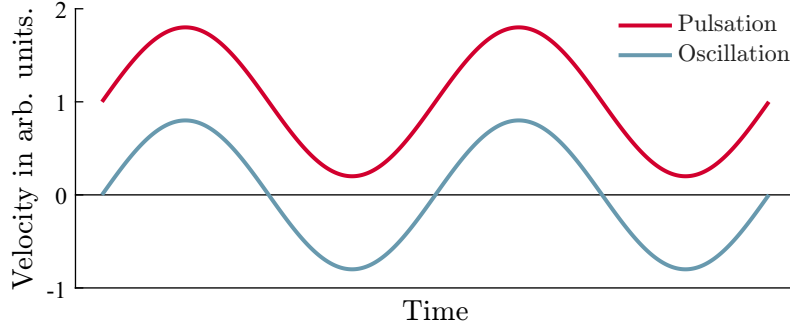


FIGURE 1.2: Comparison between an oscillatory and pulsatile flow velocity timeseries.

2014; Davidson, 2004). In other applications, turbulence can be utilized to aid in mixing or heat-transfer processes, and resonant modes can be exploited to transfer energy at specific frequencies. Despite their prevalence the prediction of resonance or turbulence onset is notoriously difficult even for simple systems, as both mechanisms can arise from complex nonlinear dynamics (Avila, Barkley, and Hof, 2023; Eckhardt et al., 2007; Ockendon and Ockendon, 2001). To understand how fluid flows can transition to turbulence or contain resonant modes, it is necessary first to introduce fundamentals of fluid mechanics in the next section, before presenting the overarching objectives of the thesis.

1.1 Governing equations of fluid flows

The Navier-Stokes (NS) equations are a set of partial differential equations to describe fluid motion. They are perhaps the most important equations of fluid dynamics and were first formulated by Claude-Louis Navier in 1822 and later extended by George Gabriel Stokes. The formulation reads,

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \frac{\mathbf{f}}{\rho}, \quad (1.1a)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (1.1b)$$

with the conservation of momentum in Eq. (1.1a) and the conservation of mass in Eq. (1.1b) as functions of the fluid velocity vector $\mathbf{u}$, time t , pressure p , fluid density ρ , kinematic viscosity ν and any external forces $\mathbf{f}$. For many systems these equations are extended with the conservation of energy, describing the thermodynamics properties of the flow, and equations of state, relating fluid properties like pressure, temperature and density.

The conservation of mass, or continuity equation (Eq. (1.1b)), expresses that the change of the mass in a system is equal to the amount of mass flowing in and out. The momentum conservation (Eq. (1.1a)) is a specific formulation of Newton's second law for fluids. The change in momentum, represented by the total derivative of the fluid velocity on the left-hand side, is equal to the sum of forces acting on the fluid on the right-hand side. These forces include the pressure gradient, which drives the fluid from high to low pressure regions, viscous forces that resist deformation and dissipate energy from the flow, and any external body forces acting the fluid, such as gravitation. What makes this equation notoriously difficult is the convective acceleration term on the left-hand side $(\mathbf{u} \cdot \nabla) \mathbf{u}$, representing the transport of momentum by the fluid flow. This term couples the velocity components with themselves, making the equation nonlinear and leads to number of complex dynamics.

The most prominent example of nonlinear interactions is the transition to turbulence of fluid flows. This effect is characterized by the dimensionless parameter of the Reynolds

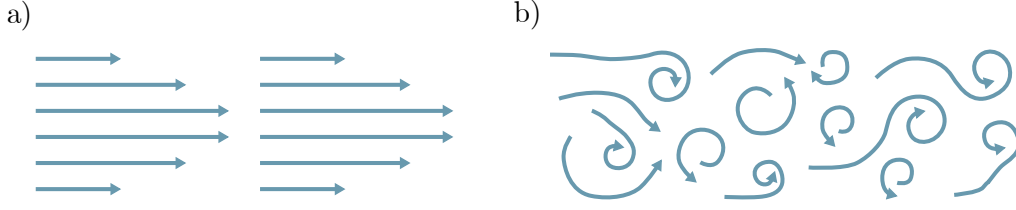


FIGURE 1.3: Sketched flow field of a laminar flow with smooth layers a) compared to chaotic turbulent flow with many different sized eddies b).

number as the ratio between inertial or advective forces F_{adv} and viscous or diffusive forces F_{diff} , or the ratio between the timescales of diffusive t_{diff} and advective t_{adv} effects,

$$Re = \frac{F_{\text{adv}}}{F_{\text{diff}}} = \frac{t_{\text{diff}}}{t_{\text{adv}}} = \frac{U L_f}{\nu}, \quad (1.2)$$

with the characteristic velocity of the flow U , the characteristic length scale of the flow L_f , and the kinematic viscosity ν . At low Re , the viscous forces dominate and the flow tends to stay laminar, with a velocity field composed of smooth, predictable, adjacent layers (see Fig. 1.3a). If the laminar flow is perturbed (e.g. with local fluctuation to the velocity), the viscous forces act faster than the inertial one ($t_{\text{diff}} \lesssim t_{\text{adv}}$), and the flow will decay back to its previous state. For higher Re on the other hand, the inertial forces are dominant and the flow becomes unstable. In this regime, any disturbances are advected faster than the viscosity can dampen them ($t_{\text{diff}} \gg t_{\text{adv}}$). This leads to a state of broad velocity fluctuations and eddies called turbulence (see Fig. 1.3b), where the flow is chaotic – highly sensitive to initial conditions – and becomes unpredictable (Davidson, 2004).

The exact dynamics of turbulence are difficult to compute analytically from the Navier-Stokes equations due to their nonlinear convective acceleration term. Exact solutions of NS equations are generally limited to laminar flows or only applicable to simplified cases or specific geometries. However, the equations can be solved numerically using computational fluid dynamics (CFD). This involves discretization of the flow system into a set of equations which are solved iteratively. These procedures often involve simplifying assumptions, such as time-averaging the flow or using empirical turbulence models, to reduce the computational demand. To fully resolve the flow, direct numerical simulations (DNS) are used. While DNS can give the most detailed and accurate description of turbulent fluid flows, it requires a spatial grid resolution that captures both integral length scales the largest eddies in the flow and the smallest turbulence scale, known as the Kolmogorov scale (Kolmogorov, 1941), where turbulence dissipates due to viscous forces. As the range between the large and small scales increases with the Reynolds number, often spanning several orders of magnitude, these simulations necessitate finer grid resolutions and more computational resources (Davidson, 2004). Consequently, DNS is limited in its feasibility for many real-world applications with high Re .

Additional challenges arise when simulating flows that interact with structures or multiphase flows (for example, suspended particles or bubbles in a liquid). Simulating fluid-structure interactions, such as resonances or deformations, requires simultaneous solving of both the fluid dynamics equations and the governing equations for structural mechanics, coupled in a single numerical grid (Bodnár, Galdi, and Necasová, 2014). Similarly, for a multiphase flow (e.g. particles in a fluid), the computations need appropriate models of the particle dynamics (particle-fluid or particle-particle interactions), and the fluid simulations must resolve the boundary and flow around each individual particle. This is often only possible if either the fluid or particle dynamics are simplified in the computational model

(Yeoh and Joshi, 2023).

For such problems, where numerical solutions are constrained by computational costs or simplifying assumptions, experimental fluid dynamics are essential. In experiments, flow behavior is measured in a physical system of a laboratory experiment or field study. These can either validate numerical models and theoretical predictions or reveal flow phenomena that are missed in analytical models or simulations (Tavoularis and Nedić, 2005).

Although the dynamics of fully turbulent flow cannot be easily described analytically and require CFD or experiments, the onset of turbulence can in some cases be predicted through a linearized analysis of the NS equations. For this purpose, the velocity is composed of a base state $\mathbf{u}_0$ (e.g. the laminar flow solution or the fluid at rest) and a perturbation to the velocity $\mathbf{u}'$,

$$\mathbf{u}(\mathbf{x}, t) = \mathbf{u}_0(\mathbf{x}, t) + \mathbf{u}'(\mathbf{x}, t). \quad (1.3)$$

Assuming the perturbation velocity is small ($\mathbf{u}' \ll 1$), the NS equation (without any external forcing) can be linearized around the base flow state by removing higher-order terms of $\mathbf{u}'$,

$$\frac{\partial \mathbf{u}'}{\partial t} + (\mathbf{u}_0 \cdot \nabla) \mathbf{u}' + (\mathbf{u}' \cdot \nabla) \mathbf{u}_0 = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u}'. \quad (1.4)$$

This gives a linear partial differential equation, where the perturbation velocity can be described with a modal solution of,

$$\mathbf{u}'(\mathbf{x}, t) = \mathbf{E}(\mathbf{x}) e^{\lambda_E t}, \quad (1.5)$$

where $\mathbf{E}(\mathbf{x})$ is the spatial eigenfunction or shape of the perturbation and λ_E its eigenvalue which determines the growth rate and frequency of the mode. Each eigenmode represents a possible pattern of the fluid flow, such as waves, vortices, or oscillations, that can develop in the system. The shape of possible eigenmodes is restricted by boundary conditions or the geometry of the flow.

If the eigenvalue of a mode is stable ($\Re(\lambda_E) < 0$), the mode decays exponentially over time due to viscous forces, and the flow reverts back to its base state. On the other hand, should the eigenvalue be unstable ($\Re(\lambda_E) > 0$), the mode grows exponentially. It is important to note that these instabilities can form coherent wave-like patterns and periodic oscillations even if the base flow is steady over time (as shown for the examples in Fig. 1.1). If an unstable mode grows significantly, the linearized assumption is no longer valid and nonlinear interactions, through the convective term $(\mathbf{u}' \cdot \nabla) \mathbf{u}'$, become important. The unstable mode can transfer energy and to other modes and excite different frequencies and wavenumbers, leading to a cascade of velocity fluctuations on different length and timescales and a transition to a turbulent flow.

This transition scenario describes the case of a linear instability with an infinitesimally small perturbation. For many systems, the transition becomes more complex. Even if all individual modal perturbations are linearly stable and predicted to decay, a linear superposition of stable eigenmodes can form a nonmodal disturbance which changes shape over time, and can experience transient growth and even lead to nonlinear interactions and turbulence (Schmid, 2007). In addition, a linearly stable flow can turn turbulent if the initial perturbation is sufficiently large (*finite amplitude perturbation*) to directly trigger nonlinear interactions, leading to a subcritical transition (Avila, Barkley, and Hof, 2023; Eckhardt et al., 2007; Strogatz, 2018). As the analysis of nonlinear perturbations requires solving the complete NS equations, such transitions to turbulence become difficult to predict (Kerswell, 2018).

So far, this description of fluid mechanics has been generalized to all fluid flows. In the scope of this thesis, I aim to investigate the specific case of oscillatory and pulsatile

driven flows, where averaged bulk velocities are periodic in time. How this will interact with the developing eigenmodes and turbulence transition is the focus of the thesis and will be discussed in the next section.

1.2 Main objectives of the thesis

In an oscillating or pulsatile flow, the system is forced with a specific frequency. When looking at the dynamics of a perturbation in the linearized NS equation, initially the already introduced approach can be used, as the base flow does not need to be steady. A periodically driven system will still develop eigenmodes, which can decay or grow into turbulence. However, when evaluating the temporal development of the flow, a periodic forcing can drastically affect its dynamics. The effect can be twofold:

As introduced through the Reynolds number, the transition to turbulence can be understood by the competition of advective and diffusive forces and timescales in the flow. Larger inertial forces with short advective timescales incentivize the growth of turbulence, while a flow dominated by viscous forces and timescales will stay laminar. If an eigenmode becomes unstable, it still requires sufficient time to grow, trigger nonlinear interactions, and break up into turbulence. In a periodically forced flow, this dynamic is extended with the third timescale of the oscillation or pulsation itself, where the instantaneous amount of inertial energy in the flow varies over time. This will periodically enhance or suppress the possible turbulence growth, depending on the phase within a period. Furthermore, oscillations or pulsations can also affect the instantaneous stability of eigenmodes throughout the period (Xu, Song, and Avila, 2021). If the flow is unstable only for part of a pulsation period, turbulence can be triggered, grow, and afterwards decay, all within a single period (Xu and Avila, 2018). On the other hand, if the pulsation frequency is much faster, then the timescale with which the turbulence decays is too slow and turbulence is sustained even if the flow becomes linearly stable during part of the period (Xu et al., 2017). The interactions of periodic flows with turbulence growth are far from understood.

A second interaction in periodic flows can arise when the forcing timescale is close to that of an eigenmode of the system. In that case, the flow will develop resonances, where the motion of the excited mode becomes strongly amplified, even if the mode itself is still stable. With this resonant amplification, the flow can grow beyond a linear regime, leading to nonlinear interactions. These can generate additional harmonics (multiples of the forcing frequency), cause modal interactions, modify the frequency of the eigenmode itself, and even cause a fully chaotic motion (Ockendon and Ockendon, 2001). The emergence and development of such nonlinear resonances are difficult to predict (Kovacic and Brennan, 2011; Strogatz, 2018).

Both of these interactions of a periodic fluid flow are governed by nonlinear dynamics and thereby are notoriously challenging to study. The effects can be summarized in the first main research objective of the thesis:

- O1** How does a periodic forcing interact with the inherent timescales of the fluid system, expressed through the eigenmodes and the turbulence growth and decay rate?

The second objective of the thesis concerns the selection of relevant observables for the nonlinear interactions of the fluid system, which can be captured experimentally. For all investigations, whether analytical, numerical, or experimental, it is essential to evaluate the dynamics of a system through relevant properties that can be used to describe the flow, known as observables. Observables are measurable quantities which provide insights into the relevant evolution of a system without requiring knowledge of the full higher-dimensional state of the system. Especially in a complex nonlinear system, where predicting the complete

dynamics is challenging or physically not possible, it can be crucial to select observables that can construct a simplified low-order model of the essential physics (Holmes et al., 2012).

For example, in fluid resonance phenomena, the response of a fluid system to a specific forcing frequency can often be described by analytically derived linear models, such as a linear harmonic oscillator approximation (Ibrahim, 2005; Torres Rojas, Pagonabarraga, and Corvera Poiré, 2017). If nonlinearities are relevant, a weakly nonlinear analysis may be employed, where nonlinear effects are assumed to be small compared to the dominant linear dynamics (Faltinsen et al., 2000). These methods can yield effective models to describe a resonance, even though the underlying fluid dynamics are governed by the nonlinear NS-equations. Although such formulations deliver simple and practical models for a complex systems, it becomes imperative to investigate their limitations and consider which aspects cannot be captured.

Also in turbulent flows, where the onset is triggered by nonlinear interactions of velocity perturbations and the dynamics are inherently unpredictable, the flow state can often be characterized using relatively simple observables. Some examples are the use of turbulent kinetic energy (as a measure of all velocity component fluctuations) (Davidson, 2004), turbulence intensity (as a measure of fluctuation magnitudes compared to the mean flow) (Mahrt, 2014), or turbulent fraction (as an indicator of how much of time or space is occupied by turbulent compared to laminar flow) (Rotta, 1956). These observables cannot reveal the mechanisms through which turbulence growth, saturates, or relaminarizes, as they do not resolve the full spectrum of fluctuations and their energy within the flow. However, they can be used to assess the strength of turbulence in a large space of flow parameters in both experiments and simulations, where a full description of the flow at each point in space and time would not be feasible. In this sense, these turbulence indicators function similarly to resonance models as a simplified description of nonlinear dynamics, allowing predictions of the loads caused by the flow or differentiations of different flow states.

The investigation of periodic flows in this thesis will involve the critical evaluation of existing observables and models and the conceptualization of new approaches where unknown aspects of fluid flows are encountered. The main research objective can be summarized as:

- O2** Which observables are necessary to describe the nonlinear interactions of a pulsatile or oscillatory fluid flow? What are the operational limits to reliably utilize the observable to model the flow?

Following the selection of suitable observables for characterizing the flow, a further challenge lies in using these observables to construct predictive models for the evolution of the flow. In a periodically forced flow system, the amplitude and frequency composition of a forcing can vary, thereby affecting the potential onset and growth of turbulence or the amplification of a resonant mode. Classically, the prediction of these effects would require a new numerical solution of the NS-equations for a given forcing. Through the selection of observables that express the dominant dynamics of the flow, a prediction can be simplified to a model of reduced complexity, with the aim of just describing the evolution of a given observable. However, their reliability will be limited to specific ranges of forcing parameters or mechanisms of the flow. Therefore, the aim is to not only determine their limitations, but also to understand how a periodic forcing affects the fluid dynamics of the system. Through which mechanisms does turbulence or resonance develop in the flow, and which of the mechanism can expressed in reduced-order model. This objective of the thesis can be summarized as:

- O3** How can selected observables and models be utilized to prediction the development

of resonances and turbulence in periodically forced flow? What are the underlying mechanisms of the flow involved in the evolution of these two effects?

These objectives are explored for two examples of fluid systems, where nonlinear resonances and turbulence emerge from a periodic forcing of the flow. The systems are the turbulence transition of a pulsatile pipe flow and the resonance of liquid sloshing with an oscillatory forcing, and will be introduced in the two following sections.

1.3 Pulsatile pipe flow

Approximately 10% of the global energy consumption is used for pumping fluid through pipe systems (Frenning, 2001). As a result, pipe flow is one of the most studied fundamental flows of fluid mechanics. In recent years, this has led to major advances in understanding the transition to turbulence in pipe flow (Avila, Barkley, and Hof, 2023). However, the turbulence onset of fluid flow through pipe networks is not limited to engineering problems, one of the most prominent examples being the human cardiovascular system. Here, the formation of turbulence in larger blood vessels like the aorta can be linked to cardiovascular disease (Ku, 1997; Smith et al., 1972), which is the leading cause of death globally according to the World Health Organisation (2017). Specifically, the increase in shear stress associated with turbulent flow can cause atherosclerosis, where the inner cell layer of the vessel walls (endothelium cells) becomes inflamed, leading to plaque formation or rupture (Cunningham and Gotlieb, 2005; Davies et al., 1986; Malek, Alper, and Izumo, 1999). Despite this physiological relevance, the mechanisms for the onset of turbulence in cardiovascular flow are largely not well understood. The reason for this is that the cardiovascular system exhibits a variety of complex properties, compared to steady flow in a straight, round, and rigid pipe, which makes it difficult to study and model accurately (Ku, 1997; Sebastian and Dittrich, 2018; Zamir, 2000). To name a few:

1. The flow is pulsatile over time, driven by the periodic pumping of the heart. Especially in the aorta, this leads to a complex waveform with rapid acceleration and deceleration of the flow during *systole* (contraction of the heart), followed by a longer resting phase with close to zero flow velocity during *diastole* (relaxation of the heart) (Bürk et al., 2012; Ku, 1997). A comparison between a simple harmonic pulsation and a physiological waveform is shown in Fig. 1.4. Such a complex pulsation will modify both the flow profiles and the linear stability of the flow (Morón, Feldmann, and Avila, 2022).
2. Blood is a multiphase fluid that contains high concentrations of particles such as

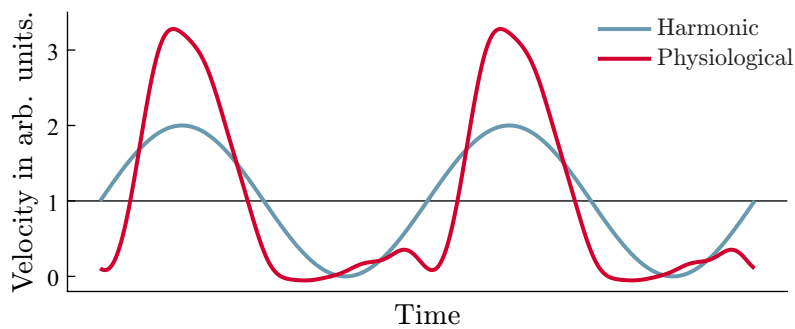


FIGURE 1.4: Comparison between the waveforms of a harmonic pulsatile flow and the waveform inside the human aorta.

blood cells (Sebastian and Dittrich, 2018) or even large particles like emboli (clusters of cholesterol, fragmented blood clots, or calcification) (Kobayashi and Karino, 2016; Mukherjee and Shadden, 2017). While sufficiently small, neutrally buoyant particles will follow the streamlines of the fluid flow, an increase in particle size or density difference will produce inertial effects that decouple the particle motion from the fluid flow. An increased inertia of a particle results in a delayed response of the particle to changes in the flow, or even significant modification of the surrounding flow field for large particles, see Fig. 1.5. This effect is commonly characterized by the dimensionless Stokes number St as the ratio between the particle relaxation time and characteristic timescale of the flow.

3. Due to its complex composition, blood as a fluid has non-Newtonian rheological properties, where the viscosity of the fluid changes, for example, based on the applied shear. In blood flow, these effects include pseudoplasticity (reduced viscosity for higher shear), viscoelasticity (combining both viscous and elastic characteristics), and thixotropy (change of viscosity over a relaxation time) (Beris et al., 2021).
4. The size of blood vessels spans over multiple orders of magnitude, from the smallest capillaries with diameters of a few microns, to the large aorta with a ~ 30 mm diameter, which is also reflected with a range of Reynolds numbers in the different vessels (Sebastian and Dittrich, 2018). In between, the vessels form a complex network of bifurcations and geometry changes (Ku, 1997; Sebastian and Dittrich, 2018), see Fig. 1.6a.
5. The vessel walls are elastic, resulting in complex fluid structure interactions, such as dynamic deformations under pressure which results in modifications to fluid flow and the pulsation waveform (Mei, Zhang, and Jing, 2018).
6. The Reynolds numbers typically observed in the human aorta lie in the transitional regime between laminar and turbulent flow (Stalder et al., 2011), where turbulence arises in pipe flow as a subcritical transition from finite amplitude perturbations (Avila, Barkley, and Hof, 2023). Even in contemporary studies, blood flow is still often considered fully laminar at all times (Saqr et al., 2020).

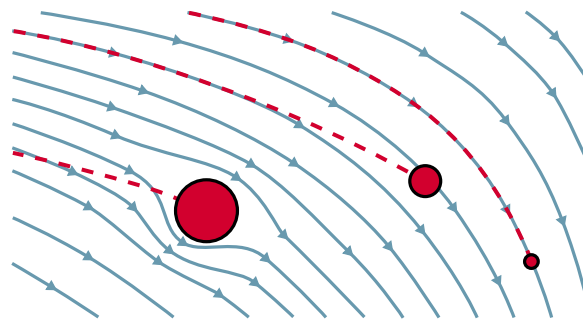


FIGURE 1.5: Interactions between a fluid flow and differently sized particles. Small particle will follow the streamlines of the flow. With increasing size the particles take longer to react to changes in the flow direction, leading to altered particle trajectory. Large inertial particle will strongly interact with and modify the surrounding fluid flow.

The sum of these aspects makes it extremely difficult to fully recreate this flow in numerical simulations or classical laboratory experiments, especially because many of the cardiovascular flow's properties will vary from patient to patient and be affected by

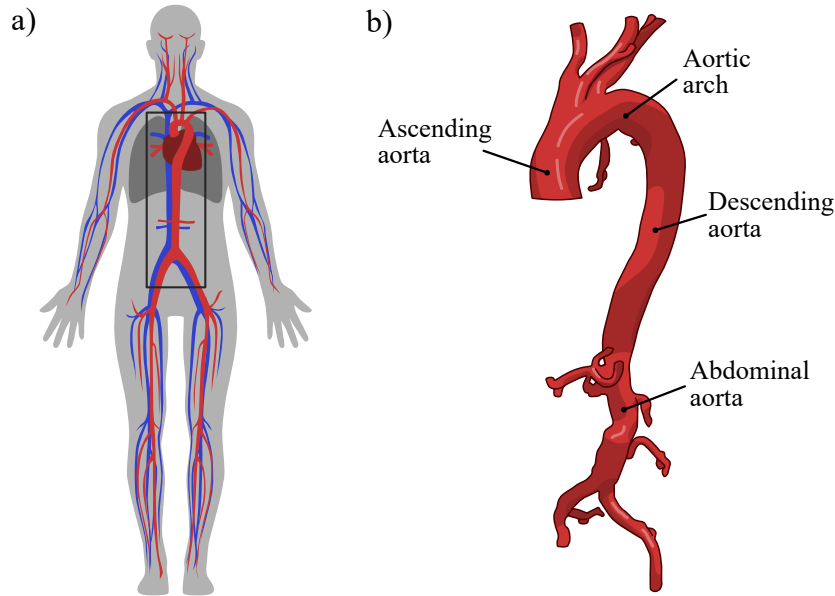


FIGURE 1.6: a) Schematic drawing of the human cardiovascular system with arteries carrying oxygen-rich blood in colored red and veins carrying the oxygen depleted blood displayed as blue. Not pictured are the smaller scale arterioles, venules and capillaries connecting the two networks. b) Sketch of the human aorta.

parameters like age or health of the individual (Bürk et al., 2012; Komutrattananont, Mahakkanukrauh, and Das, 2019). During *in vivo* (in the living organism) studies, measurements are often performed with magnetic resonance imaging methods (Binter et al., 2016; Bürk et al., 2012; Marshall et al., 2004), which can be limited in resolution and are unable to spatially or temporally resolve actual turbulence (El Sayed et al., 2023). For an understanding of cardiovascular fluid dynamics and its turbulence onset, it is crucial to perform fundamental investigations and focus on the effect of individual aspects of cardiovascular flow. This is the objective of the interdisciplinary DFG research unit² through which part of my thesis was funded. Here, multiple groups investigate different aspects of pulsatile flow, from hydrodynamic instabilities to migration phenomena with both Newtonian and complex fluids. This encompasses fundamental research of pipe flows up to the study of individual blood cell dynamics in small-scale capillaries or microchannels, using numerical simulations, laboratory experiments, and *in vivo* studies.

As part of this research unit, the studies and results presented in this thesis will focus on the fundamental study of the turbulence onset in pulsatile flow. From the complex aspects of the cardiovascular system previously listed, not all are considered. In the cardiovascular network, the effects of turbulence will most likely occur in the aorta, where the Reynolds numbers are largest (Stalder et al., 2011). For a fundamental investigation, the geometry of the descending aorta (after the aortic arch, depicted in Fig. 1.6b) can be approximated as a cylindrical straight pipe without bifurcations. The non-Newtonian properties of blood can be attributed to the high concentration of deformable red blood cells ($\sim 40\%$ volume concentration of *hematocrit*), with interactions like aggregation affecting shear stress (Beris et al., 2021). Since the focus is on larger blood vessels, where non-Newtonian effects become negligible (Liu et al., 2011), the flow properties can be considered Newtonian. Similar reasoning can be applied to the multiphase properties of blood. Relative to the diameter of the aorta ($\sim 30\text{ mm}$) the size of individual red blood cells ($\sim 6 - 8\text{ }\mu\text{m}$) (Ku, 1997; Sebastian and Dittrich, 2018) becomes negligibly small, and they will follow the fluid flow

²DFG research unit FOR 2688, <https://pulseflow.uni-saarland.de/> Grant No. AV 156/1-1.

without inertial interactions (see Fig. 1.5). For large vessels, a mesoscopic description in which blood cells contribute only to the viscosity is sufficient (Beris et al., 2021). However, particles such as emboli are much larger (~ 1 mm) and will have inertial interactions with the fluid (Kobayashi and Karino, 2016; Mukherjee and Shadden, 2017). These particles can be approximated as spherical, hard, and neutrally buoyant, to facilitate comparison with previous studies of particle-laden flow. The aspects of fluid structure interactions with the elastic vessel walls will not be considered, to better isolate and understand the dynamics that can be attributed to the pulsation and interactions with particles. To study the influence of the different perturbation mechanisms, I will also present results using a modified pipe geometry with a bend as a controlled and reproducible flow perturbation. Although geometry bends are physiologically relevant, the focus lies on the underlying transition mechanisms and not on an exact reproduction of anatomical geometries.

The resulting system in my thesis is a Newtonian pulsatile flow in a rigid straight pipe containing rigid spherical neutrally buoyant particles. This approach aims to combine the existing understanding of the turbulence transition in steady pipe flow (Avila, Barkley, and Hof, 2023; Eckhardt et al., 2007), with the newer studies on the turbulence transition in pulsatile flow and its instabilities (Morón, Feldmann, and Avila, 2022; Xu and Avila, 2018; Xu et al., 2020), and the studies of the turbulence transition in particle-laden pipe flow (Agrawal, Choueiri, and Hof, 2019; Hogendoorn, Chandra, and Poelma, 2022; Matas, Morris, and Guazzelli, 2003).

For the investigation of pulsatile pipe flow, I aim to answer the following research questions:

- RQ1** What type of turbulence transition occurs in a particle-laden pulsatile pipe flow, and how does the perturbation of particles compare to other flow perturbation mechanisms? Are particles able to trigger instabilities of pulsatile flow? Is the onset of turbulence enhanced or delayed?
- RQ2** What is the effect of a physiological pulsation waveform compared to a harmonic pulsation on the transition to turbulence? Does the shorter acceleration and deceleration contribute to turbulence growth, or is the transition inhibited?
- RQ3** How is the turbulence transition affected by the frequency of the pulsation, giving perturbations more or less time to be advected and grow in each period?

These questions will be specified in more detail, after the theoretical background (chapter 2).

The second dynamic investigated in this thesis is the development of nonlinear resonances. While a pulsatile flow in combination with elastic walls can also produce resonance effects (Torres Rojas, Pagonabarraga, and Corvera Poiré, 2017), the resonance phenomena and its emergence of nonlinear dynamics will be explored in a different flow system of resonant sloshing waves, which is ideally suited for the study of flow resonances.

1.4 Resonances of sloshing liquids

In fluid mechanics, sloshing refers to the wave motion of a free surface of liquid in a container. The sloshing waves arise due to dynamic interactions between the container and the fluid when the system is subjected to an external forcing. Based on the constraint of the tank geometry the fluid has a number of eigenmodes as different standing waves of the free surface. If the frequency of a forcing is close to a natural frequency, the system will resonate, resulting in a large amplification of the sloshing motion, see Fig. 1.7. Crucially, these sloshing waves also coincide with an oscillation of the fluid mass resulting in high dynamics loads on the container walls during resonance. In civil engineering, the effect of

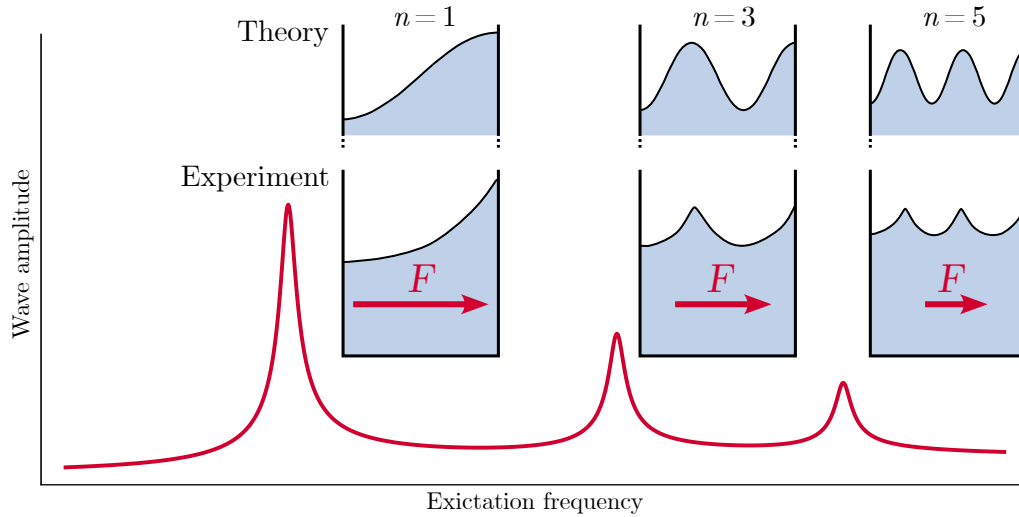


FIGURE 1.7: Wave amplitude of the first three asymmetric sloshing modes $n = [1, 3, 5]$ as a function of the excitation frequency, with sketches of the theoretical and experimentally observed wave shapes. The first mode has the largest wave amplitudes and sloshing forces.

sloshing waves can be exploited by specifically tuning the sloshing resonance to dampen the vibrations of skyscrapers during earthquakes or strong winds (Çelebi et al., 2013). On the other hand, in moving vehicles such as Liquefied Natural Gas (LNG) carriers (Godderidge et al., 2008) or tank trucks (D'Alessandro, 2011) the sloshing motion can be important for structural loads or vehicular dynamics. A specifically noteworthy example are liquid-fueled rockets, where sloshing motion can severely interfere with the guidance and control of the vehicle's flight dynamics. This was first observed during Saturn I flight SA-1 (Abramson, 1966), but even more recently during the 2nd flight of Falcon I sloshing motion led to instabilities in the control system and failure to reach orbit (Dreyer, 2009). For each of these applications, it is important to have reliable models to predict under what conditions and how strong the sloshing resonance will occur.

There are a number of challenges that complicate the investigation and modeling of the sloshing motion and its resonance (Abramson, 1966; Ibrahim, 2005):

1. Sloshing systems have a number of different wave modes, especially in shallow tanks, their natural frequencies are close together (Chester, 1968a,b). With large sloshing amplitudes near a resonance, multiple modes will be excited simultaneously and contribute to the sloshing wave (Faltinsen et al., 2000; Moiseev, 1958). The resulting wave shapes will become more complex (as shown with the theoretical and experimentally observed waves sketched in Fig. 1.7). This can lead to effects of nonlinear energy transfer between modes, with a cascading effect to a fully chaotic wave motion (Ockendon and Ockendon, 2001).
2. The container geometries of sloshing applications can be complex, with integral structures such as baffles or segmentation walls that interact with and dampen sloshing waves (Arndt and Dreyer, 2008; Dodge, 2000). Here, the geometry of the tank will affect the sloshing modes and their coupling (Faltinsen and Timokha, 2017), or even cause complex fluid structure interactions (Babu and Bhattacharyya, 1996; Hwang et al., 2016).
3. In applications like LNG tanks or for many rocket propellants, the fluids are cryogenically cooled gasses. For such fluids, the thermodynamics of the system need to be considered, with effects like external temperature gradients, thermal stratification,

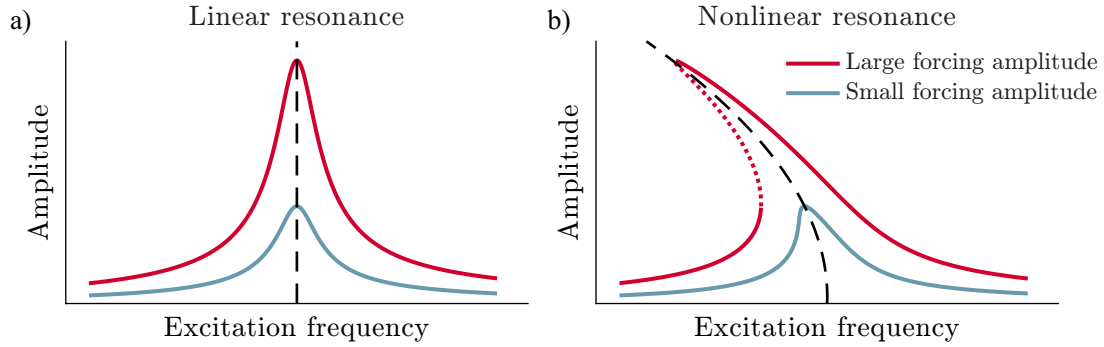


FIGURE 1.8: Comparison between a linear resonance a), where the amplitude resonance is proportional to the forcing amplitude, and a nonlinear resonance b), where larger forcing modifies the response curve.

and phase transition (Ibrahim, 2005; Monteiro et al., 2025; Zuo, Jiang, and Huang, 2018).

4. While a linearized approach is sufficient to calculate the sloshing eigenmodes, during the large amplitudes of a resonance the sloshing system will become nonlinear. The effect on the sloshing amplitude response is shown in Fig. 1.8. For a linear system (Fig. 1.8a), the response amplitude curve is simply proportional to the forcing. In the nonlinear response (Fig. 1.8b) – such as sloshing governed by the nonlinear NS-equations – the natural frequency of the eigenmode is shifted with larger amplitudes (indicated by the black dashed line) (Abramson, Chu, and Kana, 1966; Bauer, 1966). The resulting resonance curve is *folding over*, shifting the maximum away from a linear natural frequency prediction. For large forcing (red line), this leads to coexisting solutions with multiple sloshing amplitudes for the same excitation frequency. Here, the system is hysteretic and approaches one of the stable solutions based on its initial conditions. The dotted line represents an unstable solution that connects the two stable branches.

Similar to the cardiovascular system, the presence of these complex dynamics and nonlinear effects in sloshing necessitates a fundamental approach with a focus on individual aspects of the system. The focus lies on investigating the nonlinearity in the liquid sloshing resonance and its description with different models. For a long time, sloshing models incorporating multiple modes have been analytically derived for an inviscid system (Moiseev, 1958; Ockendon and Ockendon, 1973). While these multimodal models can predict the interactions of modes (Faltinsen et al., 2000), or three-dimensional wave motion (Faltinsen, Rognabakke, and Timokha, 2003, 2005) during resonance, they will become increasingly complex to apply, requiring numerical approaches to solve (Faltinsen, Lagodzinskiy, and Timokha, 2020). On the other hand, sloshing models can often not be qualitatively validated, due to the lack of accurate and well-suited experimental data, according to Hill (2003). As a result, in many engineering applications and in the design of sloshing systems, the models are often simplified to equivalent mechanical systems, like pendulum or mass-spring-dashpot models (Ibrahim, 2005). Although such approaches can incorporate nonlinear forces (Bauer, 1966; Farid and Gendelman, 2018) and multiple modes (Rubio Hervas and Reyhanoglu, 2012), in many cases these are simple linear harmonic oscillators (Deng and Tait, 2008; Li, Di, and Gong, 2012; Noorian, Haddadpour, and Ebrahimian, 2016), with the parameters tuned to fit a specific sloshing problem. While these are extremely easy to utilize and incorporate into multi-system simulations, they often fail for more complex forcing (Li and Gou, 2018) and with near-resonant sloshing (Ibrahim, 2005). In general, there seems to be a mismatch between academic models and applications.

In the thesis, I aim to investigate the nonlinear sloshing resonance experimentally and apply different model approaches for its description. By selecting a narrow rectangular container with a quasi two-dimensional wave, the entire sloshing motion can be captured and evaluated experimentally. The main control parameters of this investigation are the forcing frequency and amplitude, with a focus on the resonance of the first mode of the system, which produces the largest sloshing forces (Dodge, 2000). The resonance will be evaluated using both complex fluid mechanics models which aim to describe the full liquid surface motion (Faltinsen et al., 2000) and simplified mechanical models with only a single degree of freedom (Graham and Rodriguez, 1952).

The following research questions will be examined:

- RQ1** How does a nonlinear sloshing resonance emerge? Specifically, how does the system change from an quasi-linear response at small amplitudes, towards a nonlinear one with shifted resonance frequencies, coexisting states, hysteresis, and interacting modes?
- RQ2** Which observables of the flow are best suited to describe the dynamics of nonlinear sloshing?
- RQ3** How accurate can different model approaches describe the sloshing resonance? Using different model approaches, from the sloshing literature, mechanical models, and nonlinear dynamical systems theory, which aspects of the resonance can be captured? What are the limitations of existing resonance models?

1.5 Structure of the thesis

The main objectives of this thesis – studying resonance and turbulence in oscillating and pulsatile flows – along with the two flow systems under investigation have been introduced. The following chapters are organized by flow system, with each self-contained chapter presenting the corresponding theoretical background, specific research questions, methods, and results.

The thesis is organized into the following chapters:

- Chapter 2 covers the investigation of a pulsatile particle-laden flow. The chapter first introduces the theoretical background of steady pipe flow, pulsatile pipe flow, and particle-laden pipe flow and their turbulence transition. From this, the detailed research questions and hypothesized dynamics of pipe flow with both a physiological waveform pulsation and a particle perturbation to the flow. A specific focus of this chapter of my thesis is the conceptualization and realization of a newly constructed pipe flow experiment and the implemented measurement techniques for studying particle-fluid interactions and pulsatile flow. After these have been introduced, the results of the turbulence transition first in steady pipe flow and afterwards in pulsatile pipe flow are presented. This includes comparisons of a physiological to a harmonic pulsation, as well as comparison between a particle perturbation and geometric perturbation to the flow. The chapter concludes with a summary of the main results.
- Chapter 3 focuses on the study of nonlinear sloshing resonance in a quasi two-dimensional rectangular container under harmonic forcing. First, a description of sloshing in potential theory with the resulting eigenmodes are introduced. Afterwards the nonlinear effects of large amplitude sloshing and the models used to describe the sloshing motion, as the multimodal model by Faltinsen et al. (2000) and equivalent mechanical linear and nonlinear models, and the effects dissipation in sloshing waves are presented. Based on this theoretical framework the research questions of nonlinear

resonances are formulated in detail. Afterwards, the experiment setup, measurement techniques and evaluation methods used in this chapter are introduced. The main results contain first a description of the emerging nonlinear dynamics of large amplitude sloshing, and afterwards a comparison and evaluation of different resonance models. The chapter concludes with a summary of the main findings.

- Chapter 4 discusses the results of both the turbulence transition in pulsatile flow and the nonlinear resonance with oscillating flow in the larger context of the main research objectives from section 1.2. Afterwards, an outlook for future studies is constructed based on the findings from the research.
- Appendix A contains auxiliary material for the pipe flow chapter.
- Appendix B contains auxiliary material for the sloshing chapter.

Chapter 2

Turbulence transition in particle-laden pulsatile pipe flow

In this chapter, the transition to turbulent flow in a pulsatile particle-laden pipe flow is investigated. This research is motivated by understanding the fundamental dynamics of cardiovascular flow, where turbulence in large vessels can cause cardiovascular disease. In the blood flow system, large particles, such as emboli, which are represented by neutrally buoyant spherical particles, have inertial interactions with the fluid flow. How these interactions are affected by periodic pulsation and influence the turbulence transition remains largely unexplored.

The contents of this chapter are structured as follows:

- Section 2.1: Review of the fluid dynamics of steady, pulsatile, and particle-laden pipe flow.
- Section 2.2: Main research questions.
- Section 2.3: Methods for evaluating particle-laden pipe flows. To measure the turbulence transition in a pulsatile particle-laden flow, I designed and built a new pipe experiment specifically for the thesis. The design process of this experiment and the development of the measurement methods are also included here.
- Section 2.4: Results of the turbulence transition, starting with the mechanisms in a steady particle-laden pipe flow before introducing the pulsation with a complex physiological waveform. The particle-laden transition scenario is also compared to a geometric flow perturbation.
- Section 2.5: Summary.

2.1 Theoretical background

Understanding the transitions to turbulence in pulsatile particle-laden pipe flow requires a foundation of fundamental mechanisms of the flow system. This section outlines the theoretical background and prior research relevant to the presented study. The overview begins with the dynamics of steady pipe flow, in which the mechanisms of turbulence transition are most well understood (section 2.1.1). Afterwards, the background is extended to pulsatile pipe flows, where a periodic variation of flow velocity introduces additional phase specific and frequency dependent effects (section 2.1.2). Lastly, the influence of suspended particles in a pipe flow is considered, including the dynamics of particle migrations and the turbulence transition of particle-laden flow (section 2.1.3).

2.1.1 Steady pipe flow

The dynamics of steady pipe flow of a Newtonian fluid are dominated by the Reynolds number $Re = U \cdot D/\nu$, where U is the mean flow velocity, $D = 2R$ the inner diameter of a circular pipe and $\nu = \mu/\rho$ the kinematic viscosity of the fluid. For fully developed laminar pipe flow under a constant pressure drop Δp over a pipe segment L , the flow follows the Hagen-Poiseuille law,

$$\Delta p = \frac{8\mu UL}{R^2} = \lambda \frac{L}{D} \frac{\rho}{2} U^2. \quad (2.1)$$

In the form normalized by the dynamic pressure, the pressure drop scales with the friction factor $\lambda = 64/Re$. Inside the fully developed pipe flow, the flow velocity has the parabolic shape of the laminar Poiseuille flow profile,

$$u_z(r) = 2U \frac{R^2 - r^2}{R^2}, \quad (2.2)$$

as a function of the radial position r , depicted in Fig. 2.1. Before the flow reaches this developed state, it undergoes an entrance region where the velocity profile evolves. This development occurs as the viscous boundary layer grows radially inwards from the pipe wall. The characteristic length for the laminar flow to fully develop is known as the entrance length L_E , which increases for higher Reynolds numbers $L_E/D \propto Re$ (Durst et al., 2005).

One of the major research questions since the seminal study of Osborne Reynolds (1883) is how pipe flow transitions to turbulence. The laminar Hagen-Poiseuille solution is linearly stable to infinitesimal perturbations for Reynolds numbers up to at least $Re = 10^7$ (Mesequer and Trefethen, 2003). Therefore, the laminar flow becomes turbulent only under a sufficiently strong finite amplitude perturbation. The perturbation threshold to develop turbulence decreases with Re (Avila, Barkley, and Hof, 2023). As a result, each pipe flow experiment will naturally transition for different Re , as small disturbances in the setup will perturb the flow. Especially disturbances from the inlet geometry, where the laminar flow profile is not yet fully developed, will strongly affect the natural transition (Wignanski

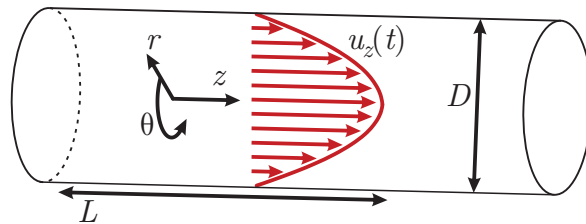


FIGURE 2.1: Laminar Poiseuille profile inside the pipe.

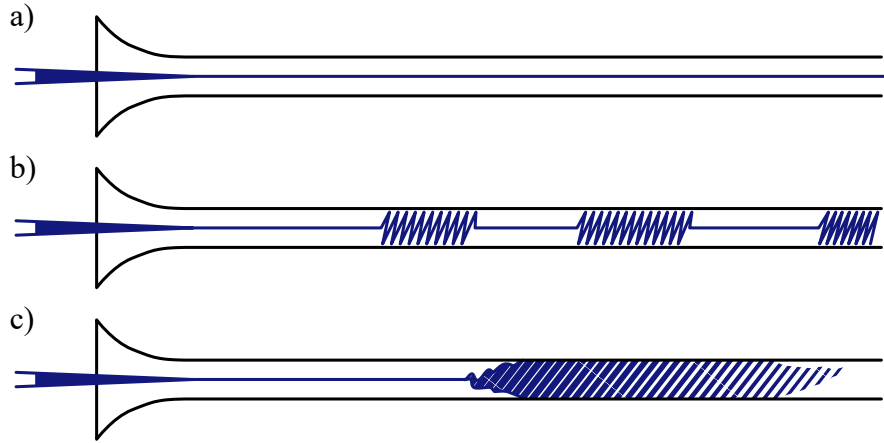


FIGURE 2.2: Flow visualizations of the onset of turbulence using an ink injection into the center of the flow. a) In laminar flow the ink line remains undisturbed. b) As localized turbulent puffs emerge the ink mixes with the surround flow intermittent with laminar regions. c) When the flow turns fully turbulent the ink gets continuously mixed into the water. The turbulence does not materialize directly at the entrance of the pipe. Figure reproduced from Figs. 3, 4 and 16 by Reynolds (1883), under public domain.

and Champagne, 1973). On the other hand, specifically tuned pipe setups can be held laminar up to very large Re (e.g. using boundary layer suction (Pfenninger, 1961)). This susceptibility to perturbation amplitude was already reported by Reynolds (1883), where reducing the disturbance at the pipe inlet would shift the turbulence onset. As a result, Reynolds proposed the critical point as the minimum Re where turbulence caused by a defined perturbation could be sustained indefinitely in the pipe, without decaying over time. To determine this threshold, one needs to look at the dynamics of transitional turbulence in pipe flow.

In the low Re regime, where turbulence can first appear, it takes the shape of localized patches of eddies – so called puffs – in the otherwise laminar flow (see Fig. 2.2). These puffs have a characteristic arrowhead-like appearance with a length of $20 - 30D$ (Eckhardt et al., 2007; Wygnanski and Champagne, 1973) (as depicted in Fig. 2.3a) and move along the pipe with approximately the mean flow velocity U (Hof et al., 2005). The temporal evolution of turbulence is generally described in advective time units D/U , as the time it takes the bulk flow to pass one pipe diameter in the streamwise direction. The shape of the puffs has been studied in detail both experimentally (Van Doorne and Westerweel, 2009; Wygnanski and Champagne, 1973) and numerically (Yakhot et al., 2019), and is independent of the perturbation type in the pipe (De Lozar and Hof, 2009). At the upstream front of the puff, the velocity profile is flattened, with a sharp velocity drop, while the downstream velocity gradually recovers back to Poiseuille profile (Van Doorne and Westerweel, 2009). Puffs require energy input from the uniform parabolic profile upstream to be sustained (Van Doorne and Westerweel, 2009). Therefore, the flattened profile downstream of the puff creates a laminar region ($\sim 20D$ in length) where no other puff can survive. The resulting state of laminar flow with intermittent turbulent patches is characteristic of the low Re transitional regime. The turbulence intensity is often described by the fraction of turbulence to laminar flow or the intermittency factor in the pipe (Rotta, 1956). Turbulent puffs are by nature transient and after sufficient time will always collapse, resulting in a decrease of the turbulence fraction. This decay of the puffs is a memoryless, stochastic process, characterized by the lifetime (Hof et al., 2008). The mean lifetime of a puff was found to be a superexponential function of the Reynolds number ($\exp(\exp(Re))$) (Avila, Willis, and Hof, 2010; Hof et al., 2008), resulting in increasingly long time scales close

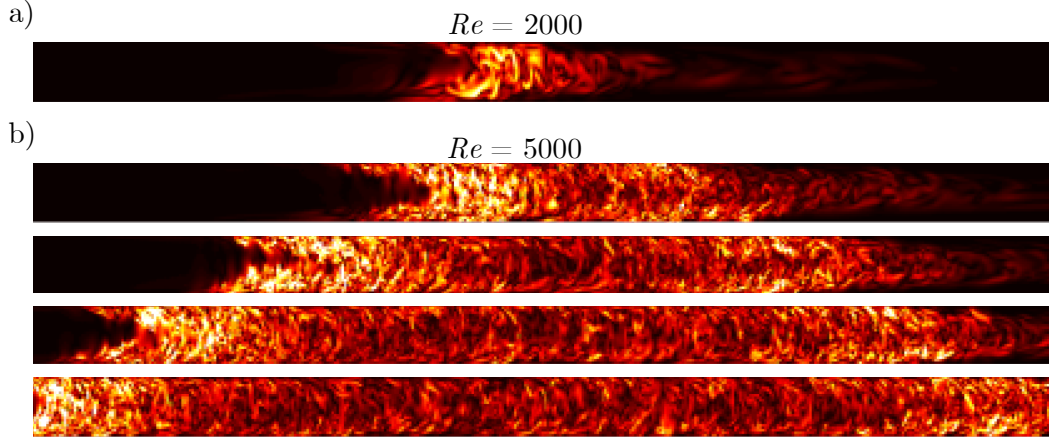


FIGURE 2.3: Snapshots from DNS of a turbulent puff at $Re = 2000$ in a) and the temporal evolution of slug at $Re = 5000$ in b) in a co-moving frame. The length scale in the vertical (radial) direction is stretched by a factor of 2 for better visualization. Dark areas correspond to small fluctuations and bright areas correspond to large fluctuations of the radial and azimuthal flow velocities. The panels in b) are separated in time by $100D/U$. Figure adapted from Fig. 1 by Song et al. (2017), with permission by the publisher. Copyright 2017 Cambridge University Press.

to critical Re , as depicted in the left half of Fig. 2.4. In contrast to its decay, a puff can also split (Wynanski, Sokolov, and Friedman, 1975), creating a new puff downstream and increasing the turbulence fraction in the pipe. This is again a memoryless process where the mean time until a puffs undergoes the first split scales with $\exp(\exp(-Re))$ (Avila et al., 2011), as shown in the right half of Fig. 2.4. The splitting process is the key mechanism for sustaining turbulence in the transitional regime. By careful measurements of the splitting time in a long pipe experiment, Avila et al. (2011) were able to determine an equilibrium time scale of the puffs decaying and spreading at the critical Reynolds number of steady pipe flow at $Re_c = 2040$ (intersection point of the two curves in Fig. 2.4). My work will show that this critical Re is also relevant to describe transition in pulsatile flows.

At higher Reynolds numbers ($Re \gtrsim 2700$) the turbulent structures become nonlocalized and continuously expand over time – these structures are called slugs (Mullin, 2011; Wynanski and Champagne, 1973). There are two different types of slugs: weak and strong (Barkley et al., 2015). At higher Re slugs are considered strong, where turbulent kinetic energy is generated at both the upstream and downstream front of the slug (Song et al., 2017). An example of the structure and growth of a strong turbulent slug over time is depicted in Fig. 2.3b. As the downstream (upstream) front is faster (slower) than the mean flow velocity, it continues feeding kinetic energy into the growth of the slug. In a weak slug at lower Re only the upstream front generates turbulence, while the downstream front dissipates turbulent kinetic energy. The growth rate of slugs monotonically increases with Re (Barkley et al., 2015). Slugs will eventually fill out the entire pipe leading to fully turbulent flow.

Since the early studies by Reynolds (1883) a prominent method for measuring the onset of turbulence is via the differential pressure drop in the pipe. This method can also be used to count the lifetime statistics of individual puff (Avila et al., 2011; De Lozar and Hof, 2009). The key characteristic extracted from these measurements is the flow's friction factor as,

$$\lambda = \Delta p \cdot \left(\frac{\rho}{2} \frac{L_m}{D} U^2 \right)^{-1}. \quad (2.3)$$

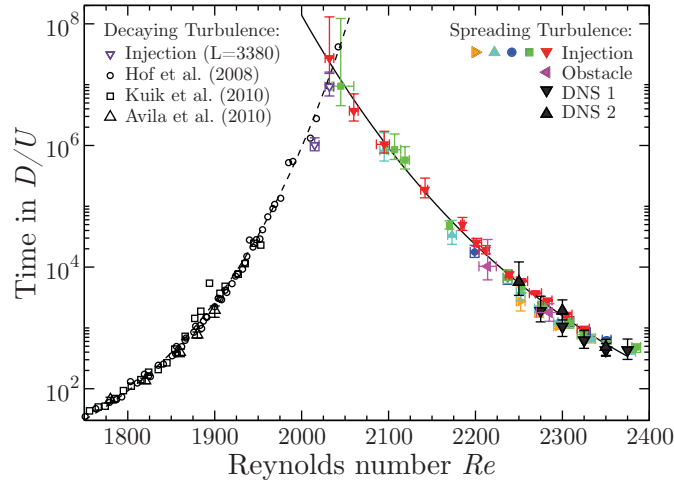


FIGURE 2.4: Mean lifetime of a turbulent puff before decaying (empty symbols) or splitting (full symbols) as a function of Re . The dashed and solid lines represent the superexponential functions of Re fitted through data points of puff decay and splitting lifetime respectively. The curves intersect as the critical Reynolds number of $Re_c = 2040$. The lifetimes include experiments with different perturbation mechanisms and numerical simulations by Avila, Willis, and Hof (2010), Hof et al. (2008), and Kuik, Poelma, and Westerweel (2010). The colors correspond to different axial distances between the jet injections and puff measurements. Figure reproduced from Fig. 5 by Avila et al. (2011), with permission by the publisher.

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In a laminar flow the pressure will follow the Hagen-Poiseuille law Eq. (2.1) with $\lambda = 64/Re$. As turbulence emerges, the pressure drop and friction factor rise. If the flow is fully turbulent, it will approach the Blasius scaling law with $\lambda = 0.316 Re^{-1/4}$. An example of the friction factor as a function of Re is shown in Fig. 2.5 in a so called Moody chart, which typically contains the dependence of the surface roughness (Moody, 1944). As the turbulent fraction in the pipe grows, the measurements continuously approach the Blasius scaling. It can be seen that different types of perturbations will have varying effectiveness in triggering turbulence. Early studies often used fixed geometric obstacles as perturbations like needles or orifices in the pipe (Reynolds, 1883; Rotta, 1956) (marked as black stars and red squares in Fig. 2.5). Although these will continuously perturb the flow, they also disrupt development of the Poiseuille profile, which is necessary for puffs to form (Kühnen et al., 2018). Additionally, if the spacing of the triggered puffs is too tight, they will cancel each other out. The most effective perturbations, for triggering turbulence in a controlled way, are pulsed injections of a small jet into the flow (Samanta, De Lozar, and Hof, 2011) with an optimal frequency, shown as green triangles in Fig. 2.5. This method can be used to trigger puff for Reynolds numbers as low as $Re \approx 1500$ (Hof et al., 2005). The jet creates a pair of counter-rotating vortices, which directly coincides with the linear optimal nonmodal disturbance of laminar pipe flow (Avila, Barkley, and Hof, 2023; Kühnen et al., 2018). These vortices mix the low velocity fluid from the wall with high velocity fluid near the center, creating a so-called lift-up effect Brandt (2014).

A similar effect of secondary vortices inside the flow can be seen if the geometry of the pipe has a bend or helical shape. A geometric change in the pipe, such as a bend, redirects the flow. Depending on the radius of the bend, centrifugal forces push the higher inertia flow from the centerline towards the outer pipe wall. This results in an asymmetric flow profile and the formation of a secondary flow with a pair of symmetric counter-rotating vortices in the pipe's cross-section, known as Dean-vortices. An analytical solution for this flow was derived by Dean (1927), and the effects of this secondary flow can be scaled with

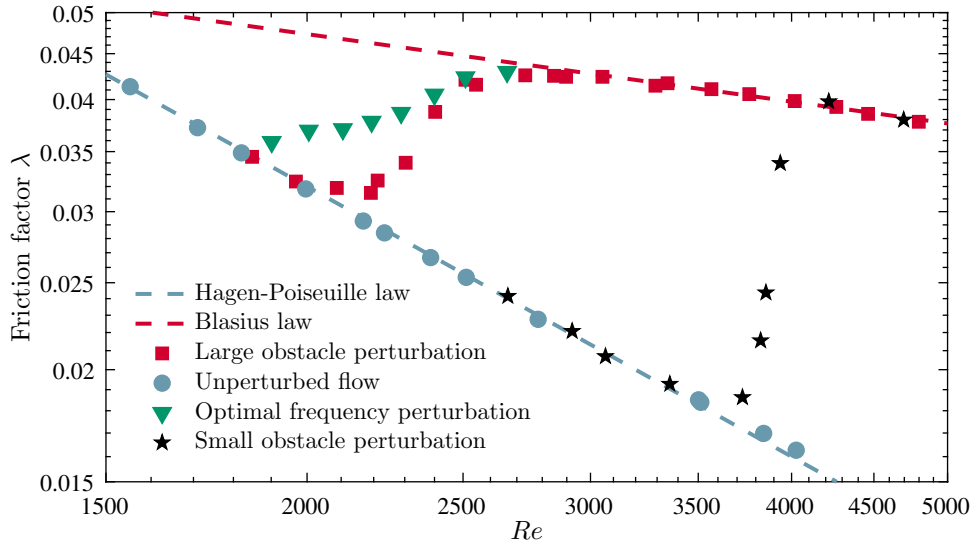


FIGURE 2.5: Friction factor as a function of Re with different types of perturbations to the flow. Figure reproduced from Fig. 5 by Samanta, De Lozar, and Hof (2011), with permission by the publisher. Copyright 2011 Cambridge University Press.

the Dean-number $De = Re\sqrt{D/D_{coil}}$, as a function of the flow's inertia, viscous forces and the diameter of the bend or coil D_{coil} . For a small curvatures $D/D_{coil} < 0.28$ the flow shows a subcritical puff-transition to turbulence qualitatively similar to a straight pipe (Kühnen et al., 2015), with increasing curvature delaying the transition (Sreenivasan and Strykowski, 1983). At larger curvature ratios, the Dean flow becomes linearly unstable, resulting in a supercritical transition (Cioncolini and Santini, 2006; Kühnen et al., 2015).

So far, this description of the turbulence transition in pipe flow is limited to stationary flows where the mass flow does not change over time. If the driving of flow changes over time, additional aspects need to be considered.

2.1.2 Pulsatile pipe flow

In a pulsatile flow, the bulk velocity oscillates around a nonzero mean value, as illustrated in Fig. 2.6. The key parameters to characterize a pulsation are the mean Reynolds number Re_m (based on the mean bulk velocity U , averaged over the pipe cross-section and pulsation period), the dimensionless amplitude $A = \hat{u}/U - 1$ (as the ratio of peak velocity $\hat{u}$ to the mean bulk velocity U) and the Womersley number $Wo = 0.5D\sqrt{\omega/\nu}$, as a dimensionless frequency, where $\omega = 2\pi f$ is the angular frequency of the pulsation (with period $T = 1/f$). In the most simple case of a single harmonic pulsation the bulk velocity (averaged over the cross-section) is expressed as a function of time t ,

$$u(t) = U + \hat{u} \cos(\omega t). \quad (2.4)$$

For many systems (like cardiovascular flow), the waveform of the pulsation will often be more elaborate. Here, the pulsation of the flow velocity can be expressed as Fourier series,

$$u(t) = U \left[1 + \sum_{n=1}^N (a_n \cos(\omega n t) + b_n \sin(\omega n t)) \right], \quad (2.5)$$

where a_n and b_n are the Fourier coefficients for each harmonic mode n . Fig. 2.6 shows an example waveform for the flow in the human aorta, with a strong acceleration during the

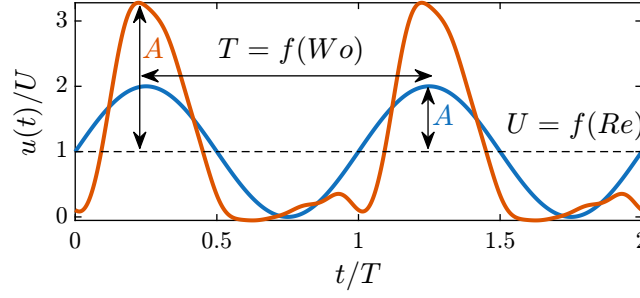


FIGURE 2.6: Velocity time series of harmonic pulsatile flow (orange, Eq. (2.4)) and the waveform of an aorta flow (blue, Eq. (2.5) with Fourier coefficients from Table 2.2).

systole phase and close to zero velocity in the diastole. The general expression of the flow velocity profile driven by a pulsatile pressure reads,

$$u(r, t) = \sum_{n=1}^N \frac{i}{\omega n \rho} \frac{\partial \hat{p}_n}{\partial x} \left[1 - \frac{J_0(i^{3/2} Wo \frac{r}{R})}{J_0(i^{3/2} Wo)} \right] e^{in\omega t}, \quad (2.6)$$

where $\partial \hat{p}_n / \partial x$ is the amplitude of the pressure gradient for each mode and J_0 is the Bessel function of the first kind and order zero. This analytical solution of the Navier-Stokes equations was derived by both Womersley (1955) and Sexl (1930) and is often referred to as Sexl-Womersley flow. Fig. 2.7 depicts the velocity profiles for different Wo , here with only a single harmonic $N = 1$. At low Womersley numbers (small frequencies) the pulsatile flow behaves like a steady flow with a parabolic profile whose magnitude changes over time (first panel, $Wo = 1$). As the pulsatile inertia increases, the Sexl-Womersley flow has two important characteristics. Firstly, the flow profile flattens in the center and there are phase specific flow reversal regions near the wall (third panel, $Wo = 12$). Secondly, the flow velocity develops a phase lag to the driving pressure gradient, approaching $\pi/2$ for $Wo \rightarrow \infty$.

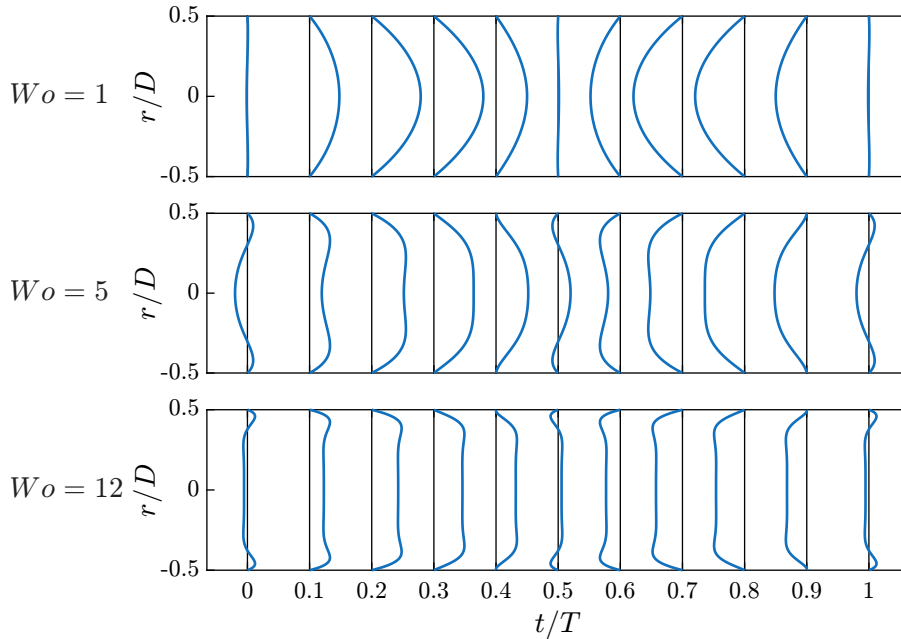


FIGURE 2.7: Pulsatile flow profiles over one pulsation period for different $Wo = 1, 5, 12$ with zero mean velocity.

The transition to turbulence in pulsatile flow is effected by all parameters of the flow: Re_m , Wo , A , and the waveform (Morón, Feldmann, and Avila, 2022; Xu and Avila, 2018; Xu et al., 2020). For sufficiently high Re the pulsatile flow will become linearly unstable (Thomas et al., 2011), but a subcritical turbulence transition will be triggered at lower Re by finite amplitude perturbations. In the regime of small amplitudes ($A \lesssim 0.4$) the transition is overall similar to steady pipe flow (Xu and Avila, 2018; Xu et al., 2017), consisting of localized intermittent puffs or growing slugs.

Different trends for the turbulence transition are reported in the literature. Studies in laboratory experiments by Peacock et al. (1998) or *in vivo* studies by Falsetti et al. (1983) and Nerem and Seed (1972) found the critical Re_c to monotonically increase with Wo and A . In contrast, studies by (Sarpkaya, 1966; Stettler and Hussain, 1986; Trip et al., 2012) showed an asymptotic approach to Re_c with increasing Wo , with all three studies naming different asymptotic limits. This was eventually explained by Xu et al. (2017), who linked the variance of transition thresholds with the experimental methods used in the different studies. While some experiments employ external perturbation to trigger turbulence in the flow (Sarpkaya, 1966; Stettler and Hussain, 1986; Trip et al., 2012), others increased Wo and A until turbulence emerges by itself (Peacock et al., 1998). For the second approach, the observed result will be specific to the intrinsic noise level and transition threshold of the experiment, instead of the underlying dynamics of the flow. Xu et al. (2017) found that for small $Wo < 4$ the laminar flow is stabilized (Stettler and Hussain, 1986; Xu et al., 2017) delaying the transition to turbulence (compared to steady flow with $Re_c = 2040$). Here, turbulence only emerge as phase-locked patches (Stettler and Hussain, 1986). These patches can appear as growing slugs, but will naturally decay and not be sustained through an entire pulsation period (Xu and Avila, 2018). In a sufficiently long pipe the critical Reynolds approaches $Re_c = 2040/(1 - A)$ with $Wo \rightarrow 0$ (Xu and Avila, 2018). With increasing Wo the transition threshold will continuously approach the steady pipe flow limit. Also, at very high $Wo > 17$ the dynamics of steady pipe flow are recovered (Sarpkaya, 1966) and the effect of the pulsation becomes negligible, as the timescale of the pulsation becomes too fast compared to the advective timescale of the flow and the turbulence patches are unaffected. The transition threshold $Re_c = 2040$ and puff lifetime characteristics matches steady flow (Xu et al., 2017).

A different picture emerges for higher pulsation amplitudes. While the puff transition is continuously more delayed, a different type of linear instability for larger $A \geq 0.5$ was recently observed by Xu et al. (2020). A helical vortex structure appeared periodically during each deceleration phase of the pulsation and collapsed into a turbulent patch. After expanding in the streamwise direction, the turbulent patch then decays in the following acceleration phase. Fig. 2.8 from Xu et al. (2020) shows the transition threshold and flow structure of the helical wave (green) compared to localized puffs (red). In the experiments this helical instability could be triggered most efficiently with geometric perturbations such as constrictions or a bending of the pipe wall and could trigger turbulence for very low $Re_m \approx 800$ (as they for example appear in cardiovascular flow). By comparison, a perturbation by a jet injection could only trigger localized turbulent puffs, with a delayed puff onset at higher Re_m for larger amplitudes. Xu, Song, and Avila (2021) were able to link this helical turbulence growth to a linear instability of the flow profile during deceleration. The growth of the helical perturbation requires the velocity profiles to have inflection points, as a result, the strongest growth appears for $Wo \approx 7$ (Xu, Song, and Avila, 2021). At lower Wo the profile becomes parabolic, making it more stable. While the helical perturbation is able to trigger phase locked turbulence during the deceleration of the flow, it cannot maintain its growth over a longer time (Feldmann, Morón, and Avila, 2021). Using transient growth analysis Xu, Song, and Avila (2021) compared the helical perturbation with the classical disturbance of a pair of streamwise vortices. They found

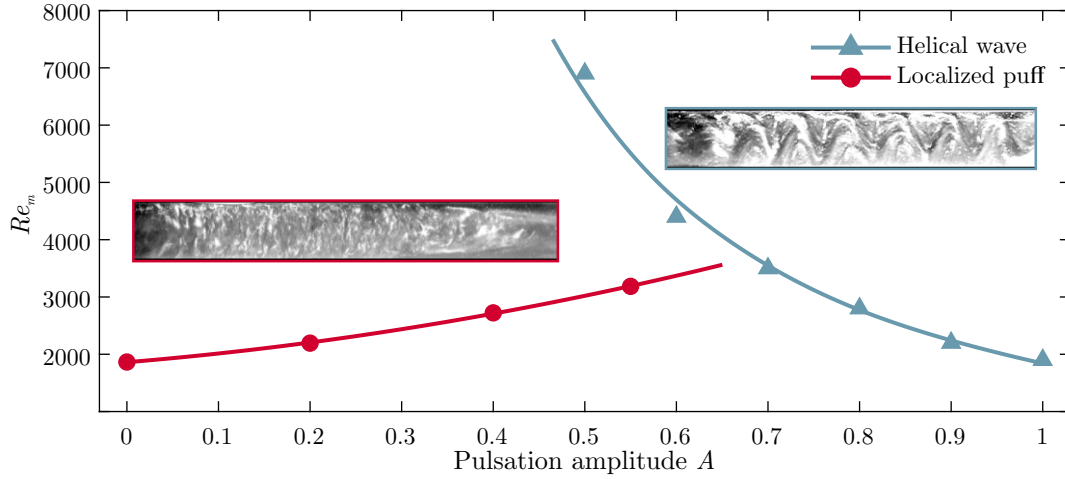


FIGURE 2.8: Threshold for the onset of turbulence as a function of A for $Wo = 5.6$. The red line represents the onset via localized puffs and the blue line the onset of the helical instability. The inset shows flow visualization images of both flow structures. The length scale in the vertical (radial) direction is stretched by a factor of 2 for better visualization. Figure reproduced from Fig. 2 in Xu et al. (2020), with permission by the publisher. Copyright 2020 National Academy of Sciences.

that for intermediate Wo the helical perturbation will exhibit the highest energy growth for a large amplitude pulsatile flow, while streamwise vortices grow strongest at very low $Wo < 4$ or very high $Wo > 20$.

In most studies only harmonic pulsations are considered, while the waveform in applications such as cardiovascular flow can be very different. Experiments by Brindise and Vlachos (2018) showed that a longer deceleration phase in the waveform will result in an earlier turbulence transition, while steep accelerations delay the transition. In a recent study by Morón, Feldmann, and Avila (2022) the effect of the pulsation waveform was systematically analyzed using transient growth analysis and numerical simulations. They determined that for the helical instability to grow, it requires instantaneously unstable inflection points in the profile (fulfilling the Fjørtoft criterion) and the instability needs to grow faster than the profile evolves. A waveform with a longer low velocity phase and a more abrupt acceleration and deceleration phase will have inflection points for a longer part of the period and exhibit stronger growth of the helical perturbation (Morón, Feldmann, and Avila, 2022). Specifically, a waveform modeled after a physiological pulsation should be very susceptible to helical perturbations, with the strongest growth for $Wo \approx 7 - 8$, as predicted by linear stability analysis of the profile shown in Fig. 2.9. The turbulence onset with a physiological waveform has so far not been systematically studied with different perturbation mechanisms.

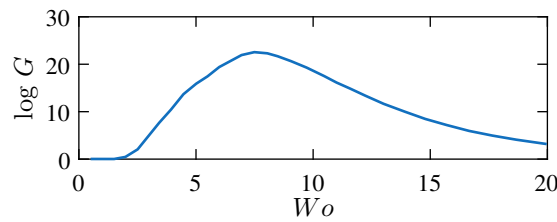


FIGURE 2.9: Energy growth G of a helical perturbation as a function of Wo from linear stability analysis with a physiological waveform. Figure reproduced from Fig. 9c in Morón, Feldmann, and Avila (2022), under Creative Commons CC BY.

Having introduced the fundamental behavior of steady and pulsatile pipe flows in single-phase Newtonian fluids, the next section will elucidate the case of particle-laden flows, where suspended solid particles interact with the carrier fluid, modifying its structure, stability, and turbulence characteristics.

2.1.3 Particle-laden pipe flow

The interaction between particles and a flow depends largely on the Stokes number $St = t_p/t_f$, which compares the characteristic time scale of a particle response t_p , to the time scale of a fluid velocity change t_f . For $St \ll 1$ the particles can be regarded as tracers and will follow the fluid motion. Larger particle sizes or strong density differences between the fluid and particles result in larger Stokes numbers, where particle and fluid interact (see Fig. 1.5) (Brandt and Coletti, 2022). If the density of the fluid is matched to the particles, buoyancy forces can be neglected. In these flows, the particle dynamics will be dictated by their inertial forces. For a hard and spherical particle, the effects of its orientation in the flow (Masaeli et al., 2012; Nizkaya et al., 2020) or deformation (Alghalibi, Rosti, and Brandt, 2019) can be neglected. In this case, the two additional control parameters of the flow are: the particle diameter D_p and volume fraction of particles inside the fluid $\phi_p = V_p/(V_p + V_f)$. The particle diameter will dictate the inertia and give a scale for interactions with the fluid. The particle concentration determines the amount of particle-particle interactions. At larger concentration, the dynamics will be dominated by more frequent particle collisions. Furthermore, the effective viscosity of the fluid suspension can increase. For lower concentrations $\phi_p < 20\%$ the effect on viscosity is assumed to be negligible (Mueller, Llewellyn, and Mader, 2010).

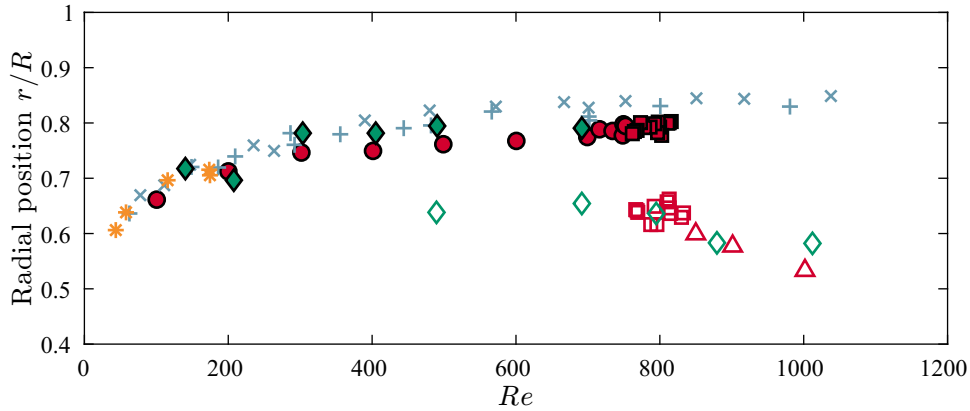


FIGURE 2.10: Radii of the particle focusing annuli as a function of Re . Red markers ($D_p/D = 0.1$; $L/D = 1000$) represent the three regimes of Segre-Silberberg focusing (circles), focusing on the inner annulus (triangle) and focusing on both annuli (square). Open symbols correspond to the inner annulus. Further experimental data by Matas, Morris, and Guazzelli (2004) (blue crosses and plus signs, $D_p/D = [0.095; 0.11]$; $L/D = 313$), Morita, Itano, and Sugihara-Seki (2017) (green diamonds, $D_p/D = 0.11$; $L/D = 250$) and Segré and Silberberg (1962) (orange stars, $D_p/D = 0.11$; $L/D = 214$). Figure reproduced from Fig. 7 in Nakayama et al. (2019), with permission by the publisher. Copyright 2019 Cambridge University Press.

The main mechanism of spherical neutrally buoyant particles in a steady laminar pipe flow, is their migration towards a stable radial position, discovered by Segré and Silberberg (1961). The slip velocity between the particle and fluid in the shear gradient of the flow lifts the particles towards the wall (Saffman, 1965), while the particles influence on the flow close to the wall creates a repelling force (Zhang et al., 2016). The so called Segre-Silberberg annulus is an equilibrium position between both effects. In inertial microfluidics, the

migration mechanism can be utilized to sort particles (Zhang et al., 2016). Fig. 2.10 depicts the stable radial position as a function of Re with measurements from multiple studies ($D_p/D \approx 0.1$). The radial position of the Segre-Silberberg annulus (full symbols and cross symbols) shifts towards the wall for higher Re . Experimental studies by Matas, Morris, and Guazzelli (2004) observed a second stable inner annulus emerging for higher $Re > 600$ (empty symbols). With increasing Re a higher concentration of particles would accumulate on this inner annulus. The same effect was shown in numerical simulations by Shao, Yu, and Sun (2008). A recent study by Morita, Itano, and Sugihara-Seki (2017) suggests that very long transients ($L > 500D$) are necessary until the equilibrium position is reached. Nakayama et al. (2019) report the Segre-Silberberg to be no longer stable for larger Re , with an intermediate regime where both the Segre-Silberberg and the inner annulus coexist. Pan, Li, and Glowinski (2021) suggest a turning-point bifurcation with a third unstable annulus in between the two. The radius of the equilibrium annuli depends on both Re of the flow and the relative particle diameter D_p/D (Matas, Morris, and Guazzelli, 2004). Simulations by Aouane et al. (2022) also suggest a change of the stable annulus for higher particle volume fractions ϕ_p , caused by more hydrodynamic interactions. In addition to the radial focusing, inertial particles also tend to self-organize into evenly spaced *particle trains* in the axial direction, as observed by Matas et al. (2004).

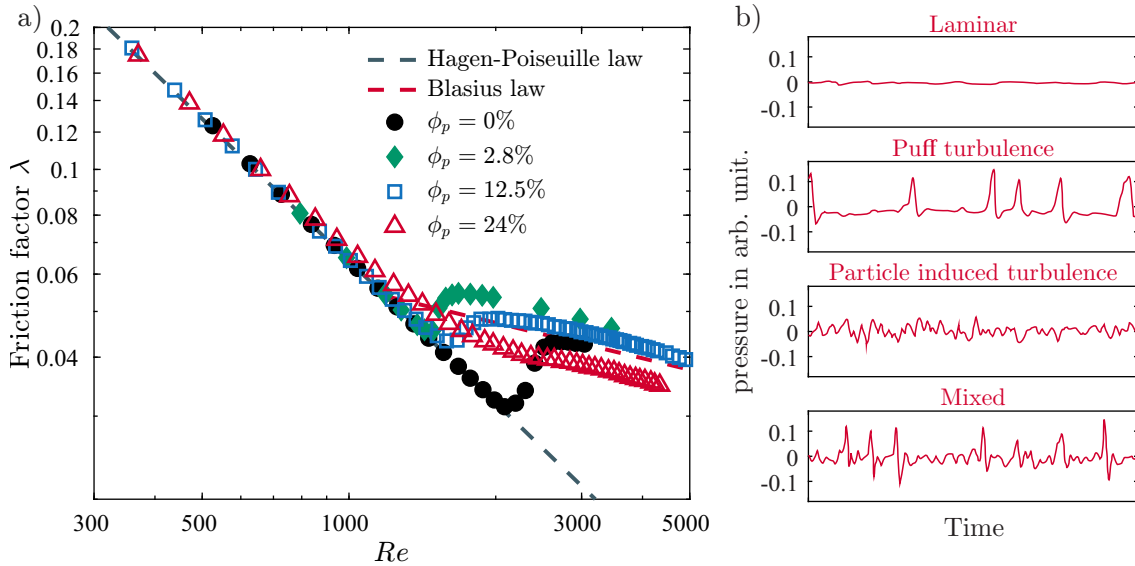


FIGURE 2.11: a) Friction factor λ as function of the suspensions Re , for different particle concentrations ϕ_p . Experiments were performed with a continuous perturbation at the entrance. b) Time series of the pressure fluctuations (top to bottom) in laminar flow, intermittent turbulent puffs, particle induced turbulence, mixed state of particle induced turbulence and puffs. Figure reproduced from Figs. 1 and 3b by Agrawal, Choueiri, and Hof (2019), with permission by the publisher. Copyright 2019 American Physical Society.

As the Reynolds number increases, the inertial effects of particles also influence the transition to turbulence. For a small concentration of inertial particles, the Reynolds number for puffs to first appear is reduced from $Re \approx 1500$ in single-phase flow to as low as $Re \approx 1000$, depending on the particle concentration and size (Agrawal, Choueiri, and Hof, 2019; Hogendoorn and Poelma, 2018; Matas, Morris, and Guazzelli, 2003). At intermediate concentrations, a different type of turbulence is triggered, with the flow exhibiting global fluctuations instead of localized intermittency (Agrawal, Choueiri, and Hof, 2019). The actual puff transition appears delayed in this regime. For even larger concentrations, only the fluctuating turbulence state could be observed. The different transition regimes can

be distinguished in the friction factor curves displayed in Fig. 2.11a, with a continuously more smooth divergence from the laminar Hagen-Poiseuille law at higher concentrations, compared to the sharp transition of a single-phase flow (see Fig. 2.5). A qualitative picture of the different regimes is given in Fig. 2.11b. The boundaries of these transitions are also highly susceptible to the diameter of the particles (Hogendoorn, Chandra, and Poelma, 2022; Leskovec, Lundell, and Innings, 2020), with larger particles further reducing the Re of the turbulence onset. Hogendoorn, Chandra, and Poelma (2022) proposed the perturbation of particles to scale proportional to $(D_p/D)^{1/2}\phi_p^{1/6}$. Measurements of the fluid flow by Hogendoorn, Chandra, and Poelma (2021) and Leskovec, Lundell, and Innings (2020) are able to quantitatively describe the fluctuating turbulence state with a gradually flattening flow profile and growing velocity fluctuations for larger Re .

So far, there are no fundamental studies combining a particle-laden flow with a pulsatile driving, either to study particle-fluid interaction or using particles as a perturbation to trigger turbulence. This leads to the research questions described in the next section.

2.2 Research questions

Although many mechanisms of particle-fluid interactions and the transition to turbulence were introduced in the previous section, so far there is very little fundamental research on particle-laden pulsatile flow. As the system inhabits both the characteristics of pulsatile and particle-laden flow, there is an immense parameter space to explore. Similarly, the turbulence transition of pulsatile flows, especially with complex physiological pulsation waveforms, and the role of different perturbation mechanisms are not yet fully understood. In this section, the specific research questions of my thesis – which were already briefly introduced in section 1.3 – are formulated in detail.

RQ1 What type of turbulence transition occurs in a particle-laden pulsatile pipe flow, and how does the perturbation of particles compare to other flow perturbation mechanisms? Are particles able to trigger instabilities of pulsatile flow? Is the onset of turbulence enhanced or delayed?

In the existing literature, pulsatile flow can transition to turbulence either by directly triggering turbulent puffs or a breaking up of the helical instability of the Sexl-Womersley flow profile (Xu, Song, and Avila, 2021). In the initial experiments in which this instability was first observed by Xu et al. (2020), helical flow patterns were triggered by small geometric defects in the pipe wall, such as constrictions or small bends, while jet injections could only trigger turbulent puffs. This suggests that the emergence of turbulence in pulsatile flow is susceptible not only to the amplitude of a perturbation but also to the type of perturbation mechanism. This diverges from observations in steady single-phase pipe flow, where turbulence always appears as localized puffs, whose structure is independent from the type of perturbation (De Lozar and Hof, 2009). The influence of the type of perturbation on the onset of turbulence in pulsatile flow is not yet understood.

Similarly, studies of particle-laden steady flow show small concentrations of inertial particles leading to an early puff transition, while higher concentrations trigger a different turbulent state, without intermittent patches (Agrawal, Choueiri, and Hof, 2019; Hogendoorn, Chandra, and Poelma, 2022). Whether these effects are also present for a pulsatile flow, remains to be explored.

I aim to explore whether the perturbation of particles is able to trigger the helical instability, or will instead induce turbulent puffs. Furthermore, it will be investigated whether particles enhance a transition, or delay turbulence. The influence of particle perturbation will be compared to a geometric perturbation mechanism.

RQ2 What is the effect of a physiological pulsation waveform compared to a harmonic pulsation on the transition to turbulence? Does the shorter acceleration and deceleration contribute to turbulence growth, or is the transition inhibited?

Most research of pulsatile flow focuses on a harmonic pulsation waveform, which is not representative of the complex physiological pulsation encountered in large blood vessels. In the study of Morón, Feldmann, and Avila (2022), transient growth analysis predicted that a non-harmonic pulsation waveform characterized by a sharp deceleration phase and a long resting phase, modeled after the flow in the human aorta (see Fig. 2.6) would energize the growth of a helical perturbation. This prediction has not been verified so far with systematic experiments or simulations of pulsatile flow driven by a physiological pulsation.

I aim to investigate the transition with both physiological and harmonic pulsations to explore the effect of the waveform on the onset of turbulence.

RQ3 How is the turbulence transition affected by the frequency of the pulsation, giving perturbations more or less time to be advected and grow in each period?

The investigation of Xu et al. (2020) only varied the pulsation amplitude A while the Womersley number remained fixed at $Wo = 5.6$. The transient growth analysis by Morón, Feldmann, and Avila (2022) and Xu, Song, and Avila (2021) shows that the development of helical perturbations is highly dependent on Wo for both harmonic and more complex pulsation waveforms, with the strongest growth at intermediate $Wo \approx 7 - 8$. In addition to the effect on the flow profile and its stability, the change in Womersley number increases or decreases the advective timescale of a pulsation period, therefore providing a perturbation more or less time to grow and turbulence patches to develop. It remains to be answered which of these effects is dominant for a physiological pulsation.

For my investigation of the particle perturbation mechanism and the different pulsation waveforms, the Womersley number will be varied as the main control parameter.

Having established the three main research objectives of this chapter, the next section will introduce the methods used for the investigation.

2.3 Methods

Some of the content presented in this section lead to publication¹ of B  uerlein and Avila (2025).

The research questions stated in the previous section, with the combination of pulsation dynamics and particle-laden flow, pose unique challenges to the applicable investigation methods.

Direct numerical simulations of particle-laden pulsatile flow would require resolving a wide range of temporal and spatial scales: The integral scale of the pulsatile flow, the Kolmogorov scale of turbulence, and the local boundary layer and interactions around each individual particle. Capturing these scales and interactions for the physiologically relevant Reynolds numbers would be extremely computationally demanding and restrict the range of examinable parameters. Simplified simulations, using reduced models of turbulence or particles, would not be able to resolve the very mechanisms of interest, such as particle induced perturbations, transient instabilities, and nonlinear growth.

Similarly, while stability analysis can resolve the growth of perturbations during a specific set of flow parameters, it is limited to idealized infinitesimal perturbations to the NS-equations. The finite amplitudes of larger inertial particles are inherently nonlinear, and their effect on fluid flows cannot easily be predicted.

These limitations make it indispensable to study the turbulence transition of particle-laden pulsatile flow with experimental methods, where the perturbation mechanism of the particle is accurately represented and its effects can be resolved through flow measurements. Despite this advantage, there are also several technical challenges in experimentally investigating such a flow. The important parameters for any experimental investigation are (Tavoularis and Nedi  , 2005):

1. The *observables* or *measurands*, as the actual physical properties that can be extracted.
2. The *measurement system*, which consists of the measurement technique and components such as sensors, data acquisition, and evaluation software.
3. The *physical system* or the experimental setup itself.

These three aspects are always coupled when performing a fluid experiment. The choice of observables in combination with a measurement system that can capture those observables will reflect the properties of the flow that can be analyzed. For a given flow system, it is important to select relevant observables that can be used to understand its underlying dynamics. The design of the experimental setup will dictate which aspect of the fluid mechanics can be created and captured. In addition, any inaccuracies inherent to the setup or its control parameters will also be reflected as errors in the measurement taken from set experiment.

The design of a pipe flow experiment from scratch for the purpose of studying particle-laden pulsatile flow was a significant challenge of the thesis. One main example being the selection and development of measurement techniques that work in a multiphase flow and can accurately describe both particle and fluid dynamics. Additionally, the pulsation requires precise driving to produce the dynamic change in flow velocity in a repeatable manner. This section provides a comprehensive description of the design and crucial elements of this setup.

For the design process, it is first necessary to review existing methods for investigating pipe flow and particle-laden flow (section 2.3.1), to select suitable techniques for the new experiment. The general overview of the experiment, along with the design and

¹B. B  uerlein carried out the measurements, the data analysis, and wrote the first draft of the manuscript.

capabilities of its driving system, are presented afterwards (sections 2.3.2 and 2.3.3). This includes an assessment of the drive's performance and accuracy to achieve different types of pulsatile flow. The measurement techniques implemented for the new setup include: A Lagrangian Particle Tracking technique which was specifically adapted for the simultaneous measurement of flow velocity and particle motion (section 2.3.4), a novel imaging method enabling three-dimensional localization and tracking particles (section 2.3.5), and a setup for differential pressure drop measurements to evaluate the fluid flow (section 2.3.6).

2.3.1 Measurement techniques for pipe flow and particle-laden flow

The selection of measurement techniques and the resulting measurands that can be captured from the flow are a key aspect of any experimental setup. For a new setup, the specific techniques and how they can be implemented are integral to the design process. In the following section, the common measurement methods of steady single-phase and particle-laden flows and their utilization in pipe flow will be reviewed. One of the most simple approaches to get a qualitative picture of pipe flow dynamics is through flow visualizations. This was already utilized by Reynolds (1883) to distinguish between laminar flow and turbulent patches in the pipe. For a more quantitative picture more sophisticated flow measurements are required. In recent studies coherent flow structures like traveling waves or localized puffs were experimentally observed with stereoscopic Particle Image Velocimetry (PIV) (Hof et al., 2004; Van Doorne and Westerweel, 2009). A cross-section of the pipe flow was radially illuminated by a laser light sheet and two cameras monitored the motion of tracers seeded in the fluid flow. In order to minimize optical distortions from the curved pipe wall a prism was built around the pipe and filled with a refractive index matching liquid (Doorne and Westerweel, 2007; Hof et al., 2004; Xu et al., 2020). The measurement data contains all three velocity components (3C) in a plane (2D), in general also referred to as a 2D3C measurement setup. By assuming that the flow features are stationary during the measurement (Taylor's hypothesis), the time resolved recordings can be extended to illustrate three-dimensional flow patterns (Doorne and Westerweel, 2007).

Optical (3D3C) measurements resolving the spatio-temporal dynamics in a full flow volume were first applied to the PIV algorithm, by using a tomographic reconstruction of the tracers in the flow (Elsinga et al., 2006). In contrast to PIV, which is based on the cross-correlation of a group of tracers (in the interrogation window or sub volume), Lagrangian Particle Tracking (LPT) algorithms identify and tracks each tracer. Early developments of such 3D particle tracking by Maas, Gruen, and Papantoniou (1993) and Malik, Dracos, and Papantoniou (1993) laid the foundation for modern LPT methods. However, compared to PIV, the 3D triangulation generally limits the image density of tracers, especially for less predictable motion in turbulent flows (Malik, Dracos, and Papantoniou, 1993). In recent years the Shake-The-Box algorithm (STB) for LPT made this approach more popular, due to the much lower computational costs and better spatial resolution (Schanz, Gesemann, and Schröder, 2016). In STB the future tracer position is predicted on the basis of the previously measured trajectory and then compared to the tracer image. By solving the the triangulation problem in the image plane only, without extensive 3D reconstructions, this methods yields a high efficiency and works with large numbers of tracers, compared to tomographic PIV and other LPT algorithms. The PIV algorithms result in an Eulerian velocity field and the LPT algorithms in Lagrangian tracer trajectories (with the possibility to extract the Eulerian velocity field from them). Despite these innovative methods, there are few examples of three-dimensional particle imaging used in pipe flow experiments (Bhattacharya and Vlachos, 2020; Walpot, Rosielle, and Geld, 2006), due to the imaging difficulties stemming from the refraction at the curved pipe wall and the high costs of typically at least three cameras and a powerful illumination.

For particle-laden flows there are more aspects of the flow to be investigated. Only observing the formation of particle clusters is straightforward and can easily be done with just the naked eye (Matas et al., 2004). For measuring the radial focusing of a particle, typically a cross-section of the pipe gets illuminated with a radially directed light sheet. From a camera recording, the 2D particle positions can be retrieved and a statistical analysis yields the preferred radial particle positions (Matas, Morris, and Guazzelli, 2004; Morita, Itano, and Sugihara-Seki, 2017; Nakayama et al., 2019). So far the actual velocity of the particles have not been measured in these types of studies.

To elucidate the underlying mechanisms of the particle-fluid interactions in a pipe flow, it is not sufficient to capture only the particle motion, but instead particle and fluid velocities need to be investigated simultaneously. There are few studies to achieve this, as large (and often opaque) suspended particles limit the optical and illumination accessibility for PIV or LPT measurements. Existing studies often use smaller particles ($D_p = 0.008D$ (Oliveira, Van der Geld, and Kuerten, 2015) or $D_p = 0.04 D$ (Kolaas, Drazen, and Jensen, 2015)) at very low volume concentrations ($\phi_p = 1.4 \times 10^{-3}\%$ (Oliveira, Van der Geld, and Kuerten, 2015) or $\phi_p = 0.1\%$ (Kolaas, Drazen, and Jensen, 2015)). One promising alternative is the usage of transparent (e.g. glass beads with $\phi_p = 0.8\%$ (Shokri et al., 2017)) or fully refractive index matched particles (like super-absorbent polymers or hydrogels), which allow optical PIV measurements with concentrations of up to $\phi_p = 18.4\%$ (Shokri et al., 2017). A secondary hurdle is the very limited availability of established multiphase processing tools for PIV and LPT. In general, tailor-made procedures with separate processing for particle and flow data are required, often only working in 2D (Kolaas, Drazen, and Jensen, 2015; Shokri et al., 2017). Only a single study reports a 3D measurement of both particles and fluid flow in a pipe (Oliveira, Van der Geld, and Kuerten, 2015).

In addition to the particle's trajectory, their rotatory motion is an important observable of multiphase flows. Rotations can be calculated using traceable patterns painted on particle surface (Mathai et al., 2016; Zimmermann et al., 2011). This can, for example, be combined with Stereo-PIV measurements of the surrounding fluid flow Tee and Longmire (2021). The most promising technique again involves the usage of refractive index matched hydrogel, with tracers grafted to the surface, in combination with 3D LPT Klein et al. (2013). The particle position and its rotation can be determined here by comparing the spacing between the individual tracer trajectories over time. These reported methods require specific preparations of each particle and so far, none of them have been used in particle-laden pipe flows.

There are several challenges to consider for measurements of particle-laden pulsatile pipe flow. The objective of capturing the perturbation caused by large particles and the subsequent development of turbulence can only be realized through volumetric flow measurements, since the effect of a particle on the surrounding flow will be inherently three-dimensional. Here, the combination of 3D3C optical flow measurements with refractive index-matched hydrogels for particles (similar to the approach by Klein et al. (2013)) seems most promising. A key technical challenge for implementation in a pipe flow, is the optical accessibility through the curved pipe wall, requiring refracting index matching, in combination with the requirement of neutral buoyant particles, limiting the choices of working fluid and particles. Furthermore, in a pulsatile flow, the measurements have to be time resolved to capture phase-specific dynamics. For the setup designed in my thesis, these measurements are realized through Lagrangian Particle Tracking, which will be discussed in more detail in section 2.3.4.

While optical flow measurements provide a highly spatially and temporally resolved evaluation of the flow, they are limited in measurement time and require elaborate and time intensive processing. Therefore, these methods are not suitable for broader parameter scans. In the setup of my thesis, the 3D measurements are supplemented by optical measurements

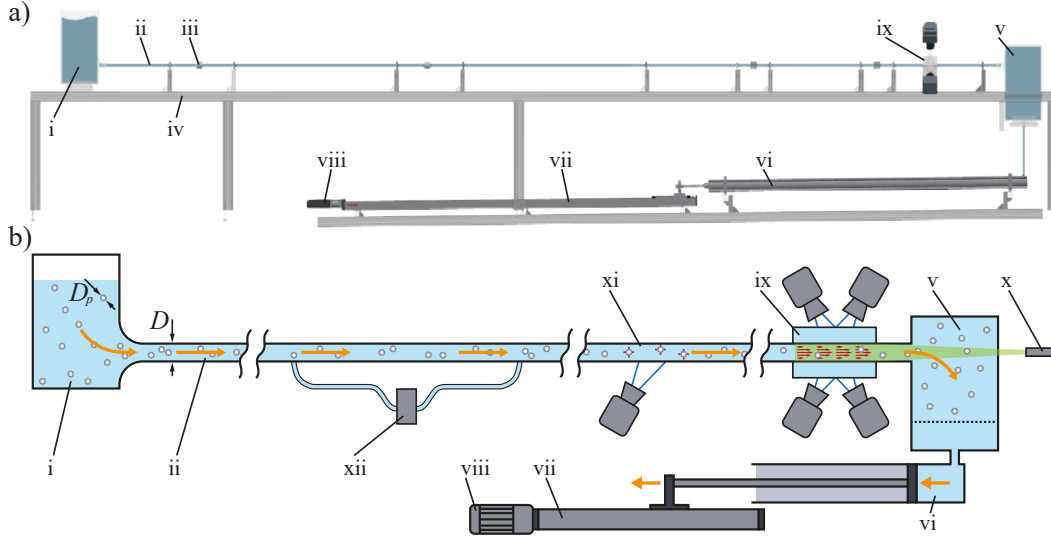


FIGURE 2.12: Full experiment setup in CAD a) and as sketch of the main components b). The particle-laden fluid is pulled from the inlet reservoir (i) into the pipe (ii). The pipe consist of individual segments which are connected (iii) and is mounted to a aluminum strut structure (iv). At the end of pipe, particle are separated from the fluid in an outlet container (v). The fluid is driven by a hydraulic piston (vi) pulled by a linear actuator (vii) with a servomotor (viii). For the measurements the pipe is outfitted with a setup for three-dimensional Lagrangian Particle Tracking (LPT) (ix) with a laser illumination (x), particle imaging and counting (xi) and differential pressure drop measurements (xii).

of just the particle motion (see section 2.3.5) and pressure measurements to evaluate the flow (see section 2.3.6).

In the next section, the overall design and the parameters of the surrounding setup to facilitate these measurement techniques are presented.

2.3.2 Experimental setup

The pipe flow experiment displayed in Fig. 2.12 is made of glass segments, has a maximum length of 20 m and an inner diameter $D = 28 \pm 0.01$ mm resulting in a length ratio of about $714D$. The mean Re can be varied between $Re_{min} = 20$ and $Re_{max} = 7000$ and steady, periodic, or other driving velocities can be generated. At Re_{max} the maximal experimental run time is 94s and at Re_{min} it is 9 h. Without an artificial perturbation, the flow becomes turbulent at $Re \approx 6100$. Particles are fed into the pipe in a controlled manner, providing a constant particle concentration. The experiment is fully automatized and controlled via LabVIEW and measurement data can be directly transferred into an electronic laboratory notebook.

The pipe, the inlet and outlet tanks, and most of the measurement equipment are mounted to a structure of aluminum strut profiles (iv). The inlet tank (i) contains 50 l of the working fluid, it is made of acrylic glass and can be opened at the top to seed particles or tracers into the fluid. A trumpet-shaped nozzle to provide a laminar inflow condition (made of acrylic glass) connects the tank to the pipe segments (made of borosilicate glass and with a typical length of 1 m, shown in (ii), manufactured by DWK Life Sciences GmbH). The individual pipe segments are connected with short acrylic glass sections (iii), where access for pressure or temperature measurements is possible through radial holes with a diameter of 1 mm. Inside these connectors, the pipes are supported using a set of O-rings that press the segments into a radial alignment with each other. The pipe itself is mounted to the structure with rubber brackets. A laser beam is used to line up the segments and

brackets, when setting up or modifying the experiment. The segmentation of the pipe allows it to easily vary the total pipe lengths (up to $L = 20\text{ m}$, $L/D = 714$) or insert specific perturbations, and the borosilicate glass of the pipe is ideally suited for optical flow measurements.

The last pipe segment is connected to the outlet tank with a volume of 13 l (v), which is sealed and degassed, so that no remaining air can act as a bellow and dampen the dynamics of the pulsatile flow. A sufficient stiffness of the tank itself was achieved with acrylic glass walls of thickness 20 mm. Single-phase flows do not need an outlet tank and can be directly connected to the piston driving. The tank is required here to collect the particles from the experiment, without clogging the pipe or entering the piston drive. A mesh screen inside the tank separates the particles from the liquid flowing through a short FEP-tubing, that connects the tank to the piston of the driving system (which is placed below the pipe due to lab space requirements). To generate flow through the pipe, the piston is pulled by a linear actuator, creating a positive displacement and sucking fluid into the piston. The actuator is driven by a servomotor, which is controlled via LabVIEW. In between two experimental runs, the piston drive is used to push the fluid and the particles from the outlet back into the inlet tank.

For investigations of single-phase flow, distilled water is utilized as a working fluid, because of the delayed growth of algae compared to tap water. In addition, a hydrochloride solution (Aqua-Stabil from Julabo) is added and the water is regularly pumped through a filter (pore size 1 mm) and illuminated with a strong UV-light to further suppress algae growth. To check whether the fluid has been contaminated, the dynamics of the laminar-turbulence transition are evaluated. The stochastic decay of turbulent puffs is observed with pressure measurements at a fixed position downstream to the puffs creation (De Lozar and Hof, 2009). At a survival probability of 50% the slope of the decay is most steep, minimizing the margin of error when sampling the number surviving puffs. Approximately 50 – 100 lifetime measurements are performed to check if the expected fluid parameters are valid.

For the investigation of particle-laden flows, three different types of particles were used: small polystyrene spheres ($D_p = 2.8\text{ mm} = 0.1D$ and density 1050 kg/m^3 , slightly larger acrylonitrile butadiene styrene (ABS) spheres ($D_p = 5.9\text{ mm} = 0.21D$ and density 1028 kg/m^3), and large transparent hydrogel spheres ($D_p = 11.5\text{ mm} = 0.41D$). To match the densities of the particles with the fluid, 10.7; 20.2 wt% glycerol is added to the distilled water (for the $0.21D$ and $0.1D$ particles respectively). The viscosity for the mixtures $\nu = 1.358; 1.662 \times 10^{-6}\text{ m}^2/\text{s}$ (for 10.7; 20.2 wt% glycerol) and their temperature dependence is calculated based on Cheng (2008) and Volk and Kähler (2018) and verified with suspended-level viscosimeter measurements. Due to variances in the density, the ABS and polystyrene particles need to be presorted before they can be used in the experiment. This is especially important in the low Re regime, where the inertial effects of the flow are reduced compared to the remaining buoyancy forces. The hydrogel particles are initially small 1.9 mm polyacrylate spheres that swell up in water to a diameter of 11.5(5) mm = $0.41D$. Due to the polymer fraction, the swollen hydrogel particles are slightly heavier than water ($\sim 1.01\text{ g/cm}^3$). In high Reynolds number turbulent flows the hydrogel can be considered as neutrally buoyant in water (Klein et al., 2013; Zade et al., 2018; Zhang and Rival, 2018), but for low Reynolds flows these remaining density differences need to be compensated. For this purpose, the fully swollen particles are added into the water-glycerol mixture, where they are initially lighter. Over several hours, the hydrogel absorbs the water-glycerol and slowly increases in density. For an intermediate period of multiple hours, the particles can be regarded as neutrally buoyant, even for small inertial flow velocities and despite their large diameter. The buoyancy can be roughly checked by driving the flow at $Re \approx 300$ and observing if the particles are properly lifted by inertial forces or roll along the (upper or

lower) pipe wall. In order to determine the viscosity of the working fluid and to recognize density variation due to temperature changes, the temperature of fluid in the inlet and outlet reservoir as well as the room temperature are continuously measured and logged.

In the following a brief overview of the most relevant employed measurement techniques to retrieve the particle-laden flow dynamics are listed:

1. **Shake-The-Box, 3D LPT:** In order to investigate the particle-fluid interaction, a three-dimensional Lagrangian Particle Tracking (LPT) system is used. The high-speed system consists of four high-speed cameras, a pulsed dual-cavity laser, and the software Davis 10 (from LaVision), including the Shake-The-Box algorithm. With this method, all three velocity components (typically referred to as 3C) are measured in a volume (3D) with a field of view length of up to 10 pipe diameters. Details of this method are explained in section 2.3.4.
2. **Particle imaging:** An newly developed efficient method to localize large particles in 3D by using a single USB camera and a mirror. The highlight of the method is to simply subtract the refraction at the curved pipe wall in the image processing instead of using an optically index matched prism (section 2.3.5).
3. **Pressure measurements:** A simple but highly efficient method for detecting the onset of turbulence or instabilities are pressure drop measurements. In this setup, these are typically measured over a length of $36D$. Special care was taken, not to disturb the flow and to retrieve a good signal-to-noise ratio during pulsatile flow (section 2.3.6).

2.3.3 Driving system

Designing a driving system to create a precise flow velocity is one of the main considerations for every flow experiment, especially for pulsatile flow, where specific mass-flux waveforms need to be produced. In steady pipe flow, experiments are often driven by a hydrostatic pressure from an elevated fluid reservoir (Barker and Gile, 1981; De Lozar and Hof, 2009; Marassi et al., 2004; Matas, Morris, and Guazzelli, 2004), a centrifugal pump (Shokri et al., 2017; Walpot, Rosielle, and Geld, 2006), or disc pump (Draad, 1996). The downside of this pressure-controlled driving is that the occurrence of turbulence directly increases the pressure drop in the pipe, which would result in a lower mass flux, mean flow velocity and thus Re . This effect has to be accounted for, e.g. by artificially increasing the overall pressure drop of the flow circuit, so that the pressure drop increase due to turbulence onset in the pipe flow is negligible (De Lozar and Hof, 2009). In order to generate a well-controlled periodically varying flow velocity (and thus Re) a mass-flux controlled driving is typically the best choice. Technically a mass-flux control is realized by a positive displacement driving. In these devices, the motion of a motor is directly coupled to a fixed volume or volume increment of the pump. Typical examples for this are piston or syringe pumps (Darbyshire and Mullin, 1995; Miura, Itano, and Sugihara-Seki, 2014; Satake et al., 2006; Xu et al., 2017). In this kind of drive, the overall volume of the piston sets the maximal experimental run time, which has to be carefully selected because the combination of piston geometry and velocity will also influence the accuracy of Re .

For the purpose of this new pipe experiment, a piston drive pulling fluid through the pipe, was selected. The motion of the piston, and therefore the velocity and pulsation of the flow are controlled with a linear actuator driven by a servomotor. There are multiple reasons to select this type of driving system. The first stems from the requirement to operate with a variable pulsatile flow and to achieve controlled flow rates in the transitional Reynolds number regime. Additionally, the experiment should be adjustable to work with different

Wo	Re_m	U in mm/s	T in s	L_{pulse}/D	T_{exp} in s	T_{exp}/T
-	100	4.9	-	-	2451	-
-	1000	48.5	-	-	245	-
-	2000	97.0	-	-	123	-
-	4000	194	-	-	61.3	-
4	1000	48.5	56.7	98	245	4.3
6	1000	48.5	25.2	44	245	9.7
8	1000	48.5	14.2	25	245	17.3
12	1000	48.5	6.3	11	245	38.9
8	100	4.9	14.2	2.5	2451	173
8	1000	48.5	14.2	25	245	17.3
8	2000	97.0	14.2	49	123	8.6
8	4000	194	14.2	98	61.3	4.3

TABLE 2.1: Maximum measurement duration T_{exp} , mean velocity U , pulsation period T and pulsation length $L_{pulse} = U \cdot T$ for steady and pulsatile flow for different pairings of Re_m and Wo . The duration is limited based on the finite fluid volume that can be pulled by the piston. The experiment time T_{exp} is calculated as the remaining time after the fluid flow has passed the entire pipe length (20 m) with mean velocity U .

pipe lengths, geometries, fluids, and most importantly particle concentrations, which will strongly affect the required driving pressure. Furthermore, the driving must not induce additional uncontrolled pulsation on the flow, as can be the case in many pumps that use a cyclical motion of gears or screws. To be able to fulfill these requirements, it was decided to use a pulling piston system with a linear actuator driven by a servomotor connected to the outlet tank of the pipe. This type of driving is already established for steady (Darbyshire and Mullin, 1995) and pulsatile single-phase pipe flows (Xu et al., 2017). In this system, the motor shaft has a closed mechanical coupling through the linear actuator spindle, piston rod, and piston head towards the fluid volume. For an incompressible liquid (e.g. without air pockets) the rotation of the motor is directly coupled to the fluid volume pulled through the pipe into the piston. Controlling the servo motor speed therefore provides direct control of the mass flux and hence the Re (including its time-dependent variations). The downside of this driving system is the limited measurement time, which is restricted by the total volume of fluid which can be pulled into the piston. This volume limitation ($\lesssim 20l$ in this setup) has to be considered in the experimental design and compared to the duration of pulsation periods and the geometric pipe dimensions. An overview of the achieved time scales of measurements of this pipe experiment is provided in Table 2.1.

2.3.3.1 Specifications

The driving system (piston and linear actuator) is mounted below the pipe on a separate structure and connected to the outlet tank of the pipe via a short FEP-tube. Mechanical vibrations from the linear actuator and the motor are therefore not transmitted to the sensitive measurement equipment (e.g. the three-dimensional STB-system and pressure sensors). The piston drive is a standard based cylinder (type: DSBC-U from Festo SE & Co. KG.) with a reduced friction piston head, to be able to operate at small velocities. Especially at low Re , the stick and slip effect inside the piston can otherwise induce sudden jumps or vibrations in the piston motion, leading to inaccuracies or perturbations of the flow. The cylinder of the piston drive has a diameter of 100 mm and an operational length of 2500 mm resulting in 19.6 l (or ≈ 1140 pipe diameters) of usable fluid volume for one experimental run.

To transmit motion, the piston rod is coupled to the carriage of the linear actuator, which can push or pull the piston. The coupling is realized as a hinged support, to guide the rod without applying radial force or torque. The linear drive is a ball screw assembly (type: MKK-80-NN-3) with a lead of 5 mm per revolution from Bosch Rexroth AG. Driving the shaft of the actuator is a permanent-magnet-excited synchronous servomotor (0.57 kW, 1.09 Nm, type: MS2N04-B0BTN also from Bosch Rexroth AG) with an integrated 20 bit multiturn encoder. As recommended by Bosch Rexroth, the use of a very powerful motor, compared to hydrodynamic forces, is able to achieve a high accuracy of the servo motor rotation rate. Mounted between motor and actuator is a coupling and a transmission with a planetary gear reduction of 20:1, to allow for the operation at small laminar velocities with still moderate motor rotation speed. The direct mechanical connection from the motor shaft over the transmission, ball screw drive, and piston rod to the piston head with minimal friction makes it possible to directly measure and control the piston head position and motion and therefore the mass flux in the experiment with the signal from the motor encoder. The rotation rate of the motor is regulated with a closed-loop servo controller (type: HCS01.1E-W0018 from Bosch Rexroth AG). The controller is connected over Ethernet to a laboratory computer where the flow velocity and pulsation is set and the motor feedback parameters are logged in a LabVIEW interface.

2.3.3.2 Performance

To evaluate the performance of the driving system, the set values of the driving velocity are compared to the feedback data of the servo motor (driving the linear displacement of the piston) for three different test cases:

- *Steady flow* with a constant command velocity.
- *Harmonic pulsation* of the flow around a mean velocity. The command waveform is defined by the Womersley number Wo and the Amplitude A , see Eq. (2.4).
- *Aorta flow* with a more complex waveform consisting of multiple harmonics. This waveform was chosen for comparative experiments with cardiovascular flow in the human aorta see Eq. (2.5) and Table 2.2.

n	b_n	a_n
1	-0.053044	1.46368
2	-0.72782	-0.171238
3	-0.09571	-0.190516
4	-0.05142	-0.143264
5	0.036842	-0.066428
6	0.014244	-0.025932
7	-0.00129872	0.008692
8	-0.0151898	-0.0043376

TABLE 2.2: Fourier coefficients a_n and b_n fitted to physiological flow in the descending aorta section, based on measurements of the mean velocity waveform for several young volunteers by Bürk et al. (2012). The Fourier coefficients are provided by Morón, Feldmann, and Avila (2022).

An example of the evaluated data for all three test cases is presented in Fig. 2.13 for $Re_m = 1000$. In the steady flow case in Fig. 2.13a the feedback velocity deviations around the set value are stochastic in nature and Gaussian distributed (with a standard deviation $\sigma_{Re} = 1.4$, corresponding to deviations in Re of 0.14%). For the pulsatile flows in

Fig. 2.13b,d the deviations are phase dependent but remain below 1.9% with respect to Re_m for the harmonic pulsation and below 6.5% during the aorta pulsation. Especially at the strong acceleration at the beginning of the systole phases in the aorta flow test case in Fig. 2.13d the deviations from the command velocity are largest, as the driving system has to produce the maximum torque to accelerate the fluid. In general, the absolute error is proportional to the current velocity of the pulsation.

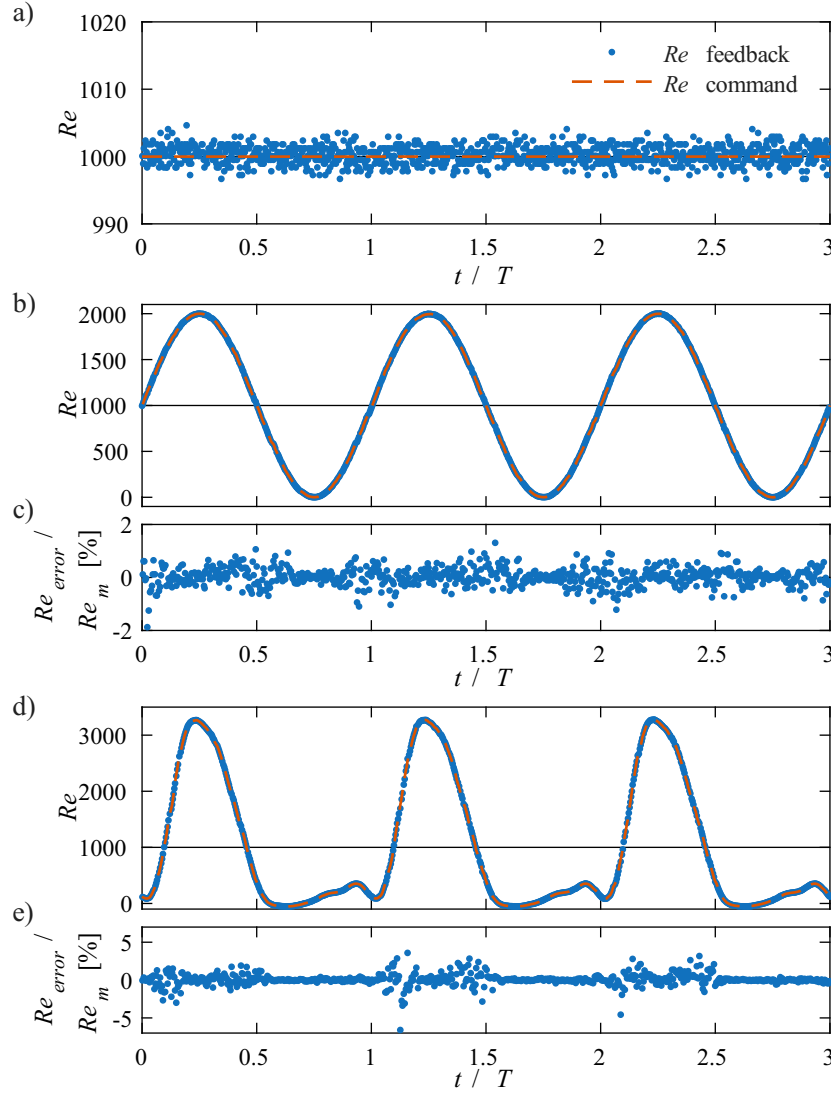


FIGURE 2.13: Three test cases for which the driving system was evaluated. A steady flow a), an harmonic oscillation b) and the modelled oscillation for the flow in a human aorta d). All three recordings were taken with a $Re_m = 1000$ using a water-glycerol mixture in the experiment (for fluid parameters see chapter 2.3.2). For the pulsatile flow the Womersley number was chosen at $Wo = 10$ and the amplitude in b) $A = 1$, in d) the amplitude is given by the waveform model. For each measurement the recorded feedback velocity and the ideal command velocity calculated as the Reynolds number in the pipe flow are compared. Due to the low visibility of the scatter in the pulsatile measurements the deviation between feedback and command velocity of the pulsatile flows is plotted in c) and e).

The relative deviations on the other hand increase for decreasing Re , as can be seen in Fig. 2.14 covering a wide range of $Re = [10; 7400]$. At $Re \lesssim 50$ the standard deviation exceeds 2%, which is caused by the corresponding slow motor rotation limiting the steering capabilities of the servo motor controller. The upper limit of the flow velocity is given

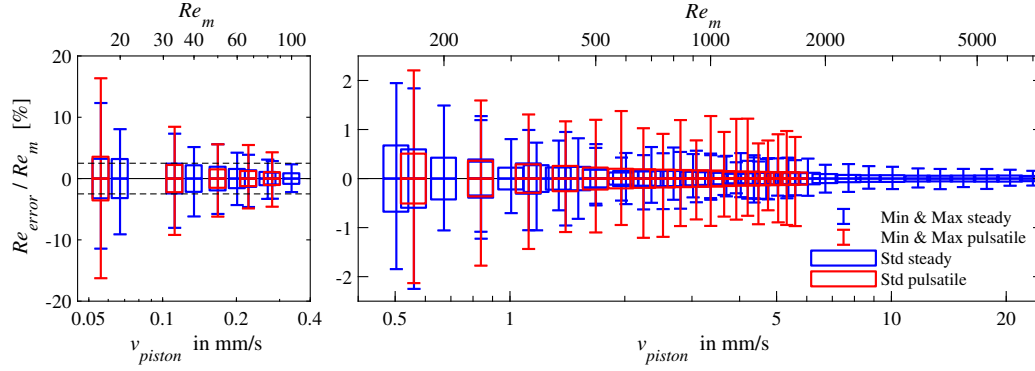


FIGURE 2.14: Relative error of the driving system over the driving velocity or Reynolds number, for steady flow and harmonic pulsatile flow ($Wo = 10, A = 1$), see Fig. 2.13a,b. In the lower Reynolds regime (left) the error increases, in the upper regime (right) the relative error decreases for higher velocities. The velocity is given as both the piston velocity which is read out from the motor controller and the corresponding Reynolds number of the pipe flow for a water-glycerol mixture

by the rotation speed limit of the motor 6000 min^{-1} which results in a flow velocity of $U = 319 \text{ mm/s}$ or a Reynolds number of about 8900 for water at room temperature or 7400 for the water-glycerol mixture. For pulsatile flows the upper limitation also depends on the waveform and Womersley number as strong accelerations during a high frequency pulsation or very sharp waveform can reach the torque limitation that can be applied to the linear actuator. Overall, the experimental setup can be reliably operated in a range of $Re = [50; 7400]$ without significant errors for the water-glycerol mixture.

A closer look at the data in Fig. 2.14 reveals that the minimum and maximum deviations for the pulsatile flow are always larger than the steady counterparts. These deviations occur during the acceleration and deceleration phases (compare Fig. 2.13), but are not visible in the time-averaged standard deviations (because here they are compensated by the low absolute deviations in the slow fluid phase). Experiments without fluid indicate that these deviations stem from the mechanics of the linear driving system and piston itself, rather than the hydrodynamics of the system.

Deviations from the servomotor control are more prominent in a pulsatile flow, but can be minimized. In a steady flow, the set velocity is constant, and the typical communication cycle time between the motor controller and LabVIEW of 25 ms (40 Hz) via an Ethernet communication is sufficient. In a pulsatile flow, on the other hand, the classical transmission of an instantaneous set velocity (marked as red line in Fig. 2.15) would cause a non-uniform motion of the piston. Especially at high Womersley numbers, the number of communication cycles per period is low (corresponding to ~ 100 points for $Wo = 20$ or ~ 45 points for $Wo = 30$) and the effect is enhanced. In order to avoid such a Womersley dependent quality of the motor control, the driver is provided both the instantaneous velocity and acceleration ramp at each communication cycle. The acceleration values are calculated as time derivatives of the waveform function to ensure that the ramp changes appropriately to the pulsation. (Furthermore, the internal command value preparation of the servo controller filters the velocity ramp through a PT1 filter with a chosen time constant of 6 ms to smooth out the ramps.) With this procedure the motor smoothly accelerates or decelerates towards the next velocity point, also in-between the communication cycles.

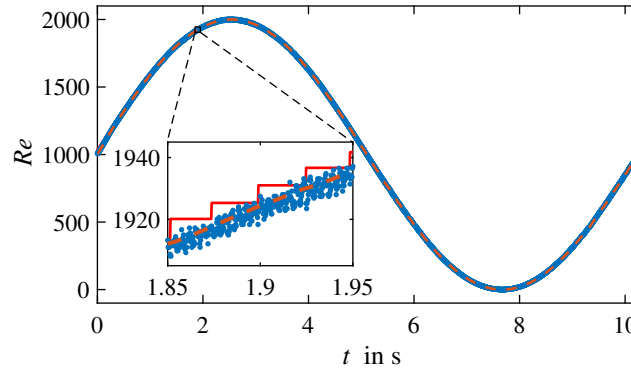


FIGURE 2.15: Comparison between the sampling steps and the interpolated command and feedback velocity of the motor in a pulsatile motion ($Re_m = 1000$, $Wo = 10$, $A = 1$). Due to the limited Ethernet communication the sampling of new command values is limited to 25 ms. The sampling steps are displayed as a red line in the zoom-in. In between the samples the motor accelerates or decelerates with linear interpolation, resulting in smoothed approximation of the harmonic motion. The higher sampling for the data in this plot was recorded with the internal logging function of the motor controller, the ordinary logging is also limited to 25 ms increments.

2.3.3.3 Further uncertainties

In the previous section, the accuracy of the driving system was discussed in detail based on the feedback values from the servo controller. To complete the evaluation of the flow accuracy, in this section further uncertainties are identified.

A systematic error might be induced by tolerances of the inner diameter of the piston. According to the manufacturer, the diameter of 100 mm has a tolerance of up to 0.35 mm (ISO 286 tolerance of H12), which could maximally result in a +0.7% larger mass flow rate and hence Re . Similarly, the pipe diameter variance ($D = 28 \pm 0.01$ mm) can affect the flow velocity, but in this setup it has a negligible influence on the local velocity in the pipe ($\pm 0.07\%$).

Further uncertainties of the mass flux that are not included in the recorded velocity signals can arise from tolerances in the gear transmission and the pitch of the linear drive shaft or misalignment between the linear actuator and the piston. According to the technical specification of the linear actuator, these are negligibly small compared to the piston tolerance.

Another source of uncertainty can arise from the variance in fluid viscosity with respect to temperature changes. A temperature change of $\pm 1^\circ\text{C}$ will change the viscosity of a water-glycerol mixture by about $\pm 2.6\%$ (based on Cheng (2008) and Volk and Kähler (2018)). In the setup, the temperatures in the inlet and outlet tank are continuously measured. During a single experiment, the temperature change over time and throughout the setup is usually smaller than 0.1°C , resulting in $\pm 0.26\%$ error to viscosity and Re . For strong temperature variances throughout a measurement campaign, the viscosity and Re of individual recordings can be recalculated in postprocessing.

Overall, it can be said that the set and feedback velocity of the piston are indistinguishable on the scale of the pulsation amplitude (see e.g. Fig. 2.13). Measurements of puff lifetimes as well as pressure drop measurements (explained in section 2.3.6) provide direct comparison to the flow physics and illustrate that the overall deviations in Re are on the order of 1%.

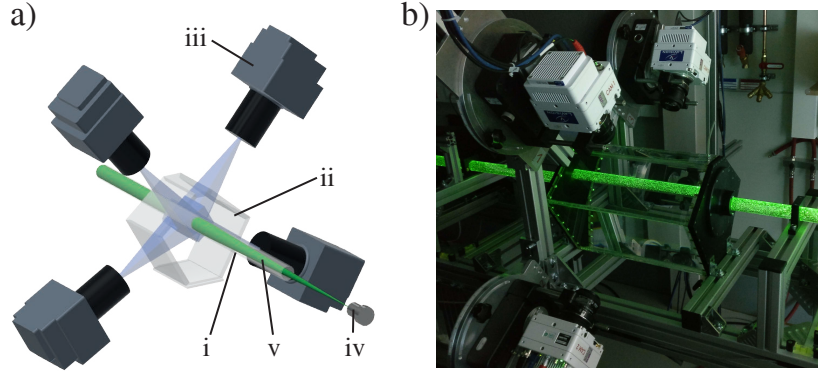


FIGURE 2.16: Measurement setup for Lagrangian Particle Tracking with the Shake-The-Box algorithm. In the sketch in a) the placing of a pipe section (i) inside a hexagonal refractive index matching prism (ii) can be seen. Four high-speed cameras are in an in-line arrangement (iii) with the laser illumination being perpendicular to all cameras. In the photo in b) the axial laser illumination of the pipe is clearly visible.

2.3.4 Lagrangian Particle Tracking

Parts of the content presented in this section led to the publication² of Bäuerlein, Schanz, Schröder, and Avila (2022)

One of the biggest experimental challenges of particle-laden flow is the simultaneous measurement of fluid flow and particle motion to investigate their interactions. In this chapter a method based on Lagrangian Particle Tracking measurements that can resolve both the three-dimensional fluid velocity field, and the trajectories and even particle rotation of large particles. This method utilizes refractive index and density matched hydrogel particles with tracers adhering to the surface (similar to the method of (Klein et al., 2013)).

2.3.4.1 Measurement setup

The developed measurement setup, displayed in Fig. 2.16, enables the Lagrangian Particle Tracking measurements in a pipe volume (of approximately $1D \times 1D \times [4 - 10] D$). The key components of the tomographic setup are four high-speed cameras, a pulsed laser, an index matching prism, and the Davis 10 software from LaVision, which uses the Shake-The-Box algorithm (Schanz, Gesemann, and Schröder, 2016).

For Lagrangian Particle Tracking, the signal of the scattered light of the tracers has to be of better quality than e.g. for a PIV analysis, because here the individual tracers are identified and tracked over time. To achieve Gaussian-distributed light intensity signals, polyamide 12 spheres with a mean diameter of $60 \mu\text{m}$ (type: Orgasol 2002 ES6 NAT3 from Arkema) are utilized as tracer particles. These tracers have a nominal density of 1.03 g/cm^3 , which matches the water-glycerol mixture very closely. Note that the signals of the typically used hollow glass sphere tracers are in contrast often non-uniform, which is caused by the recurrent breakage of the spheres into half-shells, acting as micro-lenses in the flow. To validate whether the polyamide spheres will act as tracers, the corresponding Stokes number ($St = t_p/t_f$) can be calculated, with a particle response time (Brandt and Coletti, 2022),

$$t_p = \frac{\rho_p d_p^2}{18\mu}, \quad (2.7)$$

based on the density ρ_p and diameter of the particle d_p and the dynamic viscosity of the

²B. Bäuerlein carried out the measurements, the data analysis, and wrote the first draft of the manuscript.

fluid ($\mu = \nu\rho$, see section 2.3.2 for fluid parameters). The shortest relevant fluid timescale would correspond to the Kolmogorov timescale of fully turbulent flow,

$$t_f = \left(\frac{\nu}{\varepsilon_t} \right)^{1/2} \quad \text{with} \quad \varepsilon_t \approx \frac{U^3}{D}, \quad (2.8)$$

approximated for a pipe flow (Davidson, 2004). Calculated for a maximum Reynolds number of $Re \approx 4000$ of the planned STB measurements gives a Stokes number of $St \approx 0.06$, indicating that the tracer particles could still follow the smallest eddies in the flow.

The tracers in the measurement volume are illuminated with a pulsed dual-cavity 527 nm Nd:YLF laser (2×50 mJ @ 1 kHz, DM50-527-DH from Photonics) positioned at the end of the pipe. The laser beam is first expanded with a set of telescope lenses and then adapted by an aperture to evenly illuminate the full size of the pipe cross-section. After the optics, the beam passes through the outlet tank before illuminating the measurement volume within the pipe. For measurements at different pipe lengths, the pipe gets shortened/lengthened accordingly to keep the measurement section close to the outlet tank, and hence the laser beam exit. The advantage of an axially directed laser beam in comparison to the more common radial illumination (in stereoscopic PIV measurements (Hof et al., 2004)) is twofold: Reflections on the curved pipe wall due to differences in the refractive indices are avoided, and (together with a radial in-line arrangement of the cameras) each camera receives light from the same 90° scattering angle, resulting in uniform tracer intensities. Especially for the STB-algorithm, this enhances the tracer identification because intensity differences between forward and backward scattering are omitted. The in-line arrangement of the cameras with their mountings on a stiff frame is illustrated in Fig. 2.16b. The frame consists of four traverse beams, which maintain the viewing directions of the cameras. The distance from each camera towards the pipe can be adjusted to change the size of the measurement volume. In an in-line arrangement of four cameras, an angle of 80° between the individual views is ideal and realized by the traverse beams for mounting the cameras. The precise alignment of the camera angle is enabled by in-house manufactured turntable supports, placed between the cameras and these beams (visible as round aluminum plates in Fig. 2.16b).

To minimize the optical distortion of the camera images stemming from the refraction at the round pipe wall, the pipe segment used for the three-dimensional measurements is placed inside a hexagonal refractive index matching prism made from acrylic glass. The walls of the prism are exactly matched to the camera angles provided by the traverse beams, so that each camera has an orthogonal view through the planar prism walls and hence experiences minimal refraction. Such prisms are typically filled with a fluid that approximates the refractive index of the borosilicate glass tubing ($\tilde{n}_{glass} = 1.473$). In this setup, the choice of the working fluid is limited due to the density matching with the large particles. The highest imaging quality was achieved when the working fluid (water-glycerol with $\tilde{n}_{fluid} = 1.344$, calculated based on Miessner, Lindken, and Westerweel (2007)) was also used within the prism. Since the pipe wall is rather thin (1.5 mm) the remaining refraction is weak enough to be mostly compensated with the calibration procedure. For more details on the remaining refractions, see section 2.3.4.7.

The transverse beams with the cameras, the index matching prism, and a segment of the pipe are all integrated into a single fixed structure. This allows it to easily move this measurement module (e.g. when shortening the pipe), while the alignment of the optical components changes minimally.

For the image acquisition, four high-speed cameras type VEO 640L from Phantom are mounted in the setup. The cameras can record with a maximum frequency of 1.4 kHz at the full resolution of 2560×1600 px, although due to the elongated shape of the measurement

volume, only part of the vertical resolution is used. At an axial length of the measurement volume of $280 \text{ mm} = 10D$ the resolution corresponds to 9 px/mm and for a length of $95 \text{ mm} = 3.4D$ to the maximal resolution of 27 px/mm .

2.3.4.2 Image pre-processing and calibration

A pre-processing of the raw data images is applied to enhance the contrast and visibility of the tracers. An example of a raw data image and a pre-processed one is shown in Fig. 2.17b,c. A time-averaged filter (of at least 7 frames) removes stationary signals stemming e.g. from remaining reflections and tracers sticking to the pipe wall. To achieve a similar signal-to-noise ratio over the entire field of view (despite the Gaussian intensity profile of the laser intensity) a spatial moving average (over 8 px) is used to smooth out the variations in space. As an optional last step, a Gaussian-blur smoothing and sharpening filters can be used to increase the visibility of the tracers by reducing the remaining noise and increasing the signal of the tracers.

A volumetric calibration maps the tracer positions on the camera sensors into the three-dimensional space of the measurement volume. At least three viewing directions are required to locate the tracers in 3D by triangulation. Four or more viewing directions reduce the uncertainty, but the calibration becomes increasingly challenging. In the first step of the camera calibration, a volumetric calibration plate with a defined pattern is inserted in the measurement volume. The rectangular calibration plate is mounted to a custom cylindrical shaped holder that slides along the inner pipe wall to provide the alignment in the radial direction. A reproducible orientation of the calibration plate is achieved by comparing the view from all four cameras. Note that the calibration plate as well as the holder have to fit within the pipe to be able to flood it with the working fluid for the calibration procedure. The cylindrical measurement volume has an aspect ratio of up to $1 \times 1 \times 10$, which is far larger than any commercially available calibration plates. To achieve a sufficiently accurate calibration in the radial and axial direction, a tailor-made volumetric calibration target is designed (with an $4 \text{ mm} \times 4 \text{ mm}$ even grid dot-pattern of up to 34 dots in the axial and 6 points in the radial direction), illustrated in Fig. 2.17a. The plate (made of black polyoxymethylene) has a maximal thickness of 10 mm and each side provides two different depth layers for the even grid dot-pattern. Each dot is made from a CNC drilled 1 mm hole filled with white acrylic paint. The 3D dot pattern is provided to the Davis 10 software, which detects the dots in each camera and creates a perspective calibration (using a third-order polynomial camera model). This initial perspective calibration is not accurate enough to triangulate all the tracers and needs to be further refined.

In the second step of the camera calibration, the tracers within the fluid are illuminated with the laser. The Davis 10 contains a powerful volume self-calibration algorithm (Wieneke, 2008), which uses a set of recorded flow images to reduce the remaining disparity of the initial perspective calibration. In this procedure, the recorded pipe section is split into up to $20 \times 20 \times 10$ rectangular sub-volumes. In each volume, the residual error when triangulating tracers from the four camera views is mapped. Based on this disparity map, the calibration for each camera is refined. By repeating this procedure (typically about 4-6 iterations) the averaged remaining disparity reduces to 0.25 px , which is well suited for 3D measurements. Due to the remaining refractive index differences in the experiment, there are some optical distortions close to the wall, which cannot be compensated with the polynomial camera model. This leads to larger remaining disparities of up to 2 px near the walls. Based on the final iteration of the disparity field an optical transfer function is calculated which maps how tracers are projected in each camera for each sub-volume, this also includes unevenness in shape or size of the tracers due to astigmatism or uneven illumination (Schanz et al., 2013).

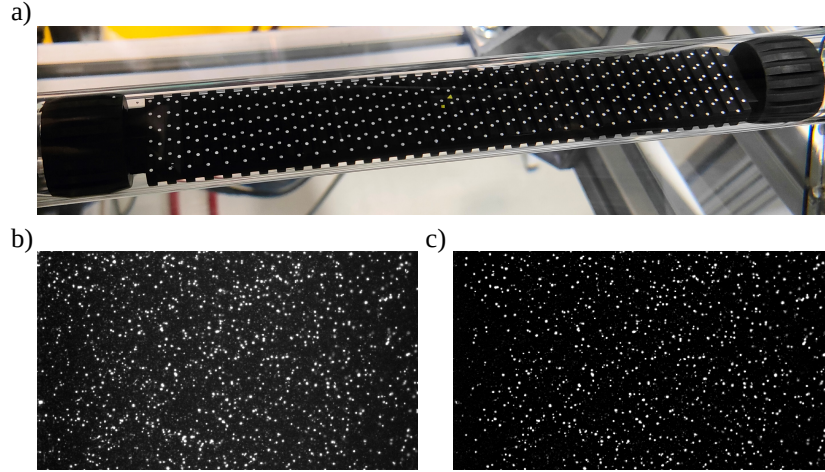


FIGURE 2.17: a) Custom double-sided volume calibration plate with two layered even grid dot patterns. Example snapshot with a section of a tracer image before b) and after c) image preprocessing.

2.3.4.3 Processing and post-processing

The pre-processed images are analyzed with the Shake-The-Box algorithm (Schanz, Gese-
mann, and Schröder, 2016) implementation in LaVision's Davis 10 to calculate the particle
trajectories. For the tracer detection, a brightness threshold of 200 counts and an allowed
triangulation error of up to 3 px is chosen. Due to the high efficiency of Shake-The-Box the
computation of a single timestep takes only 2-3 seconds (on a machine with an Intel Xeon
E5-1650 6 core processor and 32 GB of RAM). On average, approximately 5000 tracers are
triangulated and 3000 tracks are identified per timestep. The average track length is 15
timesteps. Any remaining ghost tracks can be removed using a spatial median filter applied
to the trajectory field and by masking out false tracks outside the pipe volume.

To convert the Lagrangian trajectories to an Eulerian grid velocity field, the Vortex-in-
Cell-sharp (VIC#) method (Jeon, Müller, and Michaelis, 2022) (as an updated version of
the Vortex-in-Cell-plus method by Schneiders and Scarano (2016)) is applied. This method
sets an initially zero valued velocity field estimate. In an optimization procedure, a cost
function consisting of the velocity, the velocity material derivative, the continuity law, and
the Navier-Stokes equations compares the reconstructed field to the Lagrangian particle
track data. The velocity field is iteratively updated until a convergent solution with a
minimum discrepancy to the trajectories is reached. Compared to directly calculating the
velocity field with tomographic PIV this method is able to resolve smaller flow structures
and generates a field that fulfills Navier-Stokes boundary conditions (Jeon, Müller, and
Michaelis, 2022; Schneiders and Scarano, 2016). For this experiment, the velocity field is
usually reconstructed with a 1 mm (0.036 D) grid size. Since the Davis 10 implementation
of VIC# does not support wall boundaries in the measurement volume, the flow field is
extrapolated for the volume beyond the domain of the pipe. A geometric mask is applied
in post-processing to remove these areas from the velocity field. Compared to the STB
processing, this reconstruction takes much more computational power with a processing
duration of about 60 seconds per timestep.

2.3.4.4 Three-dimensional measurements of particle-laden pipe flow

The goal of this setup is to simultaneously measure the fluid and particle motion time-
resolved in a volume (3D3C). Since large particles typically yield strong light reflections
and block out the laser illumination, transparent hydrogel particles are used. Fig. 2.18a

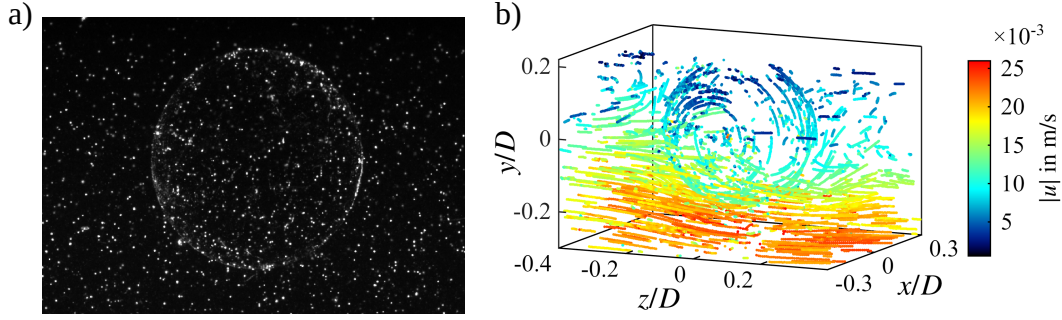


FIGURE 2.18: a) Raw data image of a hydrogel particle with tracers adhering to its surface. b) Raw data of the corresponding Lagrangian tracer trajectories illustrate the motion of the hydrogel and the surrounding flow in the pipe.

displays a tracer image with a hydrogel particle. Since the material itself has almost the same refractive index as the working fluid, it does not cast a shadow and even tracers behind the particle can be captured. Additionally, during the operation within the working fluid, some tracers adhere to the hydrogel interface. The tracers act as surface markers and their tracks can later be used to identify the particle position and its motion. These tracers accumulate over time, and their concentration can limit the usability of the hydrogel.

An important advantage of the hydrogel is that it does not require additional specific operations for image pre-processing or Shake-The-Box processing. If a successful volume-calibration was calculated, the same steps as for a single-phase fluid measurement can be used for particle-laden flow. In Fig. 2.18b the trajectory field surrounding a hydrogel particle is shown. Since no tracks are identified within the hydrogel, the particle appears as an empty sphere with the surface markers displaying the exact position of the interface. Due to remaining effects of refraction with the hydrogel, there are also some falsely identified ghost tracks at the interface and in the surrounding fluid. These usually do not match the true tracks in magnitude or directions and can therefore be easily removed with a spatial median filter or velocity thresholding.

After calculating the STB trajectory field, the trajectory and rotation of the particle can be determined. Compared to the method presented by Klein et al. (2013), where only 6 tracers were attached to hydrogel particles, a much higher tracer concentration on the interface could be achieved (200-1000 tracks). This drastically simplifies the particle identifications based on spatial distribution of the tracks in the measurement volume, without the need for long continuous trajectories or a direct track-to-track distance comparisons. For identifying the particle, the three-dimensional positions of all identified tracks for a timestep are mapped. Using the known approximate radius of the hydrogel, the entire measurement volume is searched for areas with a high concentration of tracks on a spherical shell and almost no tracks inside that sphere. Areas with the highest tracks concentration on this sphere around a void are identified as the particle center of mass positions. After the position has been determined for each time step, a moving average smoothing is applied to the hydrogel trajectory, to remove small deviations stemming from inhomogeneous track concentrations within the measurement volume.

After the hydrogel track is determined, the Lagrangian data can be separated into trajectories belonging to the particle interface and the surrounding fluid, via a sphere-shaped mask around the particles' center of mass. For the VIC# flow field reconstruction, only the tracks corresponding to the fluid flow are used, as the interface motion of the particle cannot be described by the Navier-Stokes equation and will lead to errors in the velocity field. The hydrogel interface tracks can be used, not only determine the translatory motion of the particle but also the rotation speed and axis. This is done by subtracting the mean

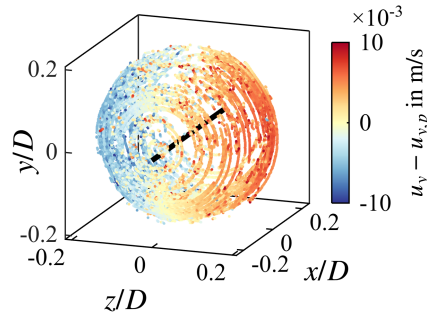


FIGURE 2.19: Particle interface trajectories with velocity relative to the particles center of mass accumulated over 100 timesteps (corresponding to 0.5 s) for the motion of a hydrogel particle in a steady pipe flow at $Re \approx 600$. The surface tracks reveal a rotation around the x -Axis with 1.45 1/s .

particle velocity from the interface track velocities. An example of this is displayed in Fig. 2.19 for the rotation of a large particle in a laminar steady flow. Following the shape of the tracks through multiple time steps here already reveals the rotational direction. For the instantaneous rotation rate, the rotation matrix between two consecutive timesteps is calculated for all tracked surface markers in both steps, using a least square quaternion-based method by Horn (1987). Since the rotation rate is not expected to drastically change in-between the steps, a moving average filter is used to smooth the rotation rate. In the displayed steady flow case, the particle rotates around the x -axis with a rotation speed of 1.45 1/s .

2.3.4.5 Example measurement of laminar flow

The laminar Hagen-Poiseuille flow is ideally suited to verify that the velocities are computed correctly. The measured flow velocities are in excellent agreement, as can be seen in Fig. 2.20 (for $Re = 2000$ at an axial position of $357D$ to achieve fully developed flow). In this measurement, the field of view had a length of 95 mm ($3.4D$) with a resolution of 27 px/mm . On average ~ 5000 particles could be tracked. In Fig. 2.20b it can be seen, that the trajectory density is mostly constant over the entire cross-section, despite the remaining refraction in the system. This could be achieved by finely dissolving the measurement domain during the volume self-calibration procedure (20×20 volumes over the cross-section). The remaining uncertainties of the measurements in single-phase and particle-laden flows will be further discussed in section 2.3.4.7.

2.3.4.6 Iterative Improvements to the LPT setup

The content presented in this section led to the publication³ of Bäuerlein, Schanz, Schröder, and Avila (2023)

During the process of using the LPT setup and measurement methods, a number of improvements were developed and implemented.

One of the main difficulties in 3D Lagrangian Particle Tracking is to avoid falsely identifying and tracking of ghost tracks. In this pipe experiment, these ghosts can arise from remaining distortions in the optical path of the cameras. This can be caused by refractive index mismatch between fluid and pipe walls, or by refractions caused by the large hydrogel particles. As the particles move through the measurement volume, their refractions cannot be compensated with a volume self-calibration procedure. Therefore, the

³B. Bäuerlein carried out the measurements, the data analysis, and wrote the first draft of the manuscript.

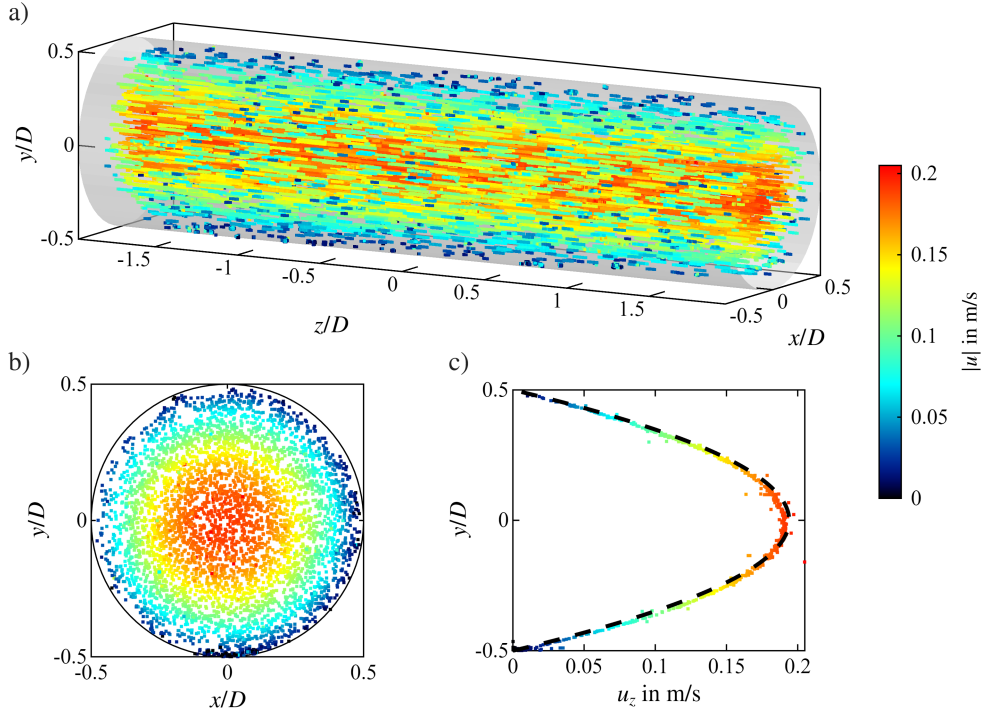


FIGURE 2.20: Shake-The-Box results of fully developed single-phase laminar flow at $Re = 2000$. a) Trajectory field over a length of 50 ms with a measurement volume of $3.9D$ (flow direction left to right). b) Detected tracer positions in the cross-section of the pipe, the color is proportional to the velocity $|u|$ with the same color scale as in a). c) Flow profile from the trajectories in the central plane of the pipe ($z \leq \pm 1$ mm). The dashed line is the expected parabolic Poiseuille flow profile.

STB algorithm will regularly identify ghost tracks for both the hydrogel interface, and in the surrounding regions of the flow, where one or multiple camera views are slightly distorted. While some of the ghost tracks can be removed in post-processing, if larger quantities of ghosts appear, due to an increased accumulation of tracers or some imperfections on the hydrogel interface, this can become more challenging.

The number of ghost tracks in three-dimensional measurement methods is most efficiently reduced by increasing the number of independent camera angles (Discetti and Astarita, 2014). As each tracer is triangulated from all the camera views, the chance of a coincidental misidentification becomes more unlikely. The refractive index matching prism has a hexagonal cross section, allowing for easy modification to include six camera views, compared to the original four. Furthermore, since the field of view of the measurements is rather long and narrow (at least $3.4D$ in length), compared to the aspect ratio of the camera sensors (4×3), a single camera can be used to capture multiple viewing angles.

The modified optical setup is shown in Fig. 2.21. For two of the cameras, a mirror prism is positioned directly in front of the lens, splitting the view by 90° into two directions. A second set of tilted mirrors directs the view through the refractive index matching prism towards the pipe. Care was taken to ensure that each view is directly orthogonal to the surface of the prism and the magnification of each view is the same. To achieve this, the split cameras are positioned on the bisecting angles between two prism surfaces, and the distance of the cameras is selected so that optical path length is the same in all six views. It is worth noticing that in the selected configuration, two of the views (from the top and bottom) are directly 180° apart. While this is usually not advisable because both views will show the same tracer pattern, making them redundant, in the pipe flow, the remaining

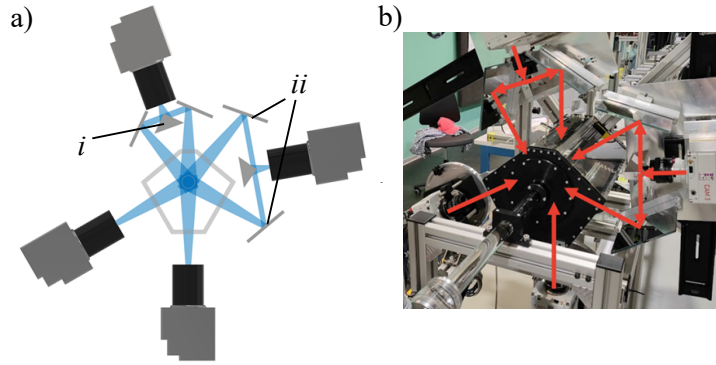


FIGURE 2.21: STB-setup with six viewing angles and four cameras. In the sketch in a) the view splitting of two of the cameras with a mirror prism in front of the lens (i) and two tilted mirrors (ii) directing the view into the refractive index matching box. The same setup is shown in b) as a photograph, with the red arrows indicating the optical path of each camera.

refractions, especially from the hydrogel, result in different projected tracers in both views, providing additional information to avoid ghost tracks. Due to addition of the mirrors, the minimum distance between cameras has to be slightly increased, resulting in a minimum axial field of view of $6D$ with a spatial resolution of $\sim 15\text{px/mm}$

A second modification to the setup was implemented for the laser illumination. For the STB measurements, it is optimal to fully illuminate the entire cross section of the pipe without any light shining onto the walls, to avoid reflections. In the previous setup, the laser was expanded with a set of telescope lenses, resulting in a slightly divergent beam. This configuration, in combination with the Gaussian light intensity profile of the laser, led to tracers close to the wall receiving little or no light compared to the centerline of the pipe. To improve on this, the beam is first expanded to a diameter larger than the pipe and then collimated with a second lens. An iris aperture is used to mask the beam, making it round and precisely matching the inner diameter of the pipe. This also effectively avoids using the outer region of the laser intensity profile, which has a much lower light intensity.

These two modifications – especially the two additional camera angles – resulted not only in a reduced number of erroneous ghost tracks but also increased the number of correctly identified tracks in both single-phase and particle-laden flows. The number of tracks per timestep was increase from around 3000 tracks with four cameras to a usually more than 10000 tracks with six camera views. This increased number of tracks also enables measurements with a low concentration particles which are not refractive index matched as will be shown for the results in section 2.4.2.5. A quantitative assessment of the measurements uncertainty and a comparison of the two configurations will be given in the next section.

2.3.4.7 Measurement uncertainty with Lagrangian Particle Tracking

One of the main sources of uncertainty is the refractive index mismatch between fluid and pipe wall. A diagram of this effect on the image distortion is shown in Fig. 2.22, for the view of a single camera. The deflection of the view is calculated based on the refraction of light at the outer and inner wall of the glass pipe, with a refractive index of glass $\tilde{n}_{glass} = 1.473$ and fluid surrounding pipe (inside the index matching prism) and inside the pipe $\tilde{n}_{fluid} = 1.344$ (see section 2.3.5 for more details). Near the top and bottom of the pipe, where the incident angles with the walls are large, the deflection will be the strongest up to $\sim 0.5\text{mm}$. With a spatial resolution of $\sim 15\text{px/mm}$ this results in deflections of $\sim 7.5\text{px}$, which would make it impossible to accurately triangulate any tracers in this region. This effect can be

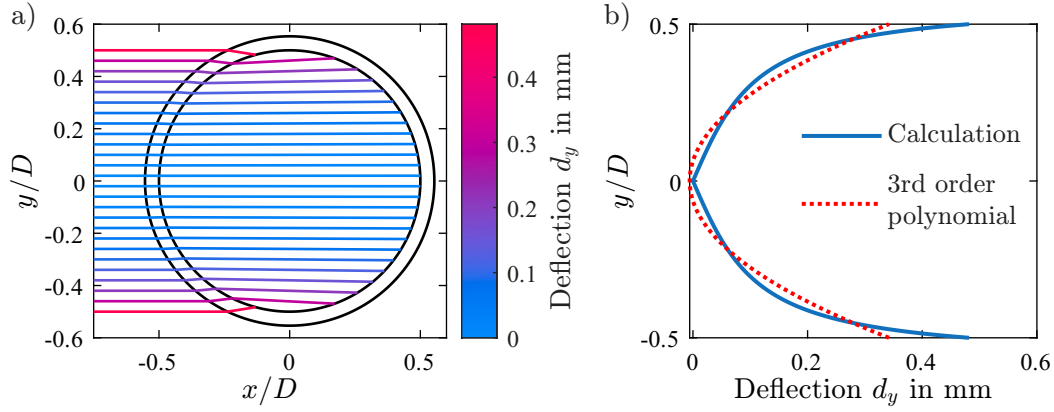


FIGURE 2.22: Effect of optical distortion in the STB setup. a) The light passing through the cross-section of the pipe, with the color indicating the amount of vertical deflection dy of each light beam. a) Vertical deflection as over the diameter of the pipe, compared to a polynomial curve used from calibration.

mostly mitigated by a finely resolved volume self-calibration procedure (Wieneke, 2008) with up to 20×20 sub-volumes over the cross section of the pipe. Although this method is limited by the camera model implemented in Davis using third-order polynomials to represent the camera distortion. While a polynomial can roughly follow the contour of the deflection (see Fig. 2.22b), it cannot follow the sharp increase near the walls, leading to some remaining disparities between the detected particle position and the calibration model (see section 2.3.4.2). The effect here is only shown for a single camera. In the real setup, the deflection of the multiple viewing angles can overlap, and further errors from reflections of the light, geometric imperfections, or vibration in the setup can cause disparities.

To quantify the actual errors of the measurement setup, the uncertainty of the tracer triangulation from the STB algorithm is evaluated. The uncertainty is calculated as the difference between the determined tracer positions and a polynomial function fitted through tracer positions in the neighboring timesteps. With this method, misidentified ghost trajectories or inaccuracies from refractions lead to a local increase of the uncertainty. This is evaluated throughout the measurement volume for an entire recording length (more than $3 \cdot 10^6$ identified tracks). For more details on this method of quantifying uncertainties in Lagrangian data, see Janke and Michaelis (2021). The spatial distribution of the uncertainties in the measurement volume is shown in Fig. 2.23 for the initial four camera setup without splitting two of the cameras. In the axial direction (Fig. 2.23a) there is very little change of the uncertainties, as the pipe walls in this direction are straight, thereby the remaining refraction will have little effect. At the ends of the measurement volume $z = \pm 2D$ the uncertainty increases. In these areas, there are few tracks identified as the field of view from the cameras only partially overlap. The regions are usually cut off during postprocessing. Compared to the radial components dx, dy the uncertainty of the streamwise coordinate dz is always smaller. This can be attributed to the refraction mainly distorting the yz -plane as shown in Fig. 2.22. In the radial direction (Fig. 2.23b) the effects of the optical distortion can be seen. In the center where all views are undistorted, the uncertainty is minimal and increases with proximity to the pipe wall, where the calibration cannot compensate for the inhomogeneous refraction. Again, the streamwise component dz is less affected.

For comparison, the uncertainty distribution with the modified six-view setup is shown in Fig. 2.24. Again, there is little visible change along streamwise direction (Fig. 2.24a). Compared to the four camera measurement, the mean uncertainty of the radial components

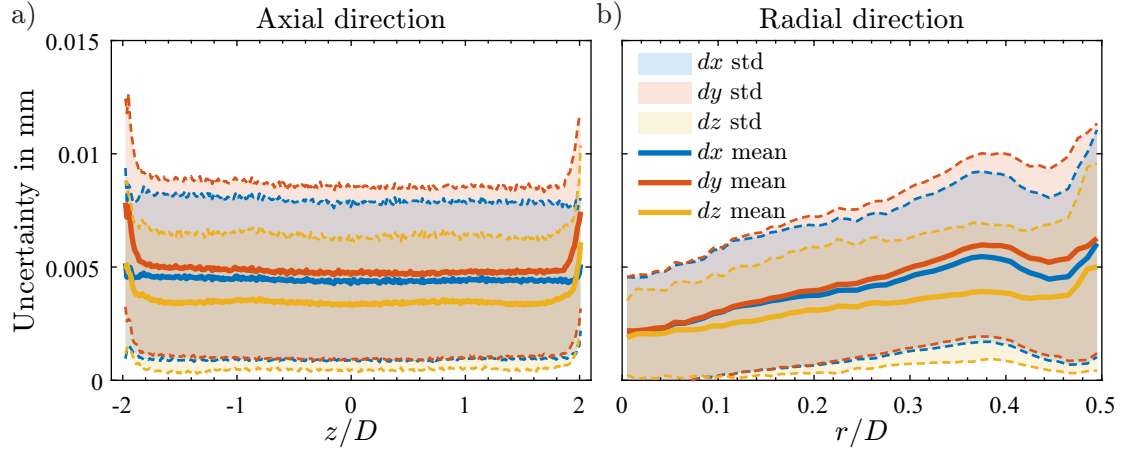


FIGURE 2.23: Uncertainty from STB particle triangulation with four camera views, along the axial coordinate a) and along the radial coordinate b).

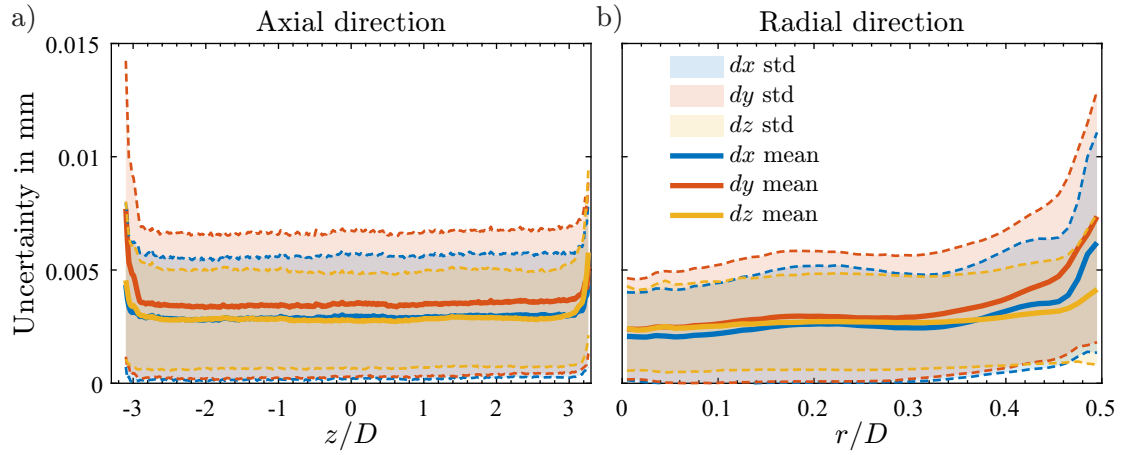


FIGURE 2.24: Uncertainty from STB particle triangulation with six camera views, along the axial coordinate a) and along the radial coordinate b).

dx, dy has been reduced by about 40% from $\sim 0.005\text{mm}$ to $\sim 0.003\text{mm}$. Now, the uncertainty of all three coordinates is of similar magnitude, as the six views provide a more accurate triangulation of each tracer without spurious matches, caused by refractions (Discetti and Astarita, 2014). This reduced uncertainty is further emphasized by the lower spatial resolution with the longer field of view in the six-view setup, where $\sim 0.003\text{mm}$ corresponds to 0.046px , compared $\sim 0.003\text{mm}$ corresponding to 0.069px in the four-view configuration. The y -coordinate always has a slightly larger uncertainty, which can be attributed to the initial plate calibration of the volume. While the plate has 6 calibration points in the x -direction it has only two different levels of dot patterns in the y -direction, which reduces the accuracy (compare Fig. 2.17)). In the radial direction the uncertainty again increases close to the wall, but remains low for a larger span of the radius (Fig. 2.24b).

When the fluid flow contains a large hydrogel particle, the refraction at the hydrogel interface will introduce further optical distortions. The effect of this on the uncertainty of the tracer triangulation can be seen in Fig. 2.25. In the axial domain of the hydrogel ($z = z_p$), the average uncertainty jumps up by about 20% on average. Similarly to the effect of the curved pipe wall, the effect is stronger closer to the edges of the particle, where the incidence angle with the particle interface and the resulting deflections are the largest. This effect will be further increased if multiple overlapping hydrogel particles are present in

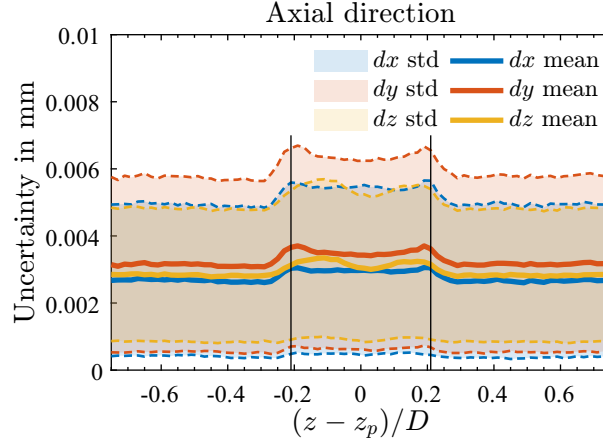


FIGURE 2.25: Uncertainty from STB particle triangulation with six camera views, along the axial coordinate in the vicinity of a hydrogel particle. The black vertical lines represent the boundaries of the particle.

the field of view.

Overall, the uncertainty of the STB measurements with values below 0.1px is in the expected accuracy range of STB results with a small amount of random noise in the images (Schanz, Gesemann, and Schröder, 2016). Therefore, the measurement data can be assumed to show accurate information of the flow dynamics. The technique is perfectly suited for detailed insights at specific flow parameters or pulsation phases of interest. For a more broad parameter scan, two additional measurement techniques will be introduced in the next two sections.

2.3.5 Efficient three-dimensional particle localization

The content presented in this section led to the publication⁴ of Bäuerlein and Avila (2021b).

The Lagrangian Particle Tracking measurements to obtain fluid and particle motion simultaneously are time-consuming to set up and evaluate, therefore, they are not suitable for large parameter scans and collecting statistics. As a result, a more efficient method for only evaluating the particle motion was developed. As both axial (Matas et al., 2004) and radial focusing (Matas, Morris, and Guazzelli, 2004; Morita, Itano, and Sugihara-Seki, 2017) of particles was reported in literature, this method needs to be capable of capturing both, under the constraints of a curved pipe wall.

For the recording of the particle motion, a single monochrome USB camera (type DMK 33UP5000 from The Imaging Source, to 2592×2048 Px) is used. In the axial direction, the field of view has a length of ~ 140 mm ($5D$). The recording rate is adapted based on the desired evaluation properties. If only particle position statistics are required, the recording rate is kept small (typically 4 Hz) to reduce the data size and processing time. To capture the particle tracks a higher recording rate is necessary (typically 10–20 Hz). Placed beneath the observed pipe section is an angled mirror, so that the camera captures the particle position from two independent and approximately orthogonal views (see Fig. 2.26). From the two views, the three-dimensional particle positions can be computed. To illuminate the particles, a LED array is positioned behind the camera. Since the light is also reflected in the mirror, the individual particles are evenly illuminated in both views. This method was tested with both, bright particles in front of a dark background, and dark particles with

⁴B. Bäuerlein developed the measurement technique, carried out the measurements, the data analysis, and wrote the first draft of the manuscript.

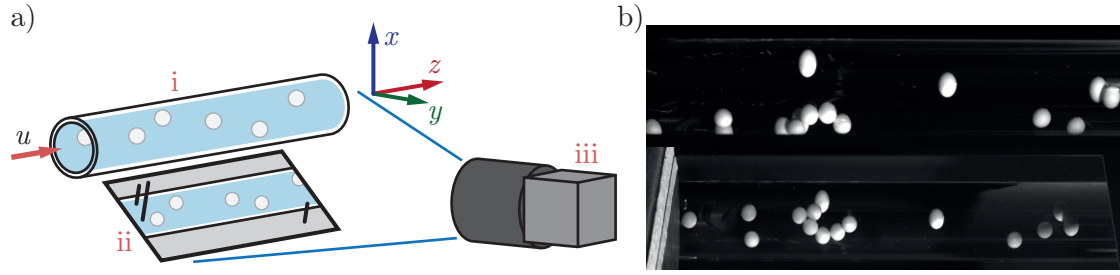


FIGURE 2.26: a) Sketch of a pipe section with the particle-laden flow (i), with the mirror underneath (ii) and the camera from the side (iii) recording both views. b) Snapshot displaying particle clusters from the direct view (upper cluster in the image) and underneath from the mirrored bottom view (lower cluster in the image).

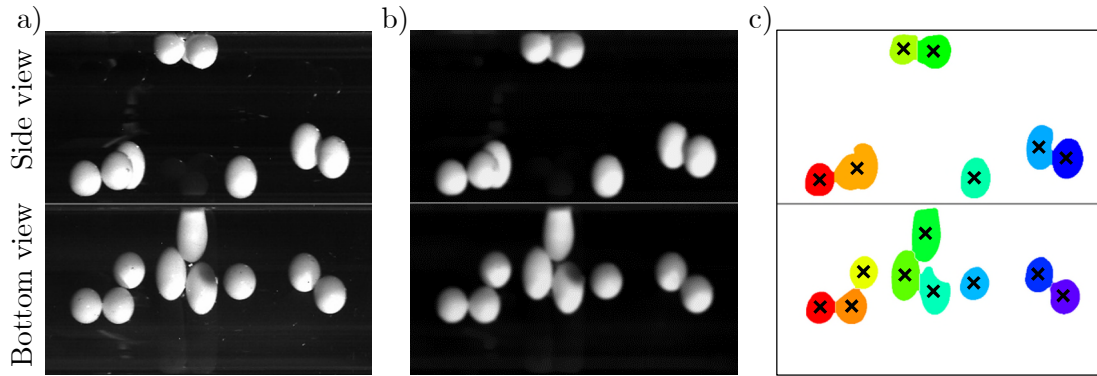


FIGURE 2.27: Preprocessing of the images from both views. a) The cropped raw image corresponding in height to the diameter of the outer wall of the pipe. b) The image after background subtraction and removal of reflections. c) The binarized image with individual particles identified as indicated by different colors. Even though many of the particle slightly overlap in the raw image they can be clearly separated in the analysis. The x-symbols mark the identified geometric center and location of each particle in the image.

a white background, and different particle diameters ($D_p = 2.8 - 5.9 \text{ mm} = 0.1 - 0.2D$). A key advantage of this optical measurement setup is its simplicity, as it only requires a single camera, a mirror, and a light source and can be easily mounted on different positions along the pipe.

In the first processing step, the raw image is cropped into the two views: directly into the pipe and through the mirror. The visible outer wall of the pipe is used for cropping and scaling the images. Note that the different magnifications necessitate separate scaling for both viewing angles (see Fig. 2.26b). In the consecutive image preprocessing steps; the contrast between particles and background is increased, to ease the identification of each particle. A Gaussian blur filter is applied to smooth out small imperfections in the images. Larger artifacts like reflection on the pipe walls or on particles themselves can be removed based on their morphology. A morphology opening in combination with large structure Top-hat transformation is used. In the resulting image, all background features are removed (see Fig. 2.27b).

From the preprocessed image, the particles can be separated from the background via a brightness thresholding. The main difficulty is to identify each individual particle, since they will often partially overlap in the image. For slightly overlapping particles, a Watershed transformation can be used. In case of strong overlap (see orange particle in Fig. 2.27c) only a single particle can be identified in both views. This limits the methods to low particle concentrations. For smaller particles ($D_p = 0.1D$) the optical distortion

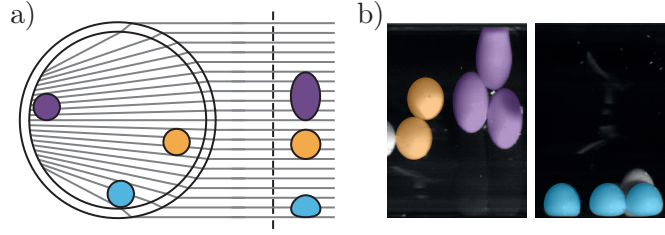


FIGURE 2.28: a) Origin of the non-uniform distortion of the particle images. The lines display the refraction of the camera view in the radial cross-section of the pipe, with the resulting deformation of the particle shape on the right. For comparison, the same deformations can be observed in the snapshots in b).

can also create blind spots close the wall, where particles cannot be observed. In the next processing step, a minimum particle size in the image is used to filter out any residual image artifacts, partially visible particles (entering or leaving the field of view), or errors from the Watershed transformation. For each remaining particle the geometric centroid is calculated and saved as the image particle position in each view. The centroid is a robust feature, even for strongly distorted particles, and allows a sub-pixel positioning accuracy.

In the previous step, the particles were located as the pixel coordinates in the images. To get the true positions, these pixel coordinates need a transformation into three-dimensional coordinate system of the pipe. This transformation is the key challenge when not using refractive index matching, as the nonuniform refraction along the radial direction makes it impossible to simply undistort the image as a whole, based on a calibration target or checkerboard pattern. The refractive index difference and round geometry causes the pipe to act like a cylindrical lens, where particles are distorted based on their position within in the cross-section of the pipe. This effect is illustrated in Fig. 2.28. Particles near the camera facing wall appear for example rather circular, whereas the ones on the opposite wall appear elongated along one axis.

To compensate for this distortion, the sight-line of each previously determined particle center of mass in the images is reconstructed. This can be achieved by calculating the refraction at both interfaces of the pipe wall, as illustrated in Fig. 2.29. For this purpose, the known material properties and geometry of the pipe flow experiment are combined with Snell's law of refraction,

$$\frac{\sin \alpha_1}{\sin \alpha_2} = \frac{\tilde{n}_{glass}}{\tilde{n}_{air}} \quad \text{and} \quad \frac{\sin \alpha_3}{\sin \alpha_4} = \frac{\tilde{n}_{fluid}}{\tilde{n}_{glass}}. \quad (2.9)$$

This experiment has the following refractive indices of the glass wall $\tilde{n}_{glass} = 1.473$, the surrounding air $\tilde{n}_{air} = 1$, and the experiment fluid $\tilde{n}_{fluid} = 1.344$ (for the water-glycerol mixture (Miessner, Lindken, and Westerweel, 2007)). Solving this equation for the change in incidence angle α provides a localized optical transfer function converting the pixel positions of images to coordinates inside the flow. For each particle a line segment (red line in Fig. 2.29a) of possible coordinates in the cross-section is determined. This transformation is only required for the radial direction, where the main distortion occurs, in the axial direction the refraction is negligible and does not require a local distortion correction.

By solving the local distortion, a range possible position for each detected particle in both camera views can be calculated. To now determine the true three-dimensional position, the information from the two orthogonal camera views is combined. The particles are paired by calculating the intersection of possible locations in the cross-section plane (see Fig. 2.29b). The algorithm determines pairings based on the minimal axial distance of two particles. False pairings, with intersection point outside of the pipe, are filtered out.

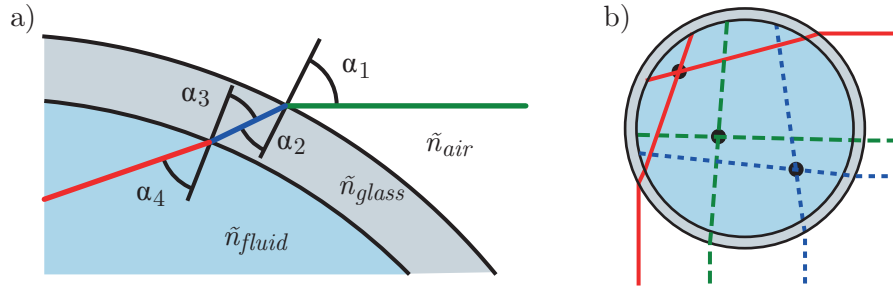


FIGURE 2.29: a) Sketch of the refraction of light at the outer and inner wall of the pipe, leading to the distorted view in the images. b) Sketch of the distortion for different vertical positions in the images. The combination of the two views reveals the true coordinates at the intersection points.

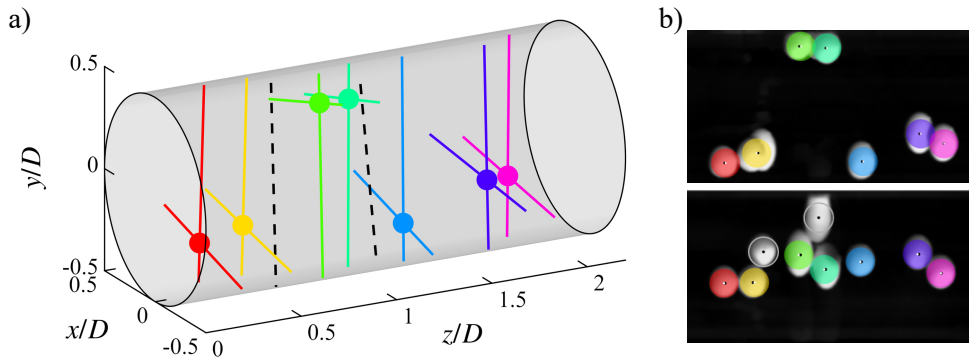


FIGURE 2.30: a) Determined particle coordinates in the recorded pipe section. The solid lines represent the distorted view directions, with the particles positions determined by the intersections. Dashed lines are particles that could not be matched. b) The particle images for the data in a) with the identified particles marked. Note that two particles visible in the bottom view are fully obstructed in the side view (at the top) and could not be paired and located.

Particles are counted as unmatched if no pair can be found, due to overlap or errors in the image processing. Fig. 2.30 displays an example of the determined coordinates for one recorded particle cluster. Unmatched particles are marked in gray.

To evaluate this method, image data with different particle concentrations is analyzed (here with $D_p = 0.1D$ particles). The remaining distance when pairing both camera views is used for quantifying the uncertainty of a detected particle, as this includes both random uncertainty from the image processing as well as systematic misalignment of the camera views. At particle concentrations up to $\phi_p = 1\%$ on average 91% of the particle can be successfully paired. For these identified particles, the uncertainty of the localization is below $0.25D_p$ (for 90% of the paired particles). When the concentration is increased to $\phi_p = 3\%$ (or up to 24 particles in the observed pipe section) the probability of overlapping particles in the images is much higher. As a result, only 64% of the particles are successfully paired and particles with similar axial position are more often mismatched. For a more robust localization at higher concentrations, the number of camera views could be increased, similar to the LPT method in the previous section. This would reduce the chance of particle overlap in multiple views and could also allow redundancy checks of the particle pair to reduce the chance of mismatches. Even if this imaging method is unable to reliably determine the positions of all particles with higher concentrations, it can still be used to tracks the instantaneous particle concentration in the flow, as the number particles identified in the images.

If the particle images are recorded at sufficiently high sampling rate, the three-dimensional location data can also be used for tracking the particles. This involves combining the determined particle position data with the given flow conditions as logged by the driving system. For each time step an expected flow velocity profile (without any particles) can be calculated, as either steady Poiseuille flow or Sexl-Womersley flow. Assuming that the particle will mainly follow the fluid flow in the axial direction, a rough prediction of a particle's position in the next time step is computed based on the flow profile. In a comparison with the recorded particle position in the next time step, minimum distances between the predicted and the actual particle cluster are identified as particle pairs. From the paired particles the trajectory and particle velocity as its time derivative can be calculated. Some preliminary results, where this method was used to evaluate the radial migration of particles in a steady pipe flow are presented in Appendix A.1.

While this particle tracking method can give insights into particle dynamics, it cannot evaluate the fluid flow surrounding the particles. For this purpose, differential pressure drop measurements (described in the next section) are implemented.

2.3.6 Pressure drop measurements

Pressure drop measurements are ideally suited to detect the onset of turbulence (Avila et al., 2011) and in a pulsatile flow they provide the instantaneous phase of the flow pressure (which differs from the driving velocity depending on the Womersley number). The friction factor is probably the most popular observable retrieved from pressure drop measurements. Its scaling in the steady laminar flow regime $\lambda_{lam} = 64/Re$ can be used to verify the determination of the fluid properties, e.g. density ρ and viscosity ν . In a pulsatile flow on the other hand, the sensitivity of pressure sensors enables the detection of undesired vibrations, stemming from e.g. air bubbles trapped within the system. Typically, pressure drop and turbulence decay (puff lifetime) measurements are regularly repeated during a measurement campaign to confirm that the fluid and sensor properties remained constant. Overall, pressure sensors are essential for accurate measurements in pipe flow experiments. They are used to calibrate and test the setup and provide important measures of the flow dynamics. In this section, the process of setting up the pressure sensors to achieve a good signal-to-noise ratio and some results from steady and pulsatile flow measurements are presented.

Although pressure drop measurements are a reliable non-intrusive technique to characterize pipe flow, they are increasingly difficult for large diameter pipes with laminar flow. The reason for this is that the pressure gradient of Hagen-Poiseuille flow (at a set Re) scales with D^{-3} . In a pipe of $D = 10$ mm the pressure drop over a length of $50D$ at $Re = 1000$ is 16 Pa, while in this pipe setup (with $D = 28$ mm) the pressure drop is only 2 Pa. The smaller the pressure drop, the more sensitive the pressure sensors have to be. In this measurement range even small oscillations in the experiment structure, the electronic noise, or other effects will influence the signal. Excellent data were obtained with the following setup (see Fig. 2.31).

A direct connection of the pressure line to the pipe flow is provided through a hole of 1 mm diameter in one of the acrylic connectors (mounted between two glass pipe segments). The hole has to be large enough to avoid an accidental blockage (e.g. by tracer particles), but small enough to not disturb the pipe flow. A tube connector threaded into the hole connects it to a flexible silicone tubing. The elasticity of the silicon allows for a more modular assembly and avoids coupling vibration from the pipe structure into the sensor. The tubing is then connected to a thin (diameter of 3 mm) and long ($\lesssim 2$ m) glass pipe, which is installed parallel to the main pipe, and ends (via another short flexible tubing) in the pressure sensor. The complete setup is flooded with the working fluid and any air

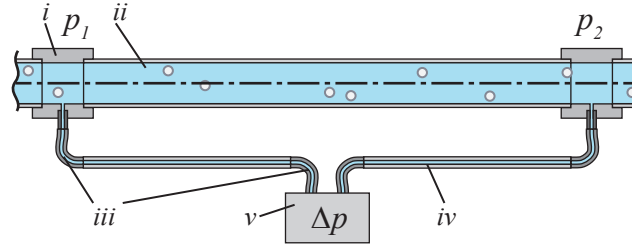


FIGURE 2.31: Sketch of the pressure measurement setup. The pressure sensor is coupled to the pipe connectors (i) in between the pipe sections (ii) via flexible tubing (iii). A small diameter glass pipe pressure line (iv) runs parallel to the main pipe. A second set of tubing (iii) connects to the sensor (v).

bubbles have to be removed. In the thin glass pipes and connective tubes, air bubbles can be easily spotted. The setup on the other side of the pressure sensor membrane is exactly mirrored, so the pressure sensor is ideally centered between the two pressure measurement holes in the pipe connectors (see Fig. 2.31). In the steady flow examples presented later in this section, the pressure drop was measured over $146D$ (4.08 m), typically for pulsatile flow a shorter length $36D$ (1.02 m) is sufficient.

Two different types of pressure sensor are used in the experiment. For preliminary measurements a pair of type HCXM010D6V and HCXM050D6V pressure sensor from Sensortech GmbH (with a pressure span of 0 Pa to 1000; 5000 Pa, respectively) allow a fast classification of the pressure magnitude. For precise measurements a DP103 differential wet-wet pressure sensor with a CD15 carrier demodulator from Validyne Engineering (with a pressure span down to ± 20.7 Pa and an accuracy of 0.25%) is ideally suited. The advantage of the Validyne sensor is the easily removable membrane (allowing different pressure ranges and measurement regimes), their long-time stability of the signals (other sensors often have a slow trend), and their easy venting of any remaining air in the pressure cavity. However, this long-time stability is only prevailed when the sensor is not overloaded. As the results the sensor has to be decoupled when not used, to avoid pressure spikes. The measurement signals are galvanically isolated from high interference devices like the motor controller, to reduce electronic noise. Special care is taken to check the pressure signals every time any additional electric device is integrated into the setup. The signal from the sensors is recorded with a National Instruments PCIe 6351 analog-digital converter at a sampling rate of 1 kHz through the same LabVIEW interface that controls the driving system. Such a time-synchronized data acquisition is required to unambiguously map phase-specific effects (like phase-locked flow instabilities) in relation to the phase of the flow driving.

2.3.6.1 Pressure measurements in steady pipe flow

To evaluate the pressure measurements, a steady single-phase pipe flow is investigated. The friction factor λ of a steady flow can be obtained from the pressure drop with Eq. (2.3). For a laminar flow this friction will follow the Hagen-Poiseuille law, and in a turbulent flow approaches the Blasius scaling law. In the experiments, the pressure drop was measured over axial distance $L_m = 146D$ (4.08 m) and is set up $164D$ (4.59 m) downstream of the pipe entrance, where the Poiseuille profile is fully developed. The fluid density was computed to $\rho = 1028 \pm 1 \text{ kg/m}^3$ based on the concentration of the water-glycerol mixture at a temperature of 21°C . The mean velocity of the fluid (and hence Re) is directly controlled by the piston drive.

The measured friction factors, depending on the Re , are plotted in Fig. 2.32a as black symbols and are in excellent agreement with the Hagen-Poiseuille law. Deviations occur at

very low $Re < 100$, because here the pressure drop is very small ($\Delta p < 1$ Pa) and minimal deviations in the pressure signal cause relatively large uncertainties in the friction factor. At $Re \approx 6100$ the flow becomes turbulent due to imperfections of the setup and the friction factor increases. Note that the natural turbulence transition occurs close to the maximum Reynolds number of the setup ($Re_{max} = 7400$) and lies well above the Reynolds number regime of interest for the transitional regime ($Re < 4000$).

In the next set of experiments, the flow is artificially perturbed by radially injecting a fluid jet (~ 5 ml over 1 second) through a hole in one of the pipe connectors, located $L = 91D$ (2.55 m) behind the pipe entrance. This jet perturbation creates a single turbulent puff in the pipe (or a slug for higher Re). When the localized turbulence enters the pipe region monitored by the pressure sensor, the pressure drop increases significantly to e.g. more than twice the value of the laminar flow at $Re = 5300$ as shown in Fig. 2.32b. When the turbulent structure exits the measurement section the pressure level drops back to the laminar pressure. For $Re \gtrsim 2000$ the jet injection triggers puffs and slugs, which are visible in the increased friction factor (plotted as orange circles in Fig. 2.32 a). The measurements are still below the friction of the Blasius scaling, because the very long measurement region still contains long stretches of laminar flow (only a single puff was triggered here).

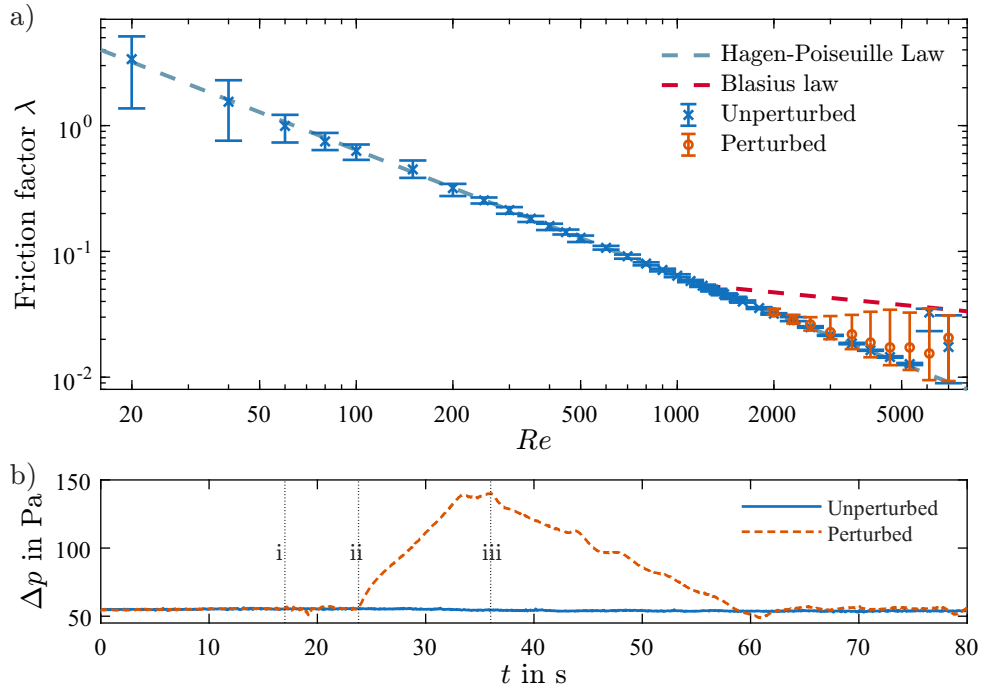


FIGURE 2.32: a) Measured friction coefficient in steady pipe flow in comparison to the laminar solution of Hagen-Poiseuille and the transitional regime with Blasius. $\circ$ symbols are measurements where an external perturbation via small fluid jet injection was given to the flow to induce the formation of turbulent puffs. For the $\times$ symbols no external perturbation was applied. The error bars represent the maximum and minimum pressure deviation from the mean values of the markers. b) Example excerpt of a pressure signal with and without a perturbation at $t = 17$ s for $Re = 5300$. After short time the turbulent structure enters the measurement section leading to a sharp rise in the pressure drop. As the turbulence exits the section the pressure drops back down to the laminar level.

2.3.6.2 Pressure measurements in pulsatile pipe flow

One characteristic of pulsatile flow is the Womersley-dependent phase lag between the mean volume flux (as produced by the driving system) and the measured pressure gradient on

the pipe. An example of this for a harmonic pulsation is displayed in Fig. 2.33a,c. Since both the pressure signal and the feedback velocity from the driving system are captured on the same laboratory computer, the phase difference can be simply calculated by comparing both signals in Fourier space using FFT.

Also noteworthy is the much larger magnitude of the dynamic pressure of the pulsatile flow, compared to the mean level of the steady flow, scaling with Wo^2 . This increased dynamic pressure requires specific selection of the measuring pipe length and the pressure range of the sensor to accurately capture the signal of the full pulsation period. In pulsatile flow with multiple harmonic components, such as an aorta-like pulsation in Fig. 2.33b,d, the Wo^2 scaling results in the pressure waveform deviating from the driving velocity waveform.

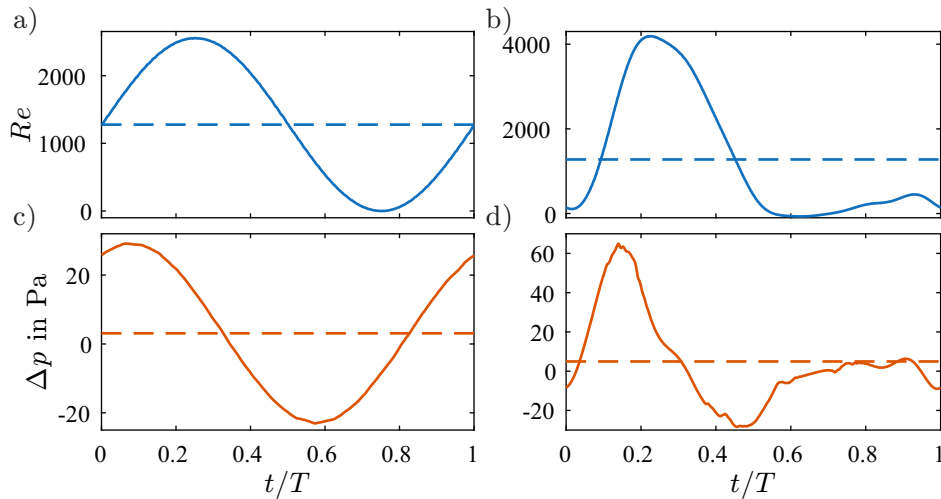


FIGURE 2.33: Pressure drop measurements for harmonic pulsatile flow ($Re_m = 1277$; $Wo = 8$; $A = 1$), with the bulk flow velocity in a) and the recorded pressure drop (over $36D$) in c). Measurements of an aorta-like pulsation waveform ($Re_m = 1277$; $Wo = 4$), with the bulk velocity in b) and the pressure drop in d). The dashed colored lines represent the time averaged mean value.

Additionally, the larger pressure oscillations can excite structural vibrations in many components of the setup, such as the tubing connection to the piston and the walls of the outlet container. And any remaining small compressible air pockets in the system (for example in between the O-rings at pipe connectors) will start to oscillate under dynamic pressure. These effects become resonant at frequencies around 1.5 Hz ($Wo = 37$), leading to additional flow oscillations when the fluid acceleration suddenly changes. In general, this frequency is outside the main Womersley number regime of interest, but for specific waveforms with multiple harmonics, the higher frequency modes can reach into this resonant regime and interact with the oscillations of the setup. In such cases, the actual flow velocity in the pipe will vary compared to the driving signals given by the motor. In addition to the dynamics of the setup, the characteristics of the pressure sensor itself are also an important aspect to consider. Although the Validyne pressure transducer has a very high accuracy for capturing small scale flow dynamics, the connective tubing and the large membrane inside the sensor result in a limited response frequency. Above this limit frequency, the transducer acts like a low-pass filter. This becomes important since this low-pass characteristic can affect frequencies as low as 1 Hz ($Wo = 30$), which complicates the detection of resonant oscillations from the setup in the pressure signals. Therefore, during pulsations with high frequencies or multiple harmonics, the pressure signal alone is not sufficient to check if the flow inside the pipe is generated as driven by the motor. In these cases, other measurements like Shake-The-Box are required for validation. A secondary dynamic of the pressure sensor

comes from the deformation of the membrane inside the transducer. For large velocity amplitudes at higher frequencies this can cause some fluid volume being pulled and pushed by the transducer. With the small opening diameter towards the pipe, this can induce a small jet disrupting the pipe flow. For these reasons, special caution is necessary when using the pressure sensor in pulsatile flow.

All three measurement methods presented in this section (STB, particle localization, and pressure measurement) can be used in tandem to evaluate different properties of the flow. When examining complex dynamics, such as the turbulence transition in particle-laden flow, a combination of all measurement techniques is necessary. This will be shown in the next section.

2.4 Turbulence transition

This section presents the results of the experimental investigation into the transitions to turbulence in pipe flow, with an emphasis on how pulsatile driving, particle suspension, and geometric perturbations influence the onset and nature of the transition. I begin by establishing the reference cases of steady single-phase and steady particle-laden pipe flow (section 2.4.1). Building on this foundation, the second part explores how a pulsatile forcing alters the transition characteristics of particle-laden flows (section 2.4.2). This includes systematic investigation of the pulsation frequency (through the Womersley number), with both harmonic and more complex physiological pulsation waveforms. To characterize the influence of the perturbation mechanism, the pulsatile flow is also examined with a geometric flow perturbation of a bend (section 2.4.3). The results conclude with a summary and discussion of the transition mechanisms (section 2.4.4).

2.4.1 Steady flow

For later investigation of pulsatile pipe flow, it is first necessary to examine how turbulence emerges in the transitional regime under steady flow conditions. This includes examination of the flow structure of puffs and slugs, as the intermitted turbulence patches in pipe flow (section 2.4.1.1). Afterwards, the transition of steady particle-laden flow is investigated for different Reynolds numbers, to determine when particles trigger turbulent patches (section 2.4.1.2).

2.4.1.1 Structure of puffs and slugs in single-phase flow

The dynamics of puffs and slugs in single-phase flow have been studied in detail Darbyshire and Mullin (1995), De Lozar and Hof (2009), Song et al. (2017), Van Doorne and Westerweel (2009), and Wygnanski and Champagne (1973, e.g.), but so far experiments were only able to extrapolate their structure using stereoscopic two-dimensional PIV measurements Hof et al. (2004) and Van Doorne and Westerweel (2009). In this study, Shake-The-Box recordings are able to capture the structures in three-dimensional flow measurements. To trigger a puff or slug, an impulsive jet injection was used to perturb the flow $\sim 200D$ upstream of the measurement position.

In the first measurement, a localized turbulent puff (at $Re = 2000$) is captured. The axial length of the field of view provides a very good spatial resolution but is too small to fully capture a turbulent puff structure with a typical length of about $> 10D$. Therefore, a part of a puff is presented in Fig. 2.34a. The typical arrowhead shape can already be seen in the vorticity surfaces. To illustrate the full shape of the structure, the stream-wise vorticity is displayed in Fig. 2.34c. Here Taylor's hypothesis is used with the time series of only a single cross-section of the velocity field to extend the field of view to a length of $30D$ (same as realized for Stereo PIV data by Doorne and Westerweel (2007)). At the upstream end of puff the vortices are only localized close to the wall. Traveling downstream the vortices are lifted towards the center of the pipe flow and smaller scale turbulence appears. Facing the downstream end of the puff, the structures are larger and extend into the center of flow. This picture is consistent with other experimental depictions of turbulent puffs Van Doorne and Westerweel (2009).

Turbulent slug structures develop at $Re \gtrsim 2700$ and unlike puffs are able to continuously grow over time. In Fig. 2.34b the first three-dimensional measurement of a part of a slug and in Fig. 2.34d the 3D reconstructions of a slug are illustrated for $Re = 3000$. Note that the length of the slug $> 70D$ exceeds the maximum measurement duration of the STB measurement, therefore, also the reconstruction only shows part of the slug. At the downstream front of the depicted slug, the vorticity is larger compared to its core. This

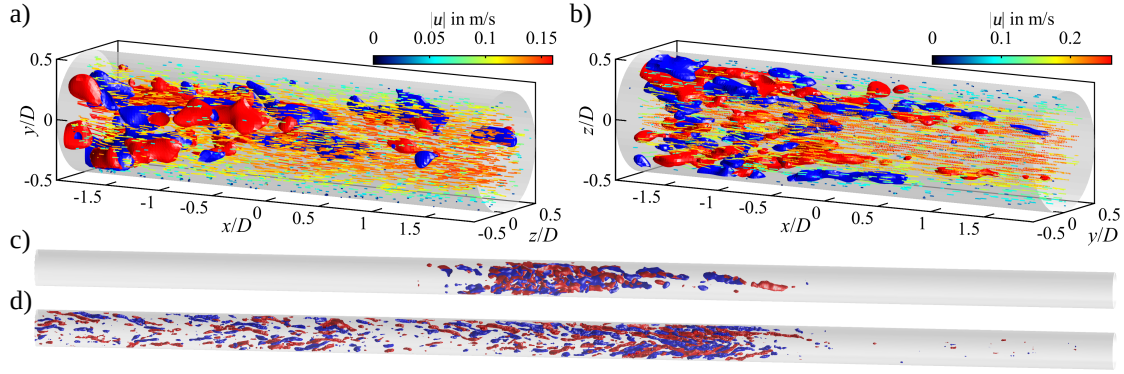


FIGURE 2.34: Section of a localized turbulent puff a) with $Re = 2000$ and a slug b) with $Re = 3000$. The trajectories from STB span over [25;17] ms respectively. The red/blue structures are isosurfaces of stream-wise vorticity $\omega_x = \pm 1.5U/D$. Taylor's hypthesis is used for a single cross-section of the velocity field, to increase the length of the field of view. The resulting full structure of puff is shown in c) and the downstream onset of the slug in d).

indicates a strong front, where turbulence is produced and flows into the core (Song et al., 2017). In a weak front, the vorticity would continuously decay with a structure more similar to the puff. It can also be observed that the turbulent vortices of the slug are much smaller compared to a puff, indicative of the higher Reynolds number.

As the formation of puffs and slugs will also appear in pulsatile pipe flow (Xu and Avila, 2018; Xu et al., 2017), the description of their structure from this section can be used to characterize the new findings of particle-laden turbulence in pulsatile flow in section 2.4.2.

2.4.1.2 Early turbulence onset in particle-laden flow

Before evaluating the dynamics of a pulsatile pipe flow, it is necessary to further investigate the case of particle-laden steady flow. While previous studies have examined the turbulence transition for various particle diameters and concentrations in the flow using an orifice as a controlled geometric perturbation (Agrawal, Choueiri, and Hof, 2019; Hogendoorn and Poelma, 2018; Matas, Morris, and Guazzelli, 2003) (similar to (Wynanski and Champagne, 1973)), in this section, the effects of the flow being only perturbed by the particles themselves are investigated. In steady flow without particles, different perturbations will always trigger localized puffs with the same flow structure (De Lozar and Hof, 2009; Mukund and Hof, 2018). However, in pulsatile flow, specific perturbations can trigger phenomena such as the helical instability at certain phases of the pulsation (Feldmann, Morón, and Avila, 2021; Xu et al., 2020). Therefore, it is crucial to differentiate how the flow is perturbed, as these observations in steady flow serve as a basis of the later understanding of pulsatile flow.

To evaluate the effects of the particles, the differential pressure drop Δp over a pipe segment with a length of $L_m = 36D$ is measured at an axial location of $230D$ downstream from the inlet (see section 2.3.6). Fig. 2.35a illustrates the friction factor λ (computed with Eq. (2.3)) as a function of Re . Without an external perturbation of particles, the flow in these experiments naturally turns turbulent at $Re = 4500$ ⁵. The particle-laden measurements are performed with $D_p = 0.1D$ diameter particles at a volume fraction of $\phi_p \approx 1\%$. For small Re , the particle-laden flow follows the Hagen-Poiseuille law, indicating fully laminar flow. It is important to note that for the large diameter of this pipe, the laminar pressure drop is extremely small ($\Delta p \approx 4.2$ Pa at $Re = 1000$) and the apparent offset between the

⁵In section 2.3.6 a different natural transition of $Re \approx 6100$ was reported. For these measurements with ($D_p = 0.1D$) particles the viscosity is ~ 1.3 times larger. It is suspected that the transition of this setup depends on the flow velocity which was $U \approx 0.26$ m/s in both measurements.

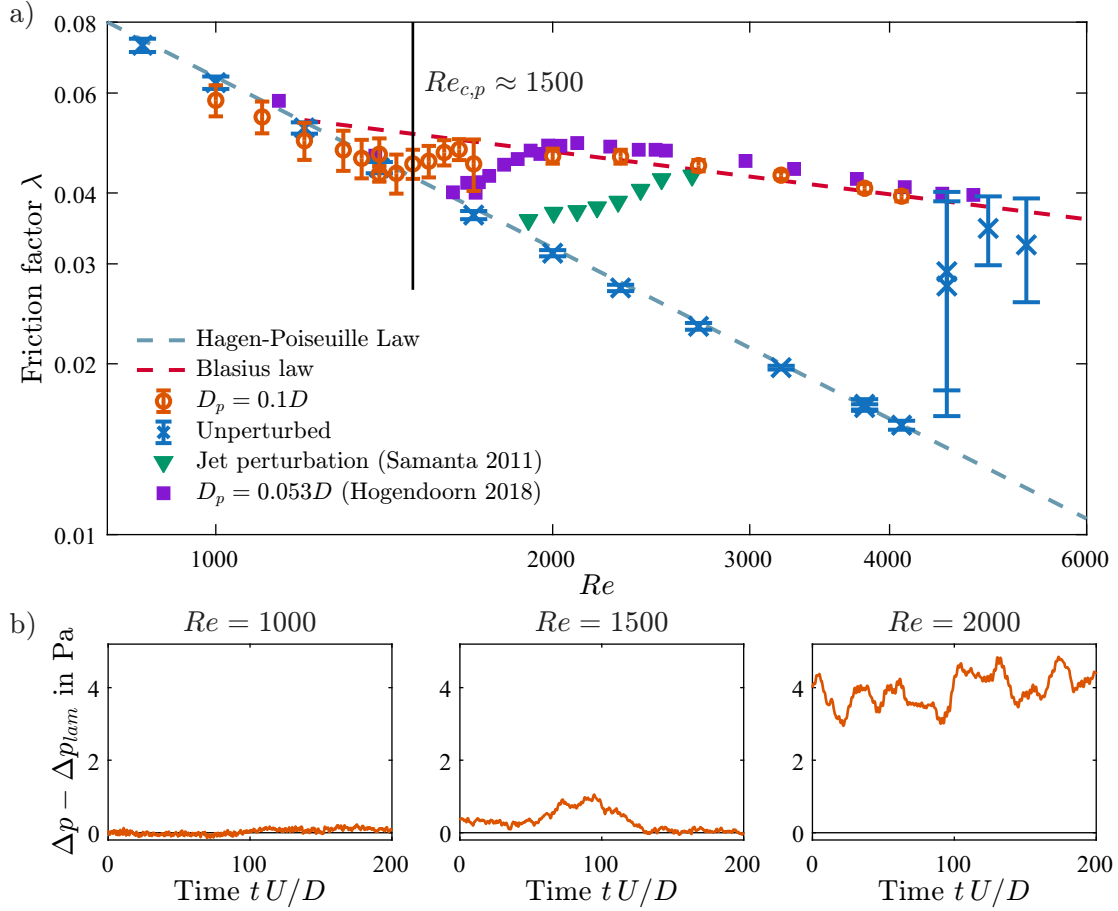


FIGURE 2.35: a) Friction factor λ as function of Re for particle-laden flow $D_p = 0.1D$; $\phi_p \approx 1\%$ (orange $\circ$) compared to unperturbed flow (blue $\times$). The blue line represents the Hagen-Poiseuille law with $64/Re$ and the black line the Blasius approximation with $0.316 Re^{-1/4}$. The errorbars indicate the recorded fluctuations in the pressure signal. For comparison, the green markers (∇) represent measurements with a jet injection perturbation by Samanta, De Lozar, and Hof (2011). The purple markers ($\square$) correspond to particle-laden flow measurements at the same concentration with slightly smaller particles $D_p = 0.053D$ by Hogendoorn and Poelma (2018). b) Timeseries of the pressure drop of particle-laden flow compared to laminar flow for three different Re .

particle measurement and the theoretical curve is within the measurement accuracy. As the Reynolds number surpasses $Re = 1500$, the particles start to trigger localized turbulent puffs, as shown in Fig. 2.35b. This corresponds to the lowest Reynolds number where puffs could be detected in a single-phase flow Hof et al. (2005). In this regime the puffs are still very weak, only elevating the pressure by about 1 Pa, therefore it is difficult to determine an exact threshold Re , where puffs are triggered. The most effective trigger in single-phase flow is an impulsive jet perturbation (indicated by ∇ with measurements by Samanta, De Lozar, and Hof (2011)). It can be concluded that particles by themselves similarly act as an ideal perturbation to flow, resulting in a subcritical transition to turbulence. At $Re \geq 2000$ the flow turns fully turbulent, indicated by the friction factor reaching the Blasius curve. In this regime, the flow already develops turbulent patches that grow into slugs, before the driving system has fully accelerated to the steady velocity, and no laminar intermittency is present (shown in Fig. 2.35b) with $Re = 2000$). For comparison, the diagram in Fig. 2.35a also includes friction factor measurements from Hogendoorn and Poelma (2018) using the same concentration ($\phi_p = 1\%$) of smaller particles ($D_p = 0.053D$). In their experiments,

the turbulence onset appears slightly more delayed ($Re \approx 1700$), which matches the effect seen in previous studies of larger particles further accelerating the turbulence transition (Hogendoorn, Chandra, and Poelma, 2022; Matas, Morris, and Guazzelli, 2003). It is noteworthy that even for the laminar particle-laden flow ($Re < 1500$) the fluctuations in the pressure drop are much larger than in the flow without particles. This can indicate that the flow profile is already significantly perturbed, although no turbulence is triggered.

Having analyzed how turbulence emerges in a steady flow, the observation of particle-laden pulsatile flow in the next section can be accurately compared.

2.4.2 Pulsatile particle-laden flow

As is known from the literature, even a small concentration of particles can drastically reduce the onset of turbulence in steady flow (Agrawal, Choueiri, and Hof, 2019; Hogendoorn and Poelma, 2018). In the human cardiovascular system, the flow is often considered to be laminar, based on the simplified assumption that the averaged Reynolds numbers are below the single-phase steady flow transition threshold of $Re_c = 2040$. Previous studies demonstrated that the transition scenario will be greatly affected by the Womersley number and amplitude (Xu and Avila, 2018; Xu et al., 2020). Specifically, the recently discovered helical instability can produce localized turbulence for Reynolds numbers as low as $Re = 800$. However, the pulsatile turbulence transition with a perturbation caused by particles has not been investigated to date. A recent study by Morón, Feldmann, and Avila (2022) indicates a stronger growth of the helical instability for waveforms with rapid deceleration followed by a long low velocity phase, as for example present in larger blood vessels. Specifically, this case will be investigated in this study, using a physiological waveform modeled after the flow in the human aorta (same waveform parameters as Morón, Feldmann, and Avila (2022)).

The following section begins by introducing the physiological aorta-like waveform and the corresponding flow profile (section 2.4.2.1). Afterwards a systematic parameter study of the effect of the Womersley number and pulsation waveform for a particle-laden pulsatile flow is presented (section 2.4.2.2). The resulting turbulence characteristics are subsequently compared to steady flow dynamics of the particle-laden flow (section 2.4.2.3). To evaluate the interactions of particles and fluid leading to development of turbulence patches, STB flow measurements are performed using two different particle sizes (sections 2.4.2.4 and 2.4.2.5).

Some of the measurements of particle-laden flow with a physiological waveform in section 2.4.2.2 were performed as part of the master's thesis by Larissa Richter (2024).

2.4.2.1 Characteristics of the physiological flow waveform

For the main investigation in this study the pulsatile flow is realized with a physiological waveform, see Fig. 2.36a (calculated with Eq. (2.5) with Fourier coefficients from Table 2.2). Due to the proximity to the heart, this waveform shows rapid acceleration ($t/T = 0 - 0.2$) and deceleration phase ($t/T = 0.2 - 0.5$) with a large peak amplitude $A = \hat{u}/U - 1 \approx 2.3$, followed by a long low-velocity or resting phase ($t/T = 0.5 - 1$) with a bulk flow velocity close to zero. For the human aorta, the typical average Reynolds numbers Re_m range from $Re_m = 700 - 1500$ ($\hat{Re} \approx 2300 - 5000$) and Womersley numbers from $Wo = 5 - 18$ (Les et al., 2010; Stalder et al., 2011), with the highest Re and Wo closest to the heart. This study is focused on $Re_m = 1277$, corresponding to the descending aorta, where the blood vessel has the closest resemblance to a straight pipe (compare Fig. 1.6b).

To characterize the flow, the instantaneous velocity profiles for $Wo = 3 - 8$ are shown in Fig. 2.36b. For smaller Wo the profile is close to parabolic during the velocity peak

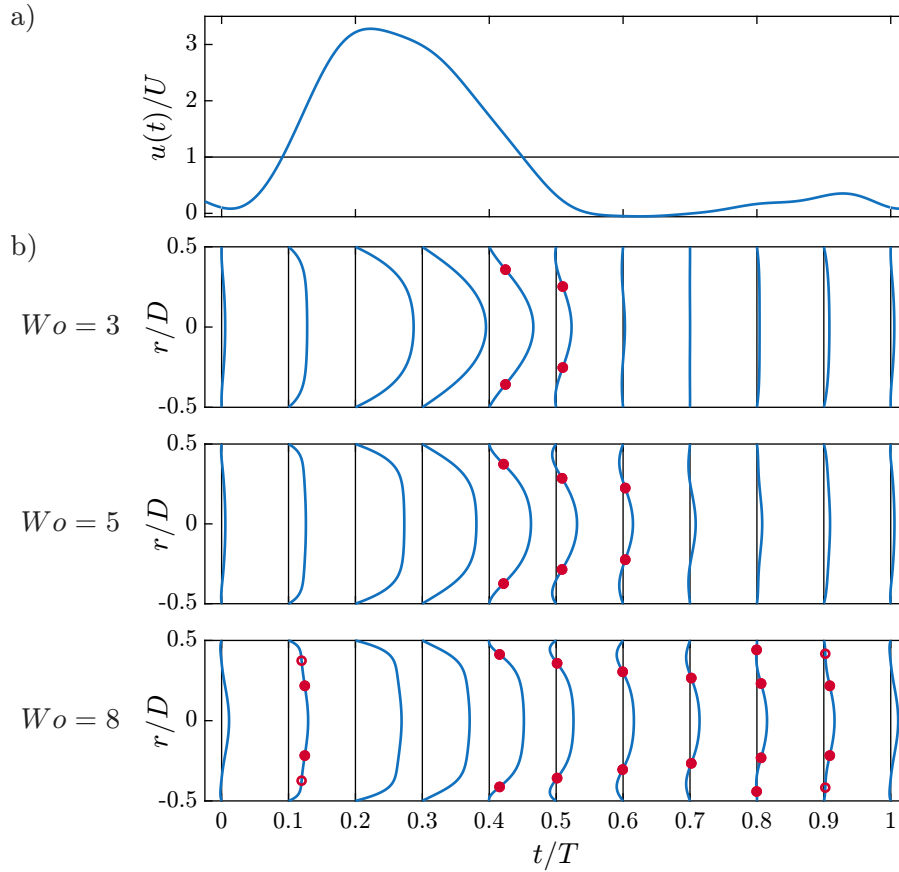


FIGURE 2.36: a) Timeseries of bulk velocity the physiological waveform modeled after the descending aorta flow. b) Instantaneous velocity profiles over one pulsation period for $Wo = [3; 5; 8]$. The red circles represent inflection points in the profile with empty circles indicating stable inflection points (not fulfilling the Fjrtoft criterion), according to Morn, Feldmann, and Avila (2022).

($t \approx 0.3T$), with nearly zero velocity during the resting phase. As Wo is increased, the profile becomes more flattened, and during deceleration a pronounced flow reversal near the walls develops. With increasing Wo the profile also contains more inflection points for longer phases of the period, making the flow more susceptible to the growth of instabilities. Linear stability analysis by Morn, Feldmann, and Avila (2022) predicts the strongest growth of a helical instability in the $Wo = 7 - 8$ regime for this waveform. In my study, I will investigate how this change of the Womersley number affects the turbulence transition, in experiments with a particle-laden flow.

2.4.2.2 Womersley dependent turbulence transition

In this section, I present results on the effect of the Womersley number on the turbulence onset in a pulsatile pipe flow perturbed by suspended particles. To systematically investigate the onset of turbulent flow, the pressure drop of the perturbed flow is analyzed and compared with an unperturbed flow (similar to the investigations in steady flow in section 2.4.1.2). The differential pressure drop Δp is measured over a pipe segment with a length of $L_m = 36D$ at an axial location of $230D$ downstream from the inlet for fully developed flow (see section 2.3.6). The instantaneous particle concentration ϕ_p is tracked using camera imaging (see section 2.3.5). Since the concentration will naturally fluctuate over time, the results are presented based on the average particle concentration during each individual measured

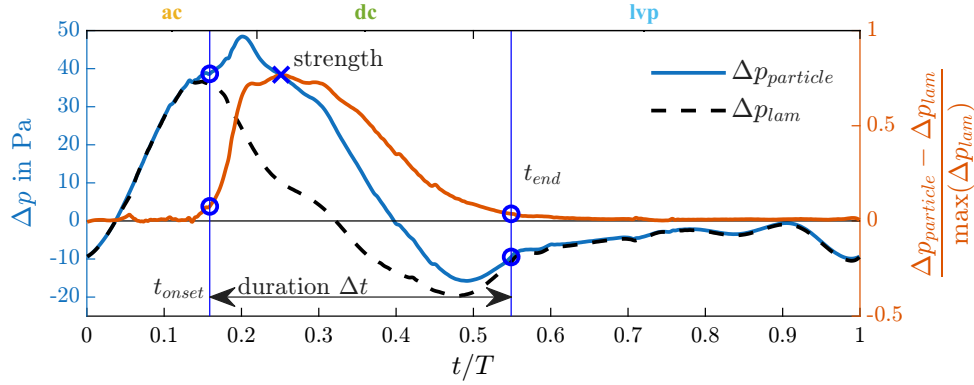


FIGURE 2.37: Time series of the measured pressure drop Δp for a physiological pulsation at $Re_m = 1277$ and $Wo = 3$ with a perturbation of a particle suspension ($D_p/D = 0.1$; $\phi_p \approx 1\%$) in blue compared to unperturbed laminar flow in black. The difference between both curves is shown in orange. The turbulence onset (blue circle) can be detected in the acceleration phase ($t_{onset} \approx 0.15T$) where the perturbed signal starts to deviate from the laminar case. In the low velocity phase the turbulence decays and the flow relaminarizes ($t_{ending} \approx 0.55T$, blue circle). The strength of the turbulence is defined as the maximum of the difference normalized by the maximum laminar pressure.

period.

A representative comparison of a pressure drop measurement is shown in Fig. 2.37. During each period of the pulsation, a turbulent patch is triggered, grows, and decays back to a laminar flow. The onset of irregular flow can be observed, where the pressure signal of the perturbed flow starts to deviate from the laminar baseline, occurring here while the instantaneous Reynolds number is still accelerating (ac) at $t_{onset} \approx 0.15T$. During the deceleration phase (dc), the intensity of the turbulence patch peaks, indicated by the maximum pressure difference between the laminar and perturbed flow. As the pressure drop in Sestl-Womersley flow increases with Wo^2 , the intensity of the pressure difference has to be normalized. For this purpose, I define the normalized difference as,

$$p_{diff} = \frac{\Delta p_{perturbed} - \Delta p_{lam}}{\max(\Delta p_{lam})}, \quad (2.10)$$

with peak of p_{diff} indicating the strength of a measured turbulent patch. In the following deceleration and low velocity phase the flow relaminarizes with the pressure difference returning to zero at $t_{ending} \approx 0.55T$. In the following period, a new turbulent structure develops during the next acceleration phase, and the cycle of turbulence and relaminarization repeats itself.

This analysis of the turbulence patches was extended to a range of Womersley numbers ($Wo = [3 - 7]$) for $Re_m = 1277$ (see Fig. 2.38a). For all shown Wo , the same pattern of intermittent turbulent patches and relaminarization is observed in each period. The strongest pressure fluctuations relative to the laminar pressure were measured at the lowest $Wo = 3$. Here, turbulent structures arise early in the acceleration phase ($t/T = 0.15$) and decay during the deceleration and low-velocity phases. As Wo is increased, the pulsation period shortens, resulting in continuously weaker turbulence patches for $Wo = 4$ to $Wo = 7$. At $Wo > 7$ turbulent patches become too weak to be reliably detected, as the difference between the laminar pressure drop and perturbed flow falls below 5 Pa and any patches of irregular flow fade into the measurement noise. In addition, for $Wo > 8$ a noticeable higher-frequency ripple appears in the pressure signals. Here, some higher harmonics of

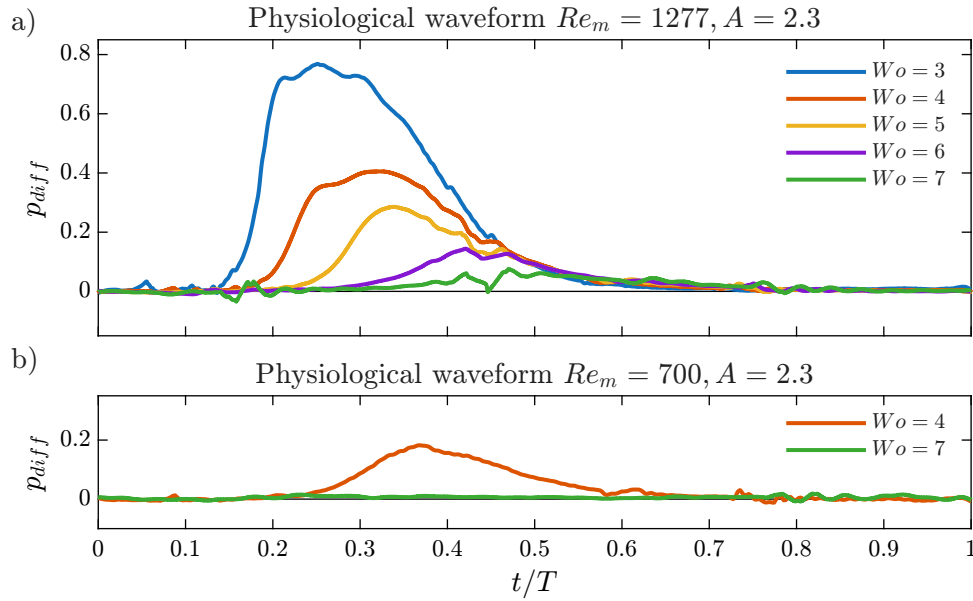


FIGURE 2.38: Time series of pressure drop difference between perturbed and laminar flow (see Fig. 2.37) of a physiological pulsation for different Womersley numbers with a particle suspension ($D_p/D = 0.1, \phi_p \approx 1\%$) at $Re_m = 1277$ in a) $Re_m = 700$ in b). The curves are averaged over multiple individual pulsation periods.

the waveform start to excite slight vibrations in the experiment (see section 2.3.6), thus limiting the range of observable Wo for this waveform. If the mean Reynolds number of the pulsation is decreased to $Re_m = 700$ (see Fig. 2.38b), the turbulence patches become weaker and also appear further delayed in the phase of the pulsation (compare $Wo = 4$). For larger $Wo = 7$, where still some turbulence was detectable in the higher Re_m measurements, the flow stays laminar throughout the period at $Re_m = 700$. For none of the investigated Wo , is the turbulence sustained for an entire pulsation period and always fully dissipates during the low velocity phase.

These observations already indicate some clear differences from previous studies of intermittent turbulence triggered from the helical instability of the flow profile. For the development of helical flow, the expected growth and thereby strength of turbulence patches would be highest at $Wo = 7-8$ (Morón, Feldmann, and Avila, 2022). To investigate whether these turbulence dynamics are specific to the physiological waveform, the measurements are repeated with a harmonic waveform. For the harmonic pulsation, a mean Reynolds number of $Re_m = 2000$ and amplitude of $A = 1$ was selected, to achieve similar properties to the physiological case of $Re_m = 1277$, with a peak Reynolds $\hat{Re} \approx 4000$ and $Re \approx 0$ in the low-velocity phase. Compared to the physiological case, the acceleration and deceleration phases are much longer, which should result in a more stable flow profile. According to linear stability analysis by Morón, Feldmann, and Avila (2022), steeper acceleration and deceleration ramps, lead to longer phase of unstable flow and stronger growth of instabilities.

The same analysis of subtracting perturbed pressure drop measurements from laminar measurements is performed, with the results shown in Fig. 2.40. Here, the same dynamic as for the physiological pulsation can be observed. For the lowest $Wo = 4$ the turbulence patches have the earliest onset and largest amplitude, with a delayed onset and decreasing strength at higher Wo . Compared to the physiological waveform (see Fig. 2.38), the turbulence patches are much stronger for the same Wo . The acceleration peaks of the harmonic wave are much longer ($Re > Re_m$ for $0.5T$ with a harmonic pulsation, compared to $0.36T$ with the physiological wave). Thereby more kinetic energy is fed into the flow

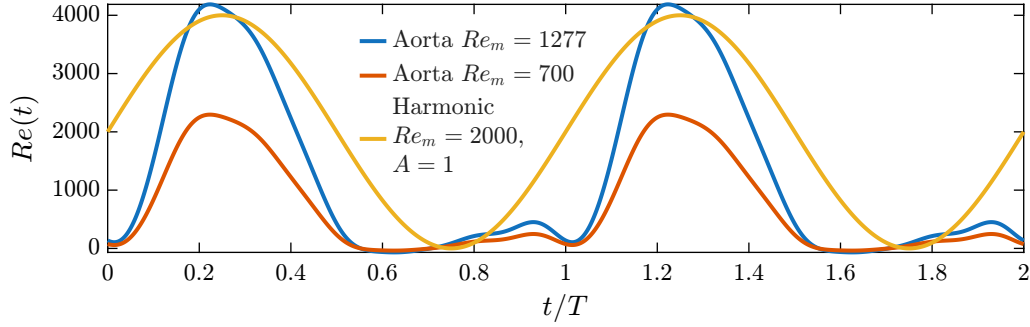


FIGURE 2.39: Comparison between a large amplitude $A = 2.3$ physiological waveform with $Re_m = 700$ and $Re_m = 1277$ and a harmonic pulsation $Re_m = 2000$, $A = 1$.

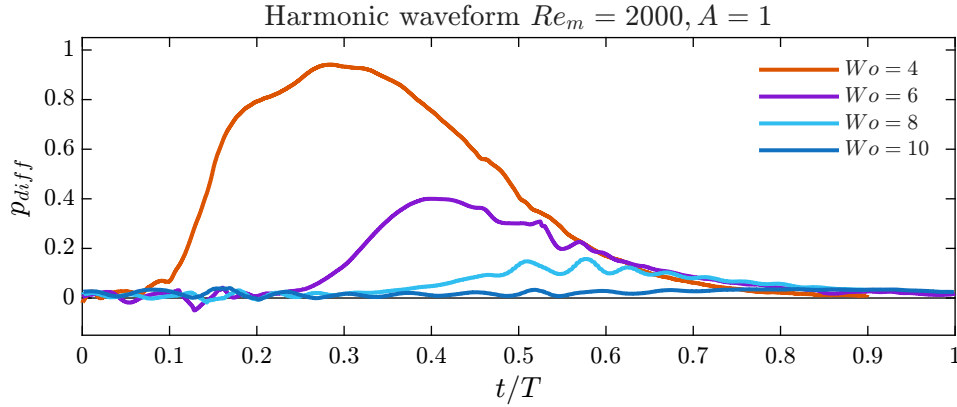


FIGURE 2.40: Time series of pressure drop difference between perturbed and laminar flow of a harmonic pulsation for different Womersley numbers with a particle suspension ($D_p/D = 0.1$, $\phi_p \approx 1\%$) at $Re_m = 2000$, $A = 1$. The curves are averaged over multiple individual pulsation periods.

and stronger turbulence patches can grow.

To directly compare the temporal evolution of the detected turbulent patches, the onset and ending time are shown in Fig. 2.41a. With the physiological pulsation at $Re_m = 1277$ the lowest $Wo = 3$ shows the earliest turbulence onset, while the pulsation is still accelerating ($t_{onset}/T \approx 0.15$). The flow relaminarizes towards the end of the deceleration ($t_{ending}/T \approx 0.5$). As previously shown in Fig. 2.38, with increasing Womersley numbers, the onset is continuously delayed. At the larger Wo the turbulence patch appears during the deceleration phase, and the flow stays turbulent well into the low velocity phase, where the instantaneous Reynolds number is close to zero. For a lower $Re_m = 700$ at $Wo = 4$ both onset and relaminarization are delayed. With the harmonic pulsation ($Re_m = 2000$, $Wo = 4$), the onset appears earlier and the ending later in phase, with a similar trend of delayed turbulence for increasing Wo .

When evaluating the turbulence duration relative to the period length (time between t_{onset} and t_{ending} in Fig. 2.41a), a slight increase from $\Delta t \approx 0.35T$ at $Wo = 3$ to $\Delta t \approx 0.5T$ at $Wo = 7$ can be seen for the physiological pulsation. However, at the highest $Wo = 7$ the onset and ending time scatters more as the smaller pressure difference is more affected by measurement noise (compare Fig. 2.38). For the harmonic waveform, the relative turbulence duration stays approximately constant at $\Delta t \approx 0.6T$. The physiological waveform seems to have overall little difference in comparison to a harmonic pulsation, with both waveforms showing similar trends of a approximately fixed turbulence duration $\Delta t/T$ over Wo . For both waveforms, the turbulence onset appears to continuously delayed for higher Wo (This

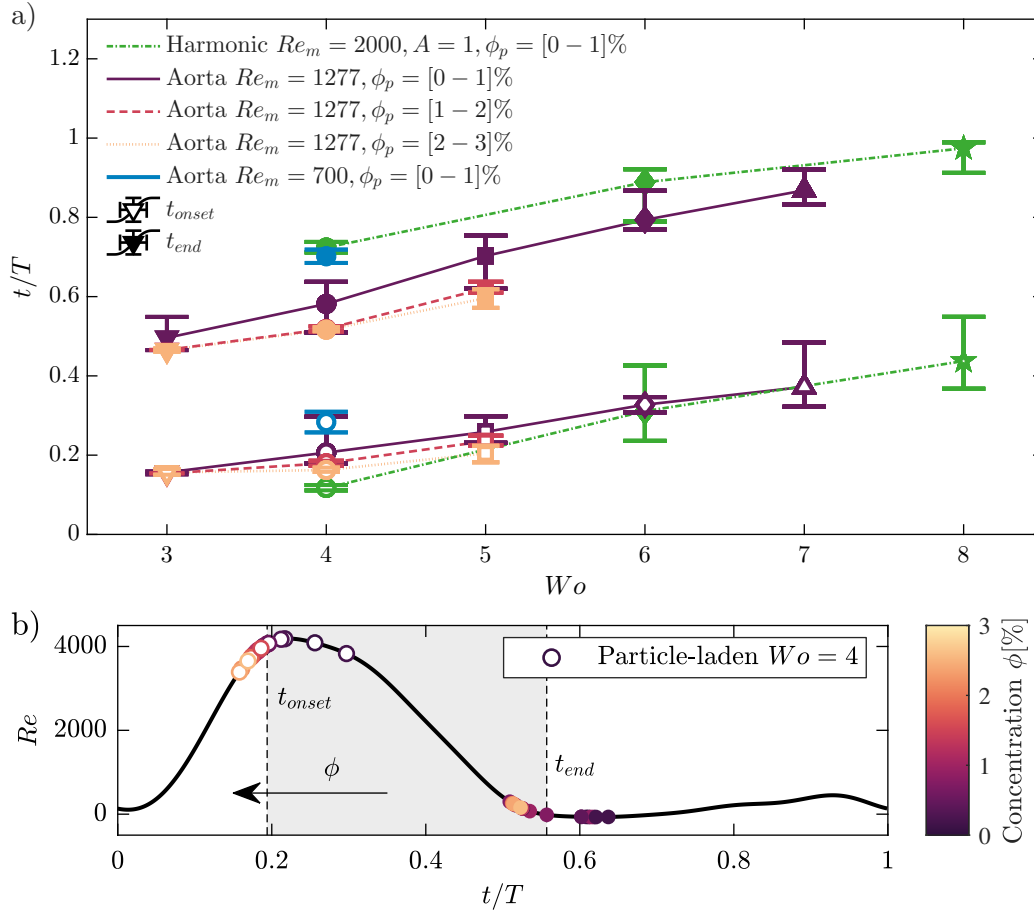


FIGURE 2.41: a) Averaged turbulence onset (empty symbols) and turbulence ending (full symbols) in the pulsation phase as function of the Womersley number for different Re_m and pulsation waveforms for a particle suspension with different concentrations ($D_p/D = 0.1, \phi_p \approx 0 - 3\%$). The errorbars indicate the fluctuations of the onset and ending time in the individual pulsation periods. b) Example of the effect of the particle concentration with the individual measurements of turbulent onset and ending with $Wo = 4$. Higher concentrations accelerate the onset and ending in phase.

delay will be further investigated in section 2.4.2.3).

To illustrate the effect of the perturbation strength, the measurements of the physiological pulsation with $Re_m = 1277$ and $Wo = 3 - 5$ were performed with different particle concentrations $\phi_p = 0 - 3\%$. It can be seen that a higher particle concentration leads to an earlier turbulence onset, but also an earlier relaminarization. This effect is shown in more detail for $Wo = 4$ in Fig. 2.41b, for individual data points of the measured turbulence patches with different concentrations. The earlier onset aligns with previous studies of the particle-laden turbulence transition in steady pipe flow (Agrawal, Choueiri, and Hof, 2019; Hogendoorn, Chandra, and Poelma, 2022), where higher concentrations can induce a transition at lower Re due to stronger flow disturbances. For a pulsatile flow, Re changes over the period, here a lower transition threshold would result in an earlier development of turbulence while the flow accelerates, as seen in the measurements. The earlier relaminarization for higher concentrations indicates a fixed timescale for the total turbulence duration (including its growth and decay) for a given Wo . When discussing the effect of particle concentration, it is noteworthy that for very small concentrations ($\phi_p \lesssim 0.2\%$) turbulence could not be measured during each period, indicating either very weak turbulent patches or the flow remaining laminar.

So far, the analysis of the turbulence development was presented in the relative timescale of the pulsation period. This description is commonly used to show the effect of instabilities, which are linked to phase specific changes of the Womersley flow profile (as shown in Fig. 2.36). For example, the helical instability can only develop from unstable inflection points in the flow profile, which appear during the deceleration phase of the period (Morón, Feldmann, and Avila, 2022). In this case, the onset of all turbulence patches would appear phase locked during deceleration with little influence of the perturbation amplitude or Womersley number (Morón, Feldmann, and Avila, 2022; Xu et al., 2020). The observed dynamics of the particle-laden pulsatile flow are distinctly different. The turbulence onset does not appear phase locked and is strongly influenced by the Womersley number. At low Wo , turbulence patches are triggered in the acceleration phase, where the flow profile is still stable. To better discuss the transition with a particle perturbation and influence of Wo , the temporal dynamics can also be evaluated on an advective timescale ($t \cdot U/D$) instead of the period relative timescale (t/T). The advective timescale represents the actual time that an instability or turbulent structure has been advected by the flow and can evolve over time. In a pulsatile flow, the advective duration of the period changes with the Womersley number ($T \propto 1/Wo^2$). With low Wo and low frequency pulsations, turbulent patches have a far longer advective time to grow, compared to higher Womersley numbers. In the presented measurements the turbulence onset and ending with a particle perturbation (Fig. 2.41a) the duration of the turbulence patches appears approximately constant relative to the period ($\Delta t \approx 0.4T$ with the physiological waveform). In contrast, the advective duration of the patches monotonically decreases over Wo for both waveforms, as the period length becomes shorter ($\Delta t \approx 76D/U$ at $Wo = 3$ to $\Delta t \approx 20D/U$ at $Wo = 7$ for the physiological waveform).

To further investigate the appearance of turbulence in a particle-laden pulsatile flow, the turbulence strength of the relative pressure difference is shown as a function of the turbulence duration in advective time units of the flow (D/U) in Fig. 2.42. This presentation reveals that low Wo turbulence patches (top right of the figure) are not only longer in advective time but also have a higher strength, compared to higher Wo turbulence with shorter and less intense turbulence (bottom left). In this presentation, the data from all the different Re_m , different particle concentrations, and the different waveforms collapse into a single linear trend, indicated by the blue line. The longer the advective duration during each period, the stronger the turbulence. Combining this with the previous observation of the change in onset and turbulence duration, the transition scenario with a particle-laden flow resembles the mechanisms of puff and slugs dynamics from steady pipe flow. During each pulsation period, the flow perturbation triggers a turbulent puff. Given the high instantaneous Reynolds number at the peak of the physiological waveform ($\hat{Re} \approx 4000$), this puff can grow and evolve into a slug (Barkley et al., 2015), similar to simulations of harmonic pulsations at low Wo by Xu and Avila (2018). An increase in period duration (lower Wo) allows the turbulence patch more time to grow, resulting in longer slugs with higher turbulence strength. On the other hand, at higher Wo , the turbulence patch is delayed and has less time to grow. This results in the approximate proportionality between turbulence duration and turbulence strength.

The observed scenario of pulsatile flow with the strong perturbation of particles differs in multiple aspects from previous studies of an instability driven transition. The onset of particle induced turbulence changes with Wo , instead of being phase locked to the deceleration phase where the helical instability can be triggered (Morón, Feldmann, and Avila, 2022). For low Wo turbulence will occur during the acceleration phase where the flow profile is linearly stable. If the turbulence patches were primarily driven by the growth of the helical instability, the expected turbulence strength should reflect the associated energy growth (as shown in Fig. 2.9), with the highest growth at $Wo \approx 7$. Instead, the

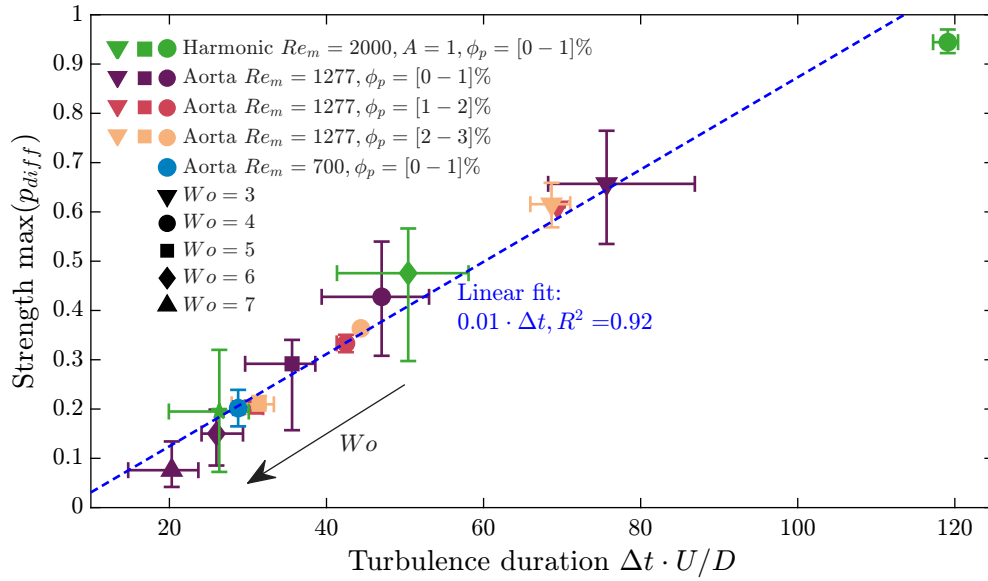


FIGURE 2.42: Turbulence strength as the difference between turbulent and laminar pressure drop measurements as a function of the turbulence duration in advective time units (D/U) for a physiological pulsation $Re_m = 700, 1277$ and a harmonic pulsation $Re_m = 2000, A = 1$ with different particle concentrations ($D_p/D = 0.1, \phi_p \approx 0 - 3\%$). The errorbars indicate the fluctuations of strength and duration in the individual measured pulsation periods.

growth is strongest at low Womersley numbers and decreases with higher Wo and shorter pulsation periods. The observed mechanism of turbulence onset in a particle-laden flow, therefore does not occur due to the helical instability.

2.4.2.3 Comparison between pulsatile and steady flow properties

In the previous section, it was shown that a particle-laden pulsatile flow transitions to turbulence in a similar manner to a steady pipe flow, with individual puffs triggered during each period. To further investigate this effect, I will try to link the observations of pulsatile flow to those from steady pipe flow in section 2.4.1.2. In a particle-laden steady pipe flow ($D_p/D = 0.1, \phi_p \approx 1\%$) turbulence emerges if Re exceeds a threshold Reynolds number of $Re_{c,p} = 1500$. Fig. 2.43a compares the pulsation periods of different combinations of waveforms, Re_m , and Wo , with the advective duration above the threshold Reynolds number $t(Re > Re_{c,p})$ highlighted in red. Looking at the waveforms investigated in this study, this limit is exceeded during each period, even if the mean Reynolds is below $Re_{c,p}$. The advective duration above the threshold is proportional to the length of the period and therefore scales with $1/Wo^2$ (see Fig. 2.43b). The mean Reynolds Re_m and waveform will also affect this duration. In a single-phase steady pipe flow Samanta, De Lozar, and Hof (2011) observed a timescale of $< 50D/U$ for puffs to fully develop. Especially for high Re_m and low Wo this timescale is greatly exceeded for many pulsation cases, causing strong and longer turbulence patches.

In Fig. 2.44 the duration of turbulent patches in a pulsatile flow is compared to the duration of the instantaneous Reynolds number exceeding the critical threshold from steady particle-laden pipe flow $Re_{c,p} = 1500$. (The measurements with particle concentrations $\phi_p > 1\%$ were omitted, as no critical Reynolds number for higher concentrations was measured in this study.) Both parameters correlate 1 to 1, showing that a longer time above the threshold will directly determine the duration of the turbulence patches. The different waveforms and Reynolds numbers collapse onto a single trend line for the investigated

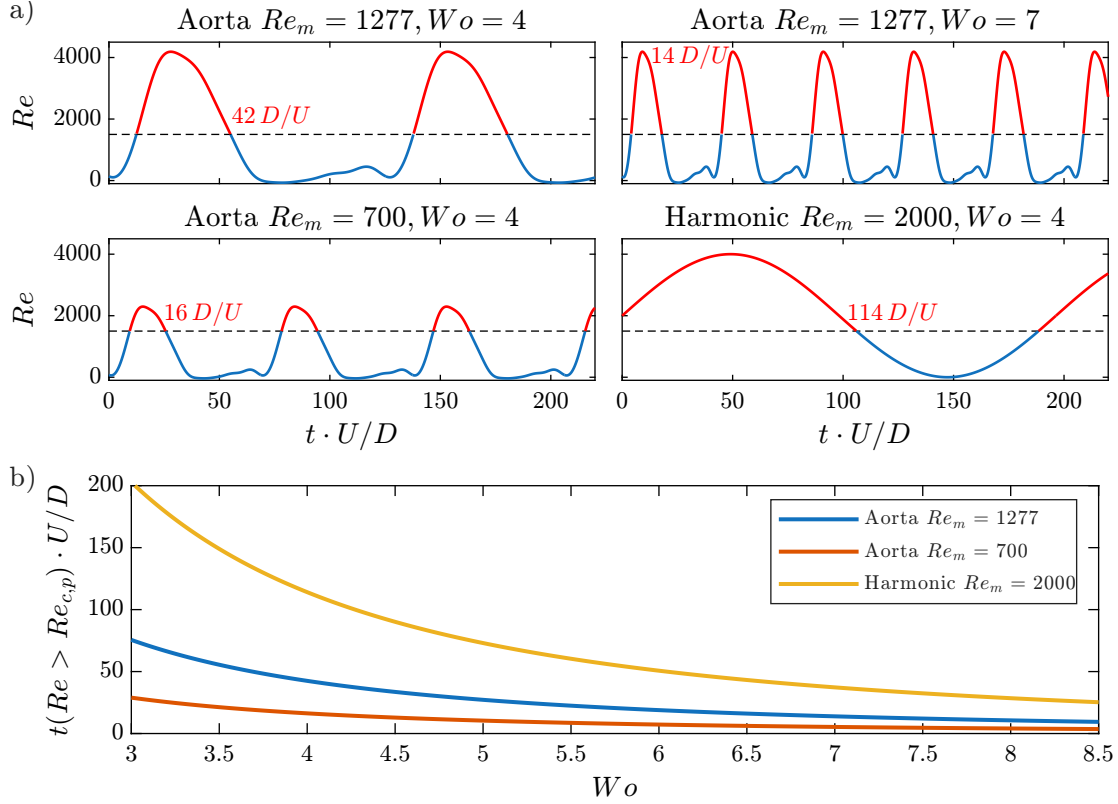


FIGURE 2.43: a) Pulsatile flow bulk velocity in advective time units D/U for different Wo , Re_m and different waveforms. The red line and text indicates the range when the velocity exceeds the critical Reynolds number $Re_{c,p} = 1500$ established for steady particle-laden flow with a particle concentration of $\sim 1\%$. b) Advective time units above $Re_{c,p} = 1500$ per pulsation period as a function of Wo for a physiological pulsation $Re_m = [700; 1277]$ and harmonic pulsation $Re_m = 2000, A = 1$.

parameter regime. This would indicate that for a given waveform, the length of the turbulence patch and therefore also its strength are correlated and can be predicted based on the characteristics of the waveform and critical Reynolds number alone (compare Fig. 2.43). There are clear limitations to this correlation. For each measurement there was a limit in Wo above which the flow stayed laminar, this shows that there is likely a minimum advective timescale for a turbulent puff to appear in the pulsation. In addition, the influence of the peak Re was not systematically investigated. For both the harmonic pulsation with $Re_m = 2000$ and the physiological pulsation $Re_m = 1277$ this peak is approximately the same, allowing the puffs to grow into slugs for lower Wo . If the peak Re is reduced, the puffs will not grow indefinitely, thereby limiting the growth of turbulence in each period.

The second comparison between the pulsatile flow and the parameters of steady pipe flow transition, is the delay between the instantaneous Re exceeding $Re_{c,p}$ and the onset of the turbulence patch displayed in Fig. 2.45. Both waveforms show a different delay that is approximately constant over Wo , with $t \approx 12 D/U$ for the physiological pulsation and $t \approx 28 D/U$ for the harmonic pulsation. This delay can explain the observed phase lag in the turbulence onset in section 2.4.2.2, where the flow will need some time before a turbulence patch can fully develop. For higher Wo with shorter periods, this results in a stronger delay of the turbulence onset. The difference between both waveforms is likely due to different acceleration ramps, where the more sudden acceleration in the physiological pulse further expedites the onset. Note that both the determination of the critical Reynolds number in steady flow and exact onset time in the pulsation will have some uncertainties

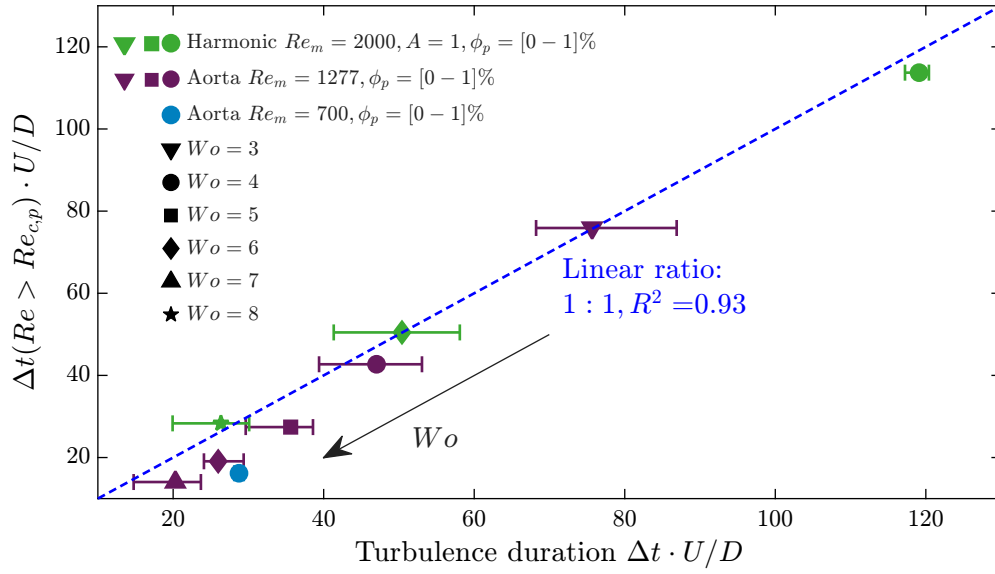


FIGURE 2.44: Duration of $Re(t)$ exceeding $Re_{c,p}$ in each pulsation period as a function of the turbulence duration in advective time units (D/U) for a physiological pulsation $Re_m = 700, 1277$ and a harmonic pulsation $Re_m = 2000, A = 1$ in a particle-laden flow ($D_p/D = 0.1, \phi_p \approx 0 - 1\%$). The errorbars indicate the fluctuations of the duration in the individual measured pulsation periods.

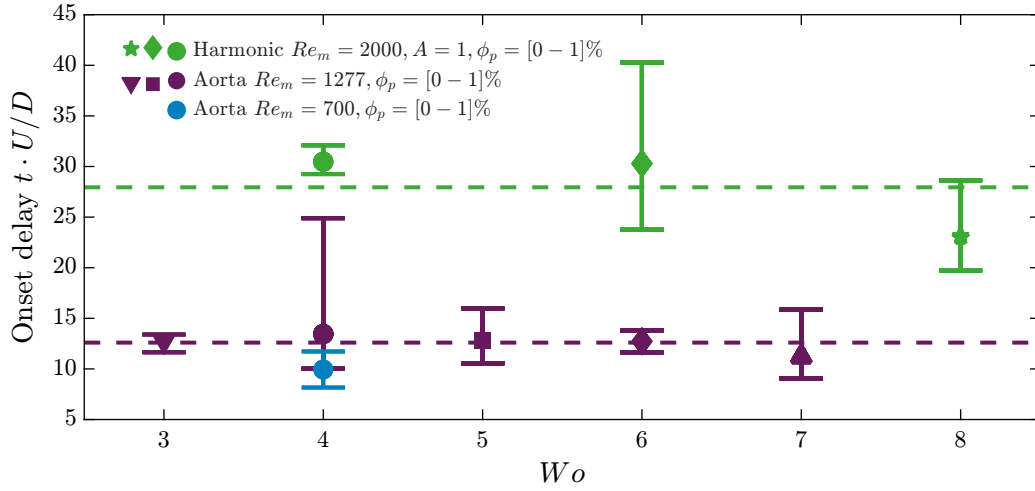


FIGURE 2.45: Time delay for the onset of turbulence t_{onset} and bulk velocity waveform surpassing the critical Reynolds number $t(Re > R_{c,p})$ in particle-laden flow. The dashed line indicate the mean value for both waveforms

not only due to the measurement noise, but also the general chaotic state of any turbulent flow. Both properties will also change with the particle concentration.

So far, the dynamics of particle induced turbulence in pulsatile flow was discussed solely based on the measured pressure drop. To analyze the actual fluid flow, three-dimensional STB measurements will be presented in the next section.

2.4.2.4 Flow interactions with large particles in pulsatile flow

Although differential pressure drop measurements are a powerful tool to collect statistics and parameter scans, the pressure signals represent an average of the flow inside the measurement section. This makes the method unable to detect the actual flow structure of the turbulent flow, smooths over smaller turbulent patches, and cannot reveal the particle-fluid interactions that cause turbulence to emerge. To observe the actual perturbation caused by the particles, Lagrangian Particle Tracking measurements using Shake-The-Box (STB) with large hydrogel particles ($D_p = 0.41D$) in the fluid were performed. From this, both the flow velocity field and the particle motion can be extracted (see section 2.3.4).

The first measurement is performed with $Re = 1277$ and $Wo = 8$ with the physiological waveform. While the previous pressure measurements were not able to reliably detect turbulence for $Wo > 7$, this is the lowest Wo where the limited recording time of STB can still nearly capture a full pulsation period. Furthermore, a helical flow pattern emerging from flow instabilities would be strongest in this regime (Morón, Feldmann, and Avila, 2022). Due to the time-sensitive process of density matching, only a few single hydrogel particles are seeded into the pipe flow. The STB measurements were specifically triggered when a particle crosses the measurement domain in the deceleration phase, where the pressure measurements showed turbulence patches developing. To evaluate the flow, the velocity profile is averaged over the azimuth angle θ with,

$$\bar{u} = \frac{1}{2\pi} \int_0^{2\pi} u(r, \theta) d\theta,$$

for different regions in the measurement volume. From the averaged profiles, the local turbulent kinetic energy is determined from the velocity fluctuations,

$$k = \frac{1}{2} ((\bar{u}_z - u_z)^2 + (\bar{u}_r - u_r)^2 + (\bar{u}_\theta - u_\theta)^2).$$

The turbulent kinetic energy profile is also azimuthally averaged with,

$$\bar{k} = \frac{1}{2\pi} \int_0^{2\pi} k(r, \theta) d\theta.$$

The complete flow field within the measurement volume is depicted in Fig. 2.46, with the corresponding azimuthally averaged profiles of streamwise velocity $\bar{u}_z$ and turbulent kinetic energy $\bar{k}$ shown in Fig. 2.47. Towards the end of the deceleration phase at $t = 0.45T$ (Fig. 2.46a) the large hydrogel particle (indicated by the gray sphere in the flow field) enters the field of view. The flow downstream of the particle (L3, yellow) is still laminar, with the fully developed Searl-Womersley flow profile and close to zero velocity fluctuations. In the region surrounding the particle (L2, orange), the flow already shows some vorticity structures and stronger turbulent kinetic energy, due to the large perturbation from the particle onto the flow. As the particle is further advected downstream ($t = 0.5T$, Fig. 2.46b), it can be seen that the upstream wake (L1, blue) of the particle has already turned turbulent, with higher turbulent kinetic energy and vorticity streaks in the flow. As expected for a pipe flow, the turbulent kinetic energy is strongest near the wall, where the flow has the strongest shear, and approaches zero in the center of the pipe, where there is no shear Wygnanski and Champagne (1973). In addition, the velocity profile in the turbulent upstream region appears to be flattened compared to the SW profile. At $t = 0.6T$ (Fig. 2.46c), both the bulk velocity and particle motion are close to zero. Here, the turbulent structure detaches from the particle and is advected further downstream. The turbulent kinetic energy in the downstream region (L3) increases. Upstream of the particle (L1) the turbulent patch has

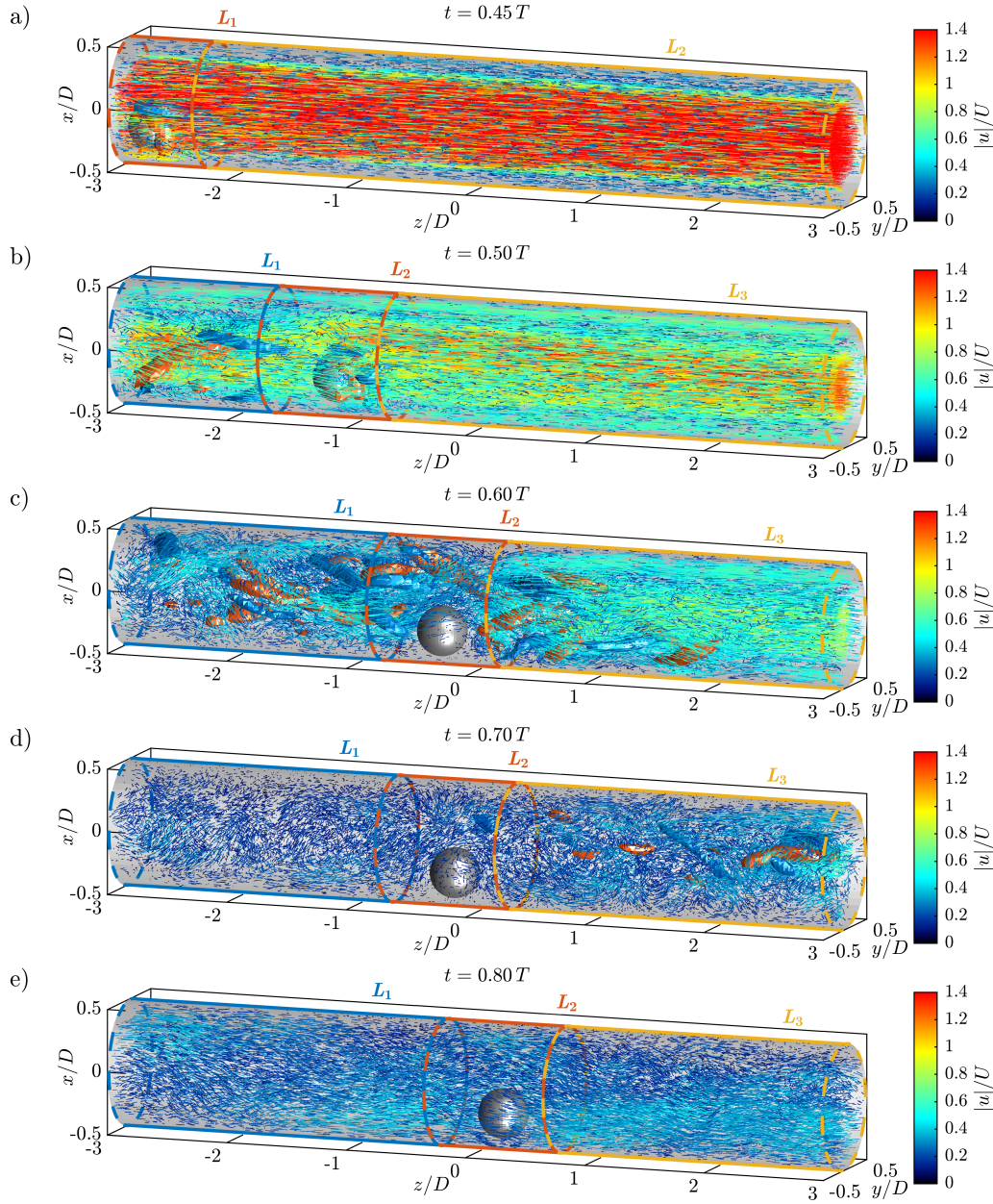


FIGURE 2.46: a)-e) Snapshots of flow trajectories from STB measurements for a physiological pulsation with $Re = 1277$; $Wo = 8$ with the perturbation by a large particle $D_p = 0.41D$ at $t = [0.45; 0.5; 0.6; 0.7; 0.8]/T$. The interface of the particle is depicted in gray. Red and blue structure represent isosurfaces of the streamwise vorticity with $\omega_z = \pm 4.5U/D$. The measurement volume is separated into three regions, upstream of the particle (L1, blue), surrounding the particle (L2, orange) and downstream of the particle (L3, yellow), for evaluating the flow profiles in Fig. 2.47.

completely flattened the flow profile, with no pronounced backflow near the walls or strong inflection points in the profile. The shape of the turbulent structure is very similar to the turbulent puff in steady flow, with vortex streaks lifted from the wall towards the center of the pipe (compare section 2.4.1.1). Further into the low velocity phase ($t = 0.7T$, Fig. 2.46d), the turbulence patch decays, with smaller vorticity streaks and reduced turbulent kinetic energy. At this point in the period, the velocity profile has been flattened to zero throughout the measurement volume. At $t = 0.8T$ (Fig. 2.46e) the flow starts to slightly accelerate due to the small secondary peak of the physiological waveform (compare Fig. 2.39), resulting in

some streamwise particle motion. Even though the flow has relaminarized, the analytically SW profile has not yet recovered. Towards the end of the pulsation period (not depicted) the particle moves out of the field of view and only at the beginning of the next period ($t = 1T$) does the analytical flow profile develop again. The observed disruption of the profile explains why the particle does not trigger the helical instability in the deceleration phase ($t > 0.5T$), as this requires the fully developed Searl-Womersley and its inflection points to grow (Morón, Feldmann, and Avila, 2022). Instead, the strong perturbation from the large particle causes a turbulent patch to appear before the helical instability can emerge which disrupts the flow profile and inhibits its development. A similar relaminarization process was observed for a lower $Wo = 4$, presented in Appendix A.2.1.

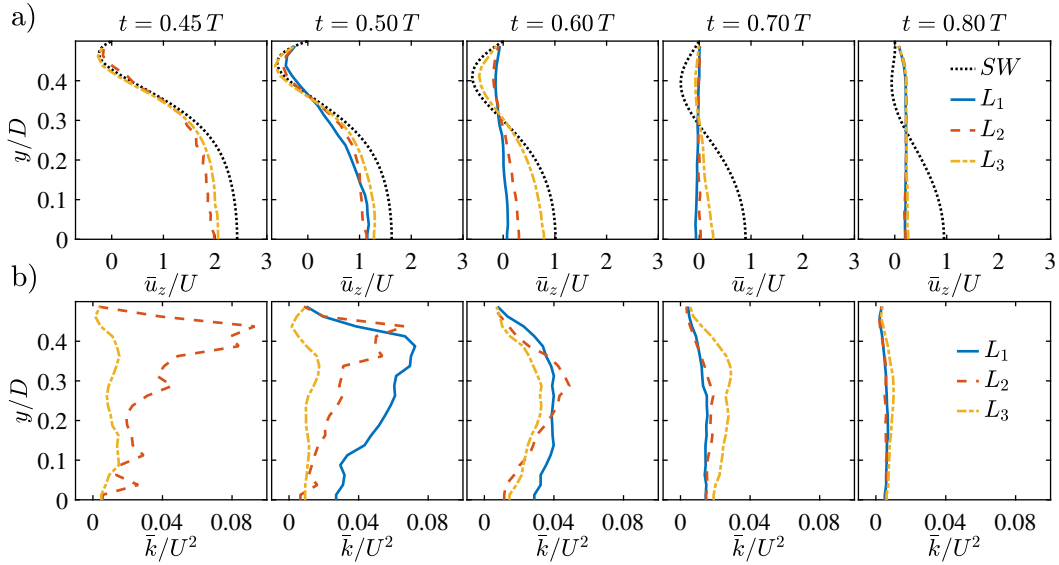


FIGURE 2.47: a) Profiles of the streamwise velocity u_z averaged over three regions of measurement volume (see Fig. 2.46), upstream of the particle (L1, blue), surrounding the particle (L2, orange) and downstream of the particle (L3, yellow), compared the flow profile without a perturbation (black). b) Profiles of the turbulent kinetic energy k in the same three regions. Both the profiles of velocity and turbulent kinetic energy are azimuthally averaged over the pipe cross section.

For the presented measurement the flow upstream of the particle already enters the field of view turbulent, therefore its onset cannot be observed. This is consistent with pressure measurements for smaller particles, where the onset at $t \approx 0.3 - 0.4T$ would be predicted at these flow parameters (compare Fig. 2.41). In contrast, the downstream perturbation by the particle can be analyzed, and is depicted in Fig. 2.48. As the particle enters the field of $t = 0.45T$ the downstream flow is still laminar, but near the particle ($z = -1.5D$) multiple pairs of counter-rotating vortices surround the particle and disrupt the flow. These vortex structures act as transient perturbation sources that can destabilize the laminar profile. With increasing distance to the particle ($z = [-0.5, 0.5]D$), these vortices fade away.

In addition to enabling the particle fluid interactions to be resolved, the hydrogel also enables the direct tracking of the particle motion and its rotation. For the same investigated parameters, the trajectory and velocity of the particle in a physiological pulsatile flow are displayed in Fig. 2.49, with the three-dimensional trajectory in Fig. 2.49a and the streamwise particle velocity $u_{z,p}$ and its angular momentum ω_p in Fig. 2.49b. As the particle initially enters the observation window, it moves in a straight line in the axial direction, decelerating in the same manner as the bulk flow velocity. By comparison, the inertia of the particle rotation decays much slower. While the particles translatory velocity has already

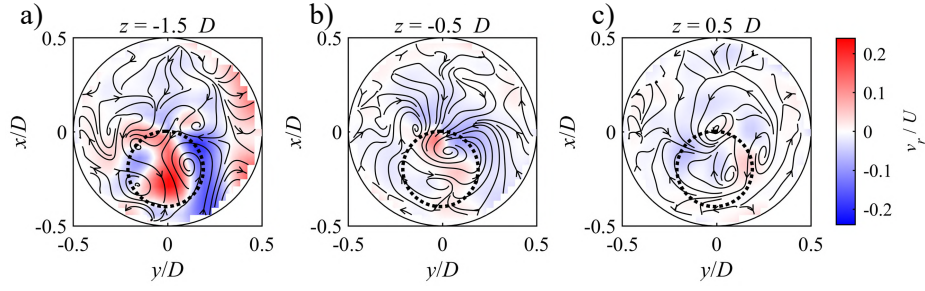


FIGURE 2.48: Radial velocity field downstream of the large particle from STB measurements as it enters the field of view $t = 0.45T$. a) Multiple strong vortices in the wake of a large particle (black circle) $3/4D$ downstream of particle position. b)-c) Increasing axial distance to the particle.

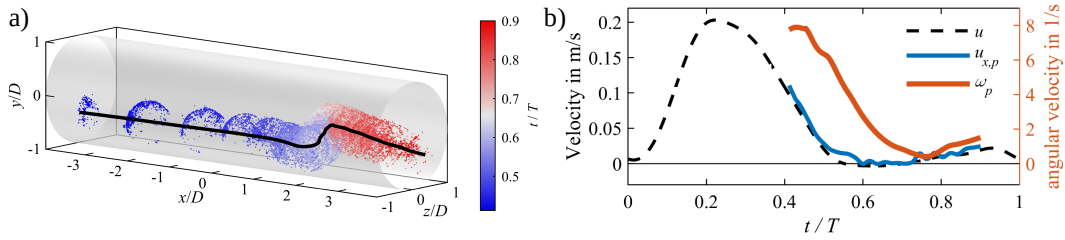


FIGURE 2.49: Motion of a large particle $D_p = 0.41D$ in physiological pulsatile flow with $Re = 1277$ and $Wo = 8$. a) The particle trajectory over time through the observation window based on the identified trajectories on the particle interface. b) Based on the interface motion the streamwise velocity $v_{x,p}$ and angular velocity ω_p of the particle can be compared to the bulk flow velocity u .

reached zero ($t = 0.6T$), it still has a significant angular momentum. This results in a slow tumbling migration through cross section plane while the flow is at rest. This migration only stops when the flow accelerates into the second smaller peak of the physiological waveform ($t = 0.8T$). This observation is not unique to this single measurement but could be reproduced during each period of the physiological pulsation. Compared to the more gradual effect of inertial migration that is known from steady flow, where radial migration occurs over multiple hundreds of pipe diameters in streamwise direction (Morita, Itano, and Sugihara-Seki, 2017), this tumbling radial migration occurs on a much shorter axial length and timescale. This effect could indicate an enhanced particle migration for pulsatile flow, although it cannot be distinguished if the migration is solely based on the flow deceleration in the pulsation or the remaining turbulent fluctuation in the flow. Further study is required to understand this effect.

2.4.2.5 Flow interactions with small particles in pulsatile flow

In the previous section, the particle fluid interactions were investigated using a large hydrogel particle, to enable simultaneous measurements of particle and fluid motion. For comparison, the STB measurement will be repeated with the small polystyrene particles $D_p = 0.1D$ that were used for the broader parameter scans in section 2.4.2 with a concentration of $\phi_p \approx 0.5\%$. Compared to the hydrogel, these particles, while slightly translucent, are not refractive index matched, and in the STB measurements they block out parts of the camera view and dim or reflect the laser illumination, see Fig. 2.50. As a result, the trajectory field will be more sparse (~ 10000 tracks per time step, compared to ~ 18000 with hydrogel) and prone to local errors from the particle reflections. However, the measurement is still sufficient to identify the onset of turbulence in the flow.



FIGURE 2.50: Snapshot of a tracer image with small polystyrene particles $D_p = 0.1D$, $\phi_p \approx 0.5\%$. Each particle has strong glare points due to the laser illumination from the right side.

Fig. 2.51 depicts the flow field inside the measurement volume of the turbulence onset with small particles in the physiological pulsatile flow with $Re = 1277$ and $Wo = 8$, with the corresponding azimuthally averaged profiles of streamwise velocity $\bar{u}_z$ and turbulent kinetic energy $\bar{k}$ shown in Fig. 2.52. At the beginning of the deceleration phase at $t = 0.3T$ (Fig. 2.51a), the flow is still mostly laminar with no vorticity streaks and only a small increase in turbulent kinetic energy near the walls. Compared to the Sexl-Womersley profile, the measured flow profile is slightly flattened. This observation is consistent with previous PIV measurements in steady particle suspensions by Hogendoorn, Chandra, and Poelma (2021). At $t = 0.4T$ (Fig. 2.51b) the first vorticity streaks appear. Although the turbulent kinetic energy in the upstream section (L1, blue) is the strongest, the initial vortices appear scattered throughout the measurement volume, due to the global distribution of multiple small particles interacting with the flow. Towards the end of the deceleration ($t = 0.5T$, Fig. 2.51c) the turbulent patch fills the entire volume, with a similar turbulent kinetic energy in all three regions. The same as in the single hydrogel particle measurement, the velocity profile is completely flattened, but here the profile is approximately the same throughout the volume as the perturbation source is not localized. During the low velocity phase ($t = [0.6, 0.7]T$, Fig. 2.51d,e) the flow relaminarizes. The flow velocity vanishes in the wake of the turbulence patch, without inflection points. Although the small particle motion could not be directly measured, the raw image data shows that even as the bulk velocity is close to zero, the particles still have some remaining inertia, similar to the observed migration of the hydrogel sphere in previous measurement. An example of turbulence development with $Wo = 4$ is shown in Appendix A.2, where the onset appears during the acceleration and the turbulent kinetic energy grows much stronger.

Compared with the single hydrogel particle, the observed turbulence dynamic with the suspension of small particles is very similar. In both measurements, the perturbation triggers the onset of turbulence during the deceleration phase of the pulsation. This patch quickly grows and fills out the measurement volume, resembling the shape of a steady flow puff. In the low velocity phase, there is no kinetic energy in the bulk flow that can feed the turbulence development, therefore, the patch decays and the flow relaminarizes. The turbulent fluctuations have a strong effect on the velocity profile, and remove the backflow

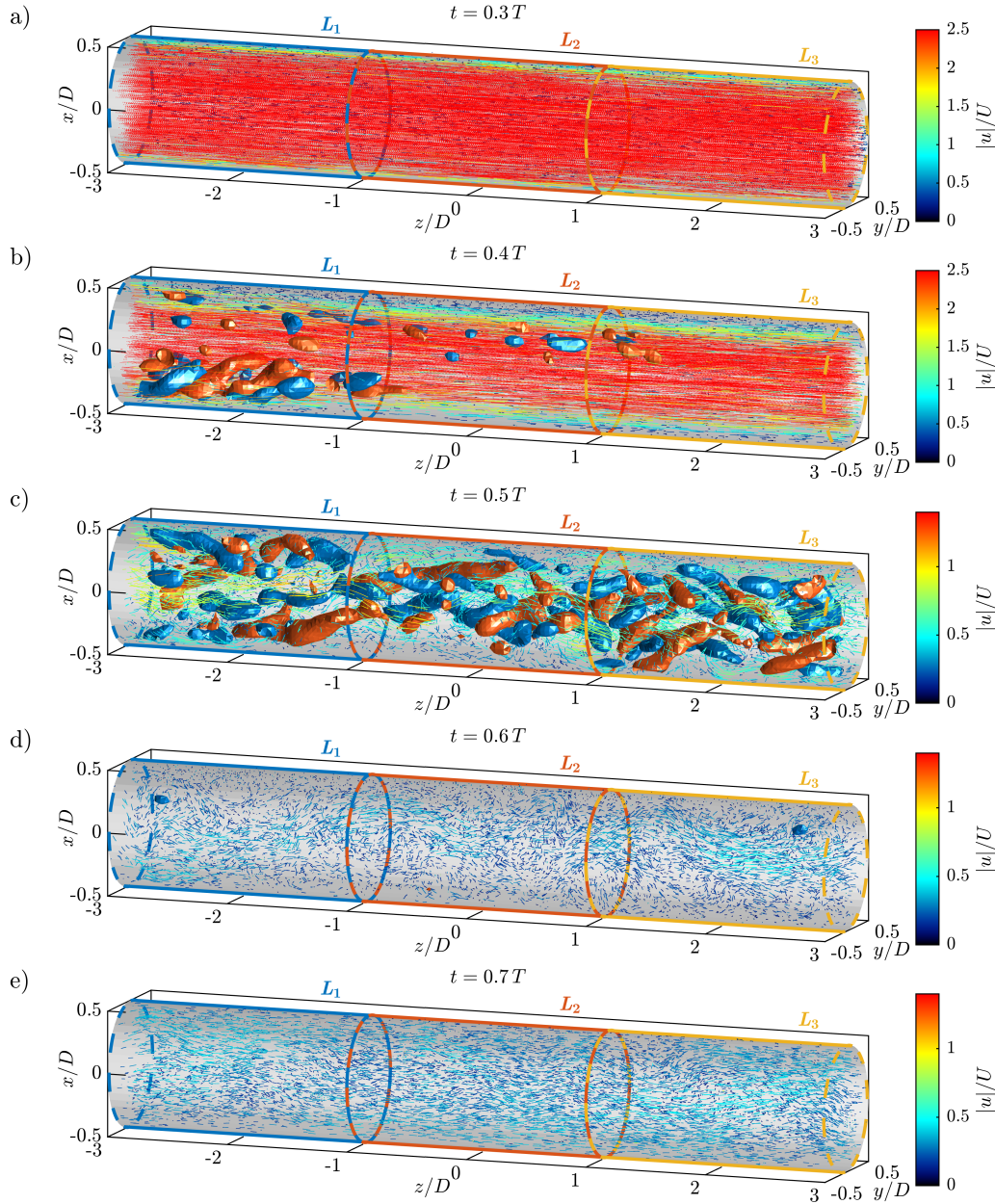


FIGURE 2.51: a)-e) Snapshots of flow trajectories from STB measurements for a physiological pulsation with $Re = 1277$; $Wo = 8$ with the perturbation in a suspension of small particles $D_p = 0.1D$, $\phi_p \approx 0.5\%$ at $t = [0.3; 0.4; 0.5; 0.6; 0.7]/T$. Red and blue structure represent isosurfaces of the streamwise vorticity with $\omega_z = \pm 4.5U/D$. The measurement volume is separated into three regions, upstream (L1, blue), center (L2, orange) and downstream (L3, yellow), for evaluating the flow profiles in Fig. 2.52.

and inflection points associated with the laminar SW profile. As a result, no instabilities can evolve in the wake of the turbulence patch.

The turbulence dynamics of the particle-laden flow with a physiological pulsation is most similar to previous studies of low amplitude and low Womersley flow (Xu and Avila, 2018), where turbulence appears as phase-locked puffs each period, which can grow into slugs and relaminarize during the low velocity phase. To investigate whether this effect is unique to the particle-laden flow, the dynamics of different perturbation mechanism are studied in the next section.

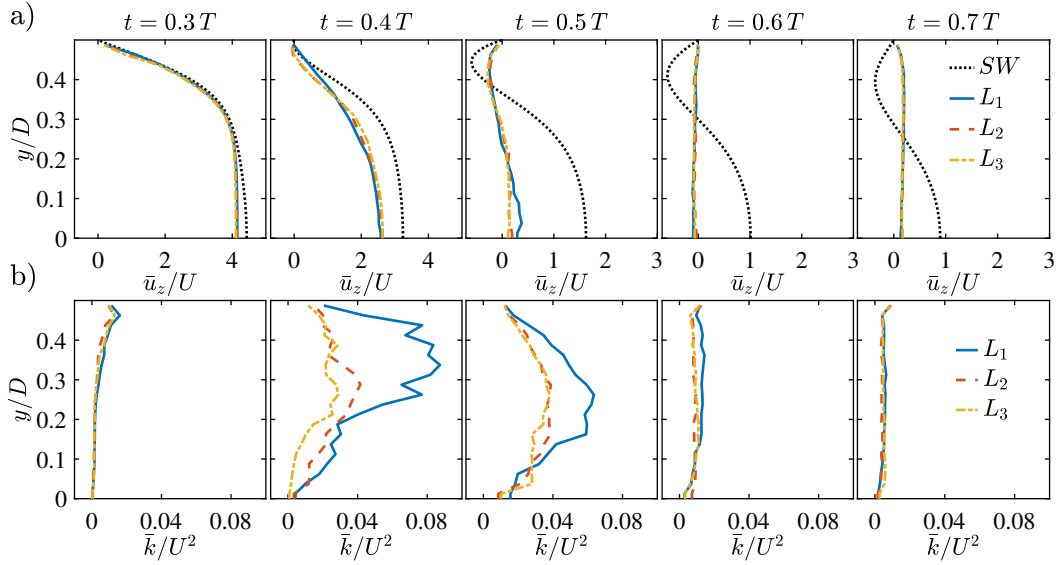


FIGURE 2.52: a) Profiles of the streamwise velocity u_z averaged over three regions of measurement volume (see Fig. 2.51), upstream (L_1 , blue), center (L_2 , orange) and downstream (L_3 , yellow), compared the flow profile without a perturbation (black). b) Profiles of the turbulent kinetic energy k in the same three regions. Both the profiles of velocity and turbulent kinetic energy are azimuthally averaged over the entire pipe cross section.

2.4.3 Pulsatile flow with a geometric bending perturbation

To compare the observed dynamics of the particle induced turbulence in pulsatile flow, additional measurements are performed with a geometric perturbation similar to the study by Xu et al. (2020). For this purpose, a section of the glass pipe is replaced with a curved segment made from stiff PVC tubing. Both ends of the tube are aligned with the straight pipe, and the center has an offset, creating a bending with an approximately sinusoidal shape, as sketched in Fig. 2.53. To avoid buckling while bending the tube, a length of $L_{bend} = 560 \text{ mm} = 20D$ for the segment was chosen. The offset L_0 or amplitude of the perturbation was selected to match the study by Xu et al. (2020) with $L_o = [5.6, 16.8, 33.6] \text{ mm} = [0.2, 0.6, 1.2]D$, leading to same offset ratios of $L_o/L_{bend} = [0.01, 0.03, 0.06]$. However, due to the different length of the bend, the curvature ratios do not match ($D/D_{bend} = [0.006; 0.017; 0.034]$). The curvature ratio of the bend (D/D_{bend}) in this setup is still sufficiently small where steady pipe flow shows a subcritical puff transition, without the Dean flow inside the bend becoming unstable (Kühnen et al., 2015). The geometry parameters of the bend and corresponding Dean number of the flow are given in Table 2.3. The bend is placed at a downstream position of more than $120D$ to achieve a sufficient entrance length for a fully developed flow entering the curvature. Pressure drop measurements and STB flow measurements are performed $36D$ and $20D$ downstream from the end of the perturbation section respectively (see Fig. 2.53a).

For a comparison to the turbulence development of particle-laden flow, the dynamics with a bending perturbation are investigated for different Womersley numbers with a physiological pulsation waveform (section 2.4.3.1). The employed pressure drop measurements with the bend for this analysis were performed as part of the master's thesis by Richter (2024). To evaluate how the bend perturbs the pulsatile flow, and how turbulence patches develop, three dimensional STB measurements are presented in section 2.4.3.2.

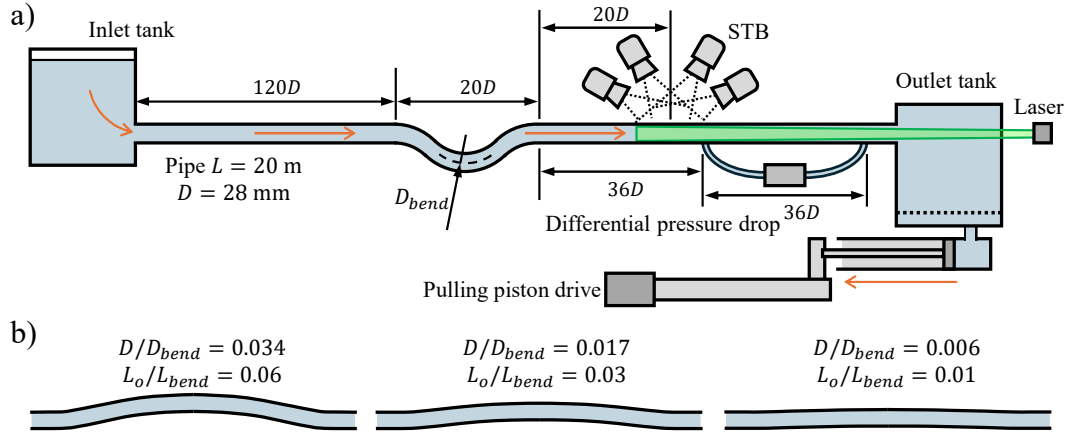


FIGURE 2.53: a) Sketch of the bend segment integrated into the pipe flow experiment. The flow is characterized via differential pressure drop measurements ($36D$ downstream of the bend) or STB flow measurements ($20D$ downstream of the bend). b) To-scale sketch of the geometry of the three different bends investigated.

L_o/L_{bend}	D/D_{bend}	De_m	$\hat{De}$
0.01	0.006	99	310
0.03	0.017	167	522
0.06	0.034	235	738

TABLE 2.3: Parameters of the geometric bend and the resulting Dean numbers in a pulsatile flow with a physiological pulsation with $Re_m = 1277$, averaged over the period De_m and during the peak of waveform $\hat{De}$.

2.4.3.1 Womersley dependent turbulence transition

To evaluate the turbulence development in pulsatile flow with a geometric bend perturbation, the same method as in section 2.4.2.2 is used, where the pressure drop is recorded with the perturbation and compared to a laminar flow with a straight pipe. This pressure difference is shown for a range of Womersley numbers ($Wo = [3 - 6]$) for $Re_m = 1277$ with a physiological pulsation in Fig. 2.54. The strongest turbulence with the earliest onset appears at the lowest $Wo = 3$. Higher Womersley numbers delay the onset and reduce the strength. For $Wo > 5$, turbulence could no longer be reliably identified with the geometry bend, as additional fluctuations in the pressure signals become too pronounced. The reason for this is the geometry change from the bend modifying the flow profile and therefore the pressure signal even during laminar phases of the period. Additionally, it is possible that the flow relaminarizes before reaching the pressure sensor (positioned $36D$ downstream of the perturbation). When comparing this perturbation with the particle-laden flow, the effect over Wo is very similar. Both perturbation mechanisms create the strongest turbulence patches at small Womersley numbers, with similar trends of reduced pressure differences for shorter pulsation periods.

In Fig. 2.55 the time of the turbulence onset and ending in the pulsation period is compared for the three different bending ratios used in the experiments. It can be observed, that for all three $Wo = [3, 4, 5]$ the variance between the bending ratios lies within the error bars, and the effect of the changing Wo is far more relevant. This indicates that in this regime of small bending ratios combined with a long curved segment in the pipe, the change in curvatures has no strong effect on the development of turbulence patches. For the largest $Wo = 5$ an increased uncertainty of the turbulence end time (horizontal error bar) can be seen. This is due to the more shallow decay of the turbulence patches compared to

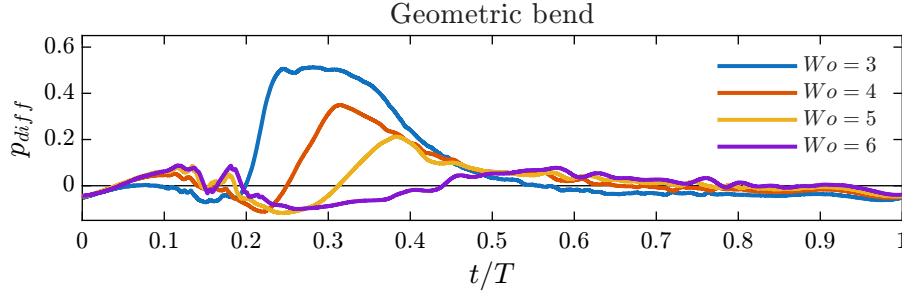


FIGURE 2.54: Time series of pressure drop difference between perturbed and laminar flow (see Fig. 2.37) of a physiological pulsation at $Re = 1277$ for different Womersley numbers with a geometric perturbation ($D/D_{bend} = 0.034$). The curves are averaged over multiple individual pulsation periods.

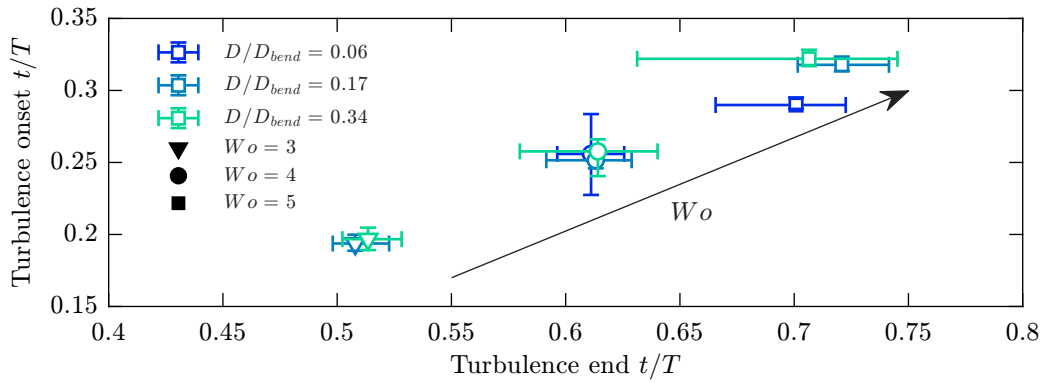


FIGURE 2.55: Turbulence onset plotted against turbulence end time relative to the pulsation phase for different bending ratios $D/D_{bend} = [0.006; 0.017; 0.034]$. For higher Wo the onset and end are delayed.

the steeper onset in the pressure signals (see Fig. 2.54), making it difficult to determine the exact time relaminarization. In the remaining discussion in this section, the data from all three bending ratios will be combined for each Wo .

The turbulence onset and ending times with the geometric perturbation are directly compared to the particle-laden flow with the same physiological waveform at $Re_m = 1277$ in Fig. 2.56. At a given Wo the onset with the bend is slightly delayed compared to the particle suspension, although both onset and ending follow the same scaling over Wo . The earlier onset of particle-laden flow indicates that the flow disturbance caused by the particles is stronger compared to the bend. This would be in line with previous observations in steady pipe flow, where particles efficiently trigger puffs (see section 2.4.1.2), compared to straight pipes with fixed geometric perturbations (Samanta, De Lozar, and Hof, 2011) or pipe flow with a small curvature (Kühnen et al., 2015).

When evaluating the turbulence strength as a function of the turbulence duration in Fig. 2.57, the bending perturbation again shows the same trend as the particle-laden flow. For longer periods at low Wo , the patches have a longer advective duration and stronger amplitude. Combined with the observed temporal dynamics, this leads to the conclusion that the geometrically perturbed pulsatile flow of this study, has the same turbulence transition as a particle-laden flow: Turbulent puffs are triggered during the acceleration peak, can then grow into slugs due to the high Re , and finally decay in the low velocity phase.

The question remains why the geometry in this study triggers puffs, while a bend used by Xu et al. (2020) with the same offset to length ratios $L_o/L_{bend} = [0.01, 0.03, 0.06]$

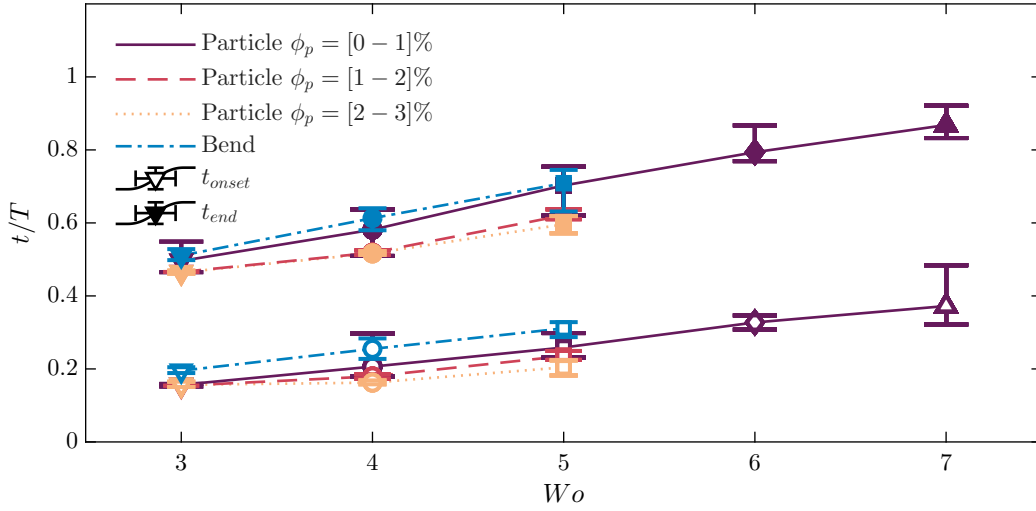


FIGURE 2.56: a) Averaged onset (empty symbols) and end (full symbols) of irregular flow patches in the pulsation phase as function of the Womersley number for a particle suspension with different concentrations ($D_p/D = 0.1$, $\phi_p \approx 0 - 3\%$) and a bending perturbation ($D/D_{bend} = [0.006; 0.017; 0.034]$). The errorbars indicate the fluctuations of the onset and end time in the individual pulsation periods. For the bending perturbation all the data is averaged over all three bending diameters (D/D_{bend}).

triggered the helical instability. The different transition is likely due to the difference in the actual bend length ($L_{bend} = 20D$ in this setup, compared to $L_{bend} = 7D$ used by Xu et al. (2020)), which will also affect the bending ratio and the strength of the Dean flow that can develop inside the segment. To confirm this, the flow is optically analyzed in the next section.

In the evaluation of pulsatile particle-laden flow, the characteristics of the turbulence patch could be directly correlated to the critical $Re_{c,p}$ determined in steady pipe flow. There, the duration above the critical threshold determined how long a turbulent patch will grow, and the delay between the critical Reynolds number in the waveform and the turbulence onset was constant at $\sim 12D/U$. For the geometric perturbation, this comparison is more difficult. For both pulsatile and steady flow, the secondary flow inside the bend will modify the development of the laminar flow profile, and change any pressure drop measurement compared to a straight pipe. As this effect increases with Re , the pressure drop continuously deviates from the Hagen-Poiseuille law, making it very difficult to determine the exact point where the flow transitions (Cioncolini and Santini, 2006)⁶. Because of these difficulties, a clear Re_c could not be established for the bend perturbation. It remains to be investigated, if the same connection between steady and pulsatile flow from particle-laden flow can also be made for the geometric perturbation.

To still compare the phase lag of the turbulent flow onset (seen in Fig. 2.56), the time delay between the maximum of the turbulence strength $t(\max(p_{diff}))$ and the maximum of the driving waveform $t(\hat{Re})$ is evaluated in Fig. 2.58. During the peak of the waveform, the flow has the highest instantaneous kinetic energy, which should correlate with the strongest turbulence in the flow. There are some fluctuations as the time of the maximum pressure difference varies for each measured period, similar to the onset and end time in Fig. 2.56. The depicted delay is of a similar magnitude in advective time units $\sim 10D/U$ for both perturbations and all investigated Womersley numbers. The magnitude of this delay is

⁶The steady flow turbulence transition with the geometric bend from this setup was studied in more detail in the scope of the Master's thesis by Richter (2024).

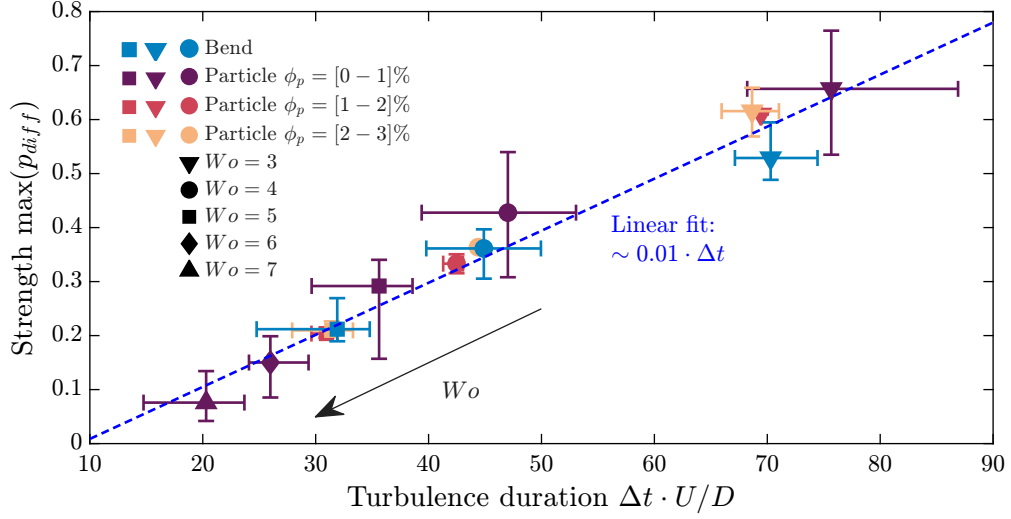


FIGURE 2.57: Turbulence strength as the difference between perturbed and laminar pressure drop measurements as a function of the patch duration in advective time units (D/U) with different particle concentrations ($D_p/D = 0.1$, $\phi_p \approx 0 - 3\%$) and a bending perturbation ($D/D_{bend} = [0.006; 0.017; 0.034]$). The errorbars indicate the fluctuations of strength and duration in the individual pulsation periods. Longer turbulence patches at lower W_o have a higher relative strength.

similar to the observed delay of the turbulence onset in particle-laden flow of $\sim 12D/U$ (see Fig. 2.45). This could indicate a inherent timescale of the turbulence development in the physiological pulsation, as it appears in both the onset delay after surpassing the critical Re and the delay between the peak of driving waveform and the maximum turbulence strength.

In the next section, the flow field of the turbulence development with a bending perturbation will be analyzed.

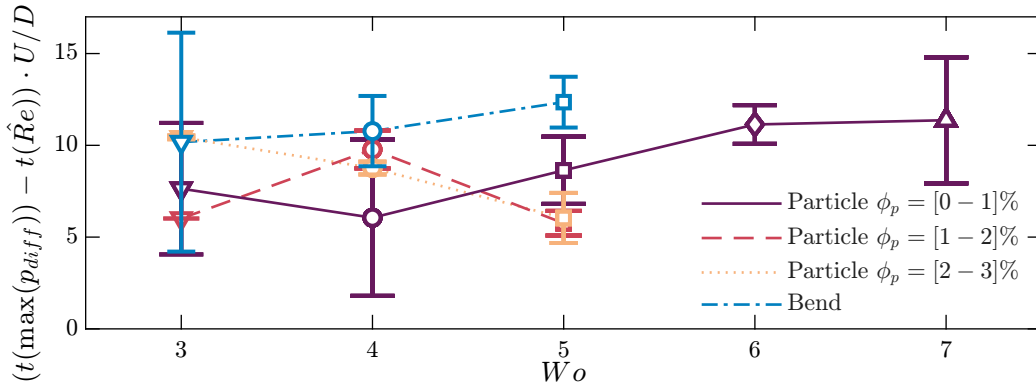


FIGURE 2.58: Time delay between the maximum of the turbulence strength $t(\max(p_{diff}))$ and the maximum of the driving waveform $t(\hat{Re})$ in advective time units (D/U) with different particle concentrations ($D_p/D = 0.1$, $\phi_p \approx [0 - 3]\%$) and a bending perturbation ($D/D_{bend} = [0.006; 0.017; 0.034]$). The errorbars indicate one standard deviation of $t(\max(p_{diff}))$, varying for each measured period.

2.4.3.2 Flow structure of bending induced turbulence

For investigating the structure of the flow, the same parameters of $Re = 1277$ and $Wo = 8$ as for particle-laden flow in section 2.4.2.4 are selected.

The complete flow field within the measurement volume is depicted in Fig. 2.59, with the corresponding azimuthally averaged profiles of streamwise velocity $\bar{u}_z$ and turbulent kinetic energy $\bar{k}$ shown in Fig. 2.60. In the presented orientation, the bending protrudes in the negative y direction at an axial location $z = -20D$ upstream of the measurement position. Since the bend breaks the radial symmetry of the pipe flow, the velocity and turbulent kinetic energy profiles are determined for two halves of the cross-section,

$$\bar{u} = \frac{1}{\pi} \int u(r, \theta) d\theta, \quad \bar{k} = \frac{1}{\pi} \int k(r, \theta) d\theta,$$

for the half facing the bend ($y < 0$) and the half facing away from the bend ($y > 0$). During the beginning of the deceleration phase ($t = 0.3T$, Fig. 2.59a) the flow is still laminar, although the velocity profile is affected by the bend, with slightly higher velocities opposite to the bend ($y > 0$). This is caused by the Dean flow pushing the higher velocity flow away from the centerline. At $t = 0.4T$ (Fig. 2.59b), the first vortex structures of a turbulence patch enter the field of view, indicated by streamwise vorticity streaks and increased velocity fluctuations in the leftmost section of the flow field (L1, blue). The asymmetry of the flow becomes more pronounced, in both the velocity profile and turbulent kinetic energy, with stronger fluctuations facing the bend ($y < 0$). As the turbulence structure is advected further downstream ($t = 0.5T$, Fig. 2.59c), the fluctuations increase throughout the field of view. Due to the turbulent vortices mixing the flow, the asymmetry is reduced. As observed in the particle-laden flow, the velocity profile appears flattened compared to the laminar SW profile. Similar to the large particle perturbation, this effect is strongest closer to the perturbation (L1). During the low velocity phase ($t = 0.6T$, Fig. 2.59d), the turbulent patch exits the field of view in the downstream direction. In the wake of the patch (L1, L2) the flow relaminarizes. Further into the low velocity phase ($t = 0.7T$, Fig. 2.59e) the turbulence patch has fully decayed. Again, the velocity profile stays flat and takes a longer time to recover. For comparison the same measurement is repeated with a smaller bending ratio of $D/D_{bend} = 0.017$ in Appendix A.2.3, where the onset and development of turbulence appears delayed due to the smaller perturbation amplitude.

To better understand the emerging turbulence from the bending perturbation, the velocity field in a cross-section closest to the bend is presented in Fig. 2.61. In the presented orientation, the bending protrudes in the negative y -direction. At $t = 0.32T$ the secondary flow starts to arise, with a pair of two counter-rotating vortices entering the field of view. With the bending perturbation these vortices are created by the Dean flow caused by the centripetal forces of the flow inside the curvature. Shortly after the snapshot, the initial Dean flow splits into multiple smaller vortices, indicating the breakup into turbulence. This initial pair of two large vortices matches the optimal nonmodal disturbance for both steady and pulsatile laminar pipe flow, when the helical instability is not dominant (Avila, Barkley, and Hof, 2023; Xu, Song, and Avila, 2021). The same mechanism creates turbulence during jet injection perturbations (Hof, Juel, and Mullin, 2003), where the vortex pair is generated by a fluid jet entering through a small hole in the wall. The vortex generates turbulence via the lift-up mechanism, where the high velocity flow from the center is mixed with low velocity flow near the walls, generating turbulent streaks (Brandt, 2014). It seems that for the physiological waveform investigated here, the short acceleration phase with a large amplitude ($A \approx 2.3$), in combination with a strong curvature of the bend, leads to a premature development of turbulence before the helical instability arises. In contrast, a smaller bend, as utilized in the experiments by Xu et al. (2020), will create a much weaker

Dean flow that does not trigger puffs and instead the helical instability is triggered during deceleration. This effect of strong perturbation can be highly relevant for physiological flows, where the bending of the aortic arch will be even more pronounced than the perturbation used here, making the flow highly susceptible to localized turbulence.

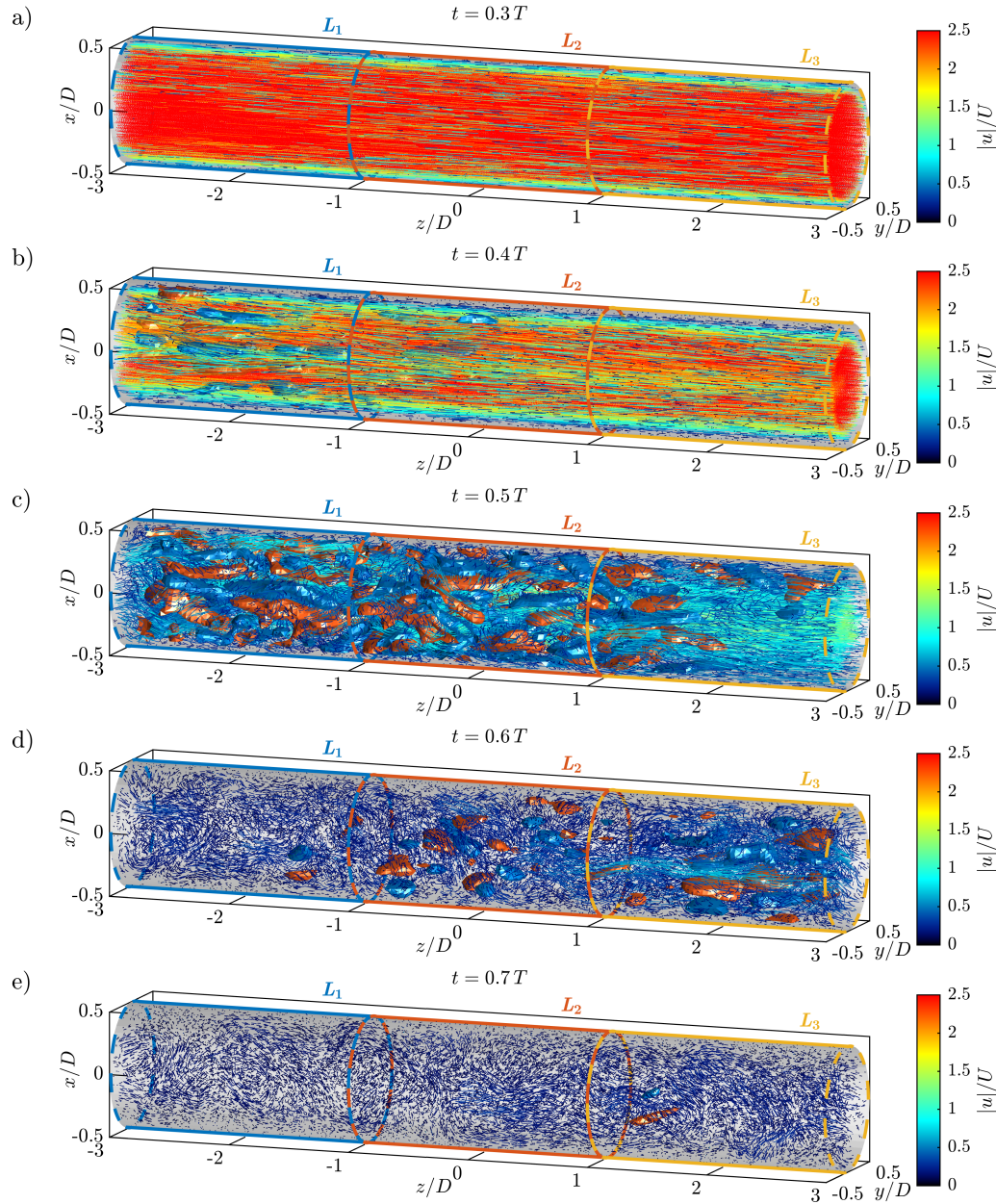


FIGURE 2.59: a)-e) Snapshots of flow trajectories from STB measurements for a physiological pulsation with $Re = 1277$; $Wo = 8$ located $20D$ downstream after a bending perturbation of $D/D_{bend} = 0.034$ at $t = [0.45; 0.5; 0.6; 0.7; 0.8]/T$. Red and blue structure represent isosurfaces of the streamwise vorticity with $\omega_z = \pm 2U/D$. The measurement volume is separated into three regions, three regions of measurement volume, upstream (L_1 , blue), center (L_2 , orange) and downstream (L_3 , yellow), for evaluating the flow profiles in Fig. 2.60.

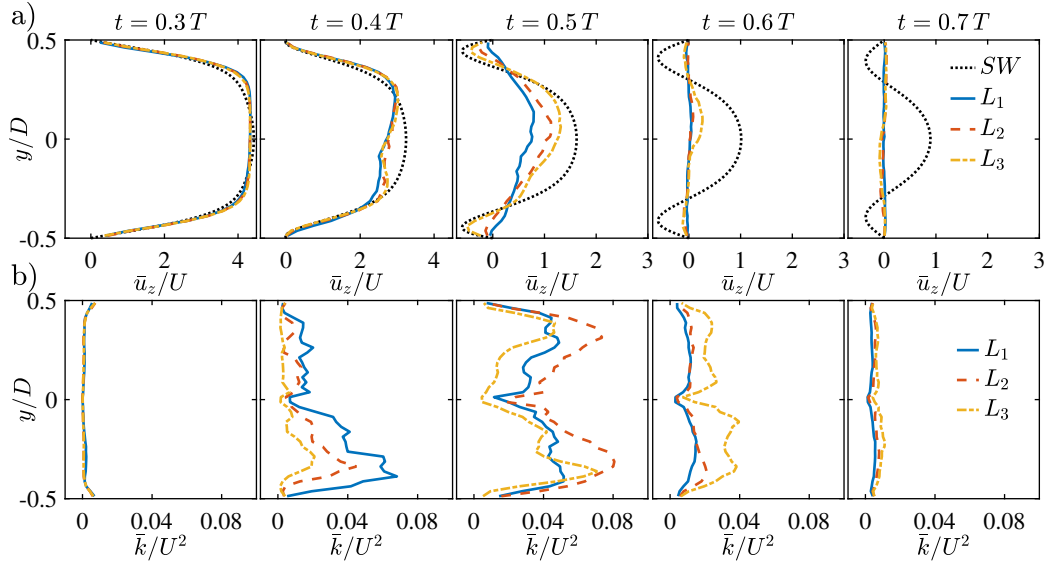


FIGURE 2.60: a) Profiles of the streamwise velocity u_z averaged over three regions of measurement volume (see Fig. 2.59), upstream (L_1 , blue), center (L_2 , orange) and downstream (L_3 , yellow), compared the flow profile without a perturbation (black). b) Profiles of the turbulent kinetic energy k in the same three regions. Both the profiles of velocity and turbulent kinetic energy are axially averaged over $2D$ in the three colored sections and azimuthally averaged for cross-section half facing the bend ($y < 0$) and the half facing away from the bend ($y > 0$).

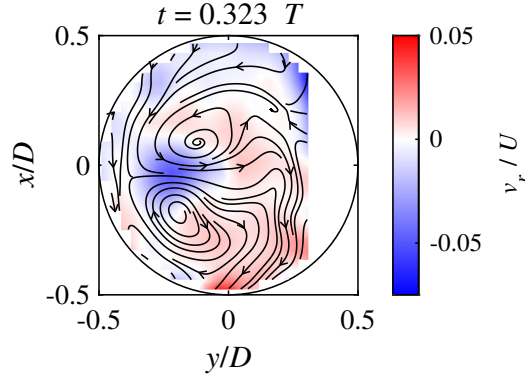


FIGURE 2.61: Radial velocity field from STB measurements in the section closest to the bend (section L_1 in Fig. 2.59 at $t = 0.32T$, shows a pair of Dean vortices).

2.4.4 Comparing different turbulence transition scenarios

In this study, two different types of perturbations were used to trigger turbulence in a pulsatile pipe flow: a suspension of particles and a geometric bend perturbation. For both perturbation types, the transition is mainly affected by the advective timescale of the flow, instead of a phase-locked onset stemming from instabilities in the corresponding instantaneous velocity profile. This results in the earliest turbulence onset (during the pulsation period) with the longest turbulence patches, and highest turbulence strength at low Wo . As Wo increases and the advective duration of the periods shortens, the onset is delayed, and the turbulence intensity is weaker. This trend was observed for a

physiological pulsation as well as a large amplitude harmonic pulsation with particle-laden flow, and with both perturbation types and a physiological pulsation. For the particle-laden flow, the turbulence onset and duration could be directly correlated to the dynamics of a particle-laden steady flow, with the timescale of the waveform above a critical $Re_{c,p}$ determining the duration to which turbulence patches will grow.

To evaluate the effects on the flow field, STB measurements were performed with a single large hydrogel particle, a suspension of small particles, and a flow perturbed by a geometric bend (with $Re_m = 1277$, $Wo = 8$ and a physiological waveform). For all three cases, the temporal development is similar, with an initial onset and turbulence growth during deceleration ($t \approx 0.4T$). In the long low-velocity phase of the physiological waveform, where no new energy is fed into the flow, the turbulent structures start decaying. The flow fully relaminarizes ($t \approx 0.7T$) before the flow accelerates again, resulting in temporally intermittent turbulence patches during each period. The turbulence patch completely flattens the flow velocity profile in all three cases, inhibiting the growth of other instabilities during the low velocity phase.

Looking at the flow structure, the onset mechanisms can be better distinguished. For the single-phase flow with the geometric bend in the pipe geometry (see section 2.4.3.2), the initial flow perturbation arises from a pair of Dean-vortices caused by the curvature. These vortices split-up and continuously break down into a turbulent patch. As the secondary Dean-flow emerges inside the bending, the turbulence structure appears spatially localized behind the curved segment. Overall, this is qualitatively similar to the perturbation by a single large hydrogel particle. Here, too, the turbulence is spatially localized around a perturbation source. Initially, as the flow decelerates, only the upstream wake of the particle is turbulent. Only after the bulk velocity is close to zero, does the turbulent structure detach from the particle. The downstream flow is characterized by multiple vortex pairs. Switching from a single large particle to a suspension of smaller particles causes a globally acting perturbation. As the concentration is homogeneously spread throughout the entire pipe, the onset appears everywhere, with each particle perturbing the flow.

Overall, both geometric bends and particles are attributes of the cardiovascular system, and the observed dynamics of a large perturbation turbulence onset can be highly relevant for physiological flows.

2.5 Summary

Investigating particle-fluid interactions is essential for understanding and predicting the dynamics of complex flow systems like human cardiovascular flow. Despite the complexity and relevance of this system, there are few fundamental studies on the dynamics of particles in pulsatile flow, especially its turbulence transition. The goal of this chapter was to understand turbulence transition mechanisms in a pipe flow with a physiological pulsation and the perturbation of neutrally buoyant particles. For the purpose of this investigation, a new pipe flow experiment had to be designed and constructed. This section will first summarize the pipe experiment design and methods developed in this chapter (section 2.5.1). Afterwards, the main findings on the turbulence transition in pulsatile pipe flow are summarized (section 2.5.2).

2.5.1 Pipe flow experiment

The primary objective in designing the new pipe flow experiment was to create a setup capable of realizing and investigating particle-laden pulsatile flow in the transitional regime between laminar and turbulence. Existing experimental pipe flow studies of turbulence transition focus on either single-phase flows both steady (Avila et al., 2011; Doorne and Westerweel, 2007; Hof et al., 2004) and pulsatile (Peacock et al., 1998; Trip et al., 2012; Xu et al., 2020), or particle-laden steady flows (Agrawal, Choueiri, and Hof, 2019; Hogendoorn and Poelma, 2018; Matas, Morris, and Guazzelli, 2003). The aim of the new experiment was to bridge this gap and create a setup capable of creating precise pulsatile flow conditions and performing flow measurements in a particle-laden flow. The pipe experiment is constructed of multiple glass segments with a larger diameter of $D = 28$ mm to facilitate optical flow measurements and has a maximum total pipe length of $L = 20$ m $= 714 D$ for a sufficient development length for both flow and particles. To produce the flow, a high-precision piston driving system was implemented, providing the flexibility to generate steady flows or pulsations with variable frequencies and complex waveforms. The experiment was tested with different particle materials, sizes, and concentrations added to a water-glycerol fluid flow.

For a particle-laden flow, one of the challenges in traditional experimental fluid mechanics is measurements which capture both the motion of solid particles while also resolving the surrounding fluid velocity field. Most often these two dynamics have to be captured separately through different measurements. In the pipe flow experiment, both aspects can be captured simultaneously using three-dimensional Lagrangian Particle Tracking measurements using Shake-The-Box (STB), with the key innovation of utilizing transparent, refractive index matched hydrogel as the particle-phase. The implementation of STB in a pipe flow required a custom optical setup and calibration procedure. The hydrogel particles enables optical access and tracer illumination without distortion or occlusion. As some tracer naturally adhere to the hydrogel interface, a commercially available STB algorithm can determine trajectories of the fluid flow and the particle surface. In a custom developed post processing method, the interface trajectories are identified and used to determine particle's translation and rotation dynamics. In combination with the temporally and spatially resolved fluid flow field, such measurements were previously not possible in pipe flows.

Complementary to STB, additional measurement techniques are implemented for larger scans of flow parameters. A novel single camera particle imaging method was developed to track the three-dimensional position of particles in the pipe using a calculated refraction model of the pipe geometry, without the need for refractive index matching hardware. This allows simultaneous tracking of the axial and radial particle clustering and a time-resolved

evaluation of the local particle concentration at different positions along the pipe. Lastly, a high-resolution differential pressure drop measurement can be used to detect the onset of turbulence or flow instabilities.

The modular design of the experiment enables the addition of additional measurement techniques or other structural modifications, such as the integration of bends or other flow perturbations to induce secondary flow structures and trigger specific flow patterns (Samanta, De Lozar, and Hof, 2011; Xu et al., 2020). The piston driving mechanism, with a separation of particles and fluid, makes it possible to implement different particles, or more complex fluids, to reproduce different flow applications.

2.5.2 Turbulence transition

In this section, the research questions formulated at the beginning of the chapter (see section 2.2) on the turbulence development in a pulsatile particle-laden pipe flow are addressed.

RQ1: Mechanisms of a particle perturbation in pulsatile flow

The turbulence transition in particle-laden pulsatile flow with a large pulsation amplitude is characterized by the emergence of intermittent turbulent patches during each pulsation period. These patches grow rapidly during the velocity peak of the pulsation waveform and subsequently relaminarize in the low-velocity phase. In contrast to previous studies of large-amplitude pulsatile flows, where turbulence originates from the modal growth of helical instabilities during the deceleration phase (Morón, Feldmann, and Avila, 2022; Xu et al., 2020), the particle-laden flow shows the same growth mechanisms of puffs and slugs in steady pipe flow.

Central to this process is the interaction between the particles and the flow field. STB measurements reveal the formation of vortices in the wakes of individual particles, which act as finite-amplitude perturbations and trigger the formation turbulent patches. Crucially, this growth can occur during the acceleration phase of pulsation period, before the Sexl-Womersley flow profile would naturally become unstable to helical perturbations. For a sufficiently long pulsation period, the triggered turbulent puffs will expand to turbulent slugs. Once a puff has been triggered, the Sexl-Womersley profile is disrupted, inhibiting any subsequent growth of a helical perturbation.

Furthermore, a direct link could be established between the dynamics of particle-laden pulsatile flow and the transition criteria in steady pipe flow. The delay in turbulence growth and the resulting duration of turbulent patches are directly coupled to the critical Reynolds number ($Re_{c,p}$) required for a puff onset in steady particle-laden flow. Independent of the Womersley number, there is a constant delay (corresponding to the advective timescale) between the moment where the bulk velocity surpasses $Re_{c,p}$ and the onset of turbulence. Consequently, the duration of the turbulence patch within a single period directly corresponds to the duration of the waveform exceeding the critical Reynolds number ($\Delta t \approx t(Re > Re_{c,p})$). Similar to previous observations in steady pipe flow (Hogendoorn, Chandra, and Poelma, 2022; Matas, Morris, and Guazzelli, 2003), a higher concentration of particles will expedite the turbulence onset within a period.

When comparing these findings with a different perturbation mechanism of a bend in the pipe geometry, a remarkably similar transition mechanism is observed. In this case, the Dean flow that develops inside the bend serves as a perturbation to trigger puffs, mirroring the role of the particle wake. This suggests a separation for turbulence transition regimes in pulsatile flow: while small perturbations may allow for the development of modal instabilities, sufficiently large flow perturbations (such as strong bends or particles suspensions) will directly trigger puffs and slugs.

RQ2: Influence of the pulsation waveform

The investigation in this chapter was mainly focused on a pulsatile pipe flow with a physiological waveform based on the flow in the descending human aorta. This waveform is characterized by a sharp acceleration and deceleration phase followed by an extended low-velocity phase. These features should make the flow susceptible to stronger growth of flow instabilities (Morón, Feldmann, and Avila, 2022). However, a comparison of the physiological waveform to a harmonic pulsation with the same peak Reynolds number revealed the same turbulence transition mechanism of puffs for both waveforms. For a given Womersley number, a harmonic pulsation shows longer-lasting turbulence patches, compared to flow driven by a physiological waveform. This is due to the longer velocity peak in a harmonic waveform, which provides a higher duration above the critical Reynolds number ($Re_{c,p}$) for turbulent patches to be sustained. Although the specific shape of the waveform does result in a slightly different phase-dependent delay for the onset of turbulence, these differences are secondary to the much more pronounced influence of the Womersley number and timescale of the pulsation.

RQ3: Womersley effects and the advective timescale

In pulsatile pipe flow the Womersley number affects both the flow velocity profile, which is associated with the modal instabilities, and the period duration, providing more or less advective time for turbulence patches to develop. This investigation finds the latter effect and the advective timescale to be the dominant factor for the observed turbulence transition mechanism. As previously established, the duration of the velocity peak in the pulsation waveform directly dictates the duration of turbulence patches. Consequently, a change in Wo and period length T will correspond to a change in both the advective duration and strength of the developing turbulence patches. As a result, a low Womersley flow (long T) will exhibit the strongest turbulent patches. The extended duration of the velocity peak allows for maximal kinetic energy injection, providing sufficient time for the development of large turbulent slugs. In contrast, as Wo increases and the pulsation period is shortened, there is a proportional decrease in both patch duration and strength. Notably, this linear relationship between pulsation period duration, turbulence patch duration, and turbulence strength, was observed regardless of the investigated pulsation waveform (physiological or harmonic) or perturbation mechanism (suspended particles or geometric bend). This underscores the importance of the advective timescale in this turbulence transition scenario of large perturbations.

Summary of impact

The findings of this chapter demonstrate that the features typically found in cardiovascular flows, such as strong geometric deformations or suspended particles, are able to trigger turbulence in pulsatile pipe flow. A novel regime of turbulence development in large amplitude pulsatile flow with strong flow perturbations could be established and characterized. Here, the turbulence onset does not develop from the flow instabilities, but instead has a growth of puff-like turbulent patches, where both duration and strength of turbulence will be dictated by the advective timescale of the pulsation. In physiological flows with much stronger perturbation, such as the aortic arch, it is likely that the same transition mechanism will be dominant.

Chapter 3

Nonlinear resonant dynamics of sloshing liquids

Sloshing describes the motion of a free liquid surface within a container subjected to external forcing. The most prominent applications are propellant tanks in rockets and spacecraft. Here, vibrations during flight can trigger resonances, where even minimal forcing amplitudes lead to drastic wave amplification characterized by nonlinear effects such as frequency shifts and coexisting states, which are notoriously challenging to accurately predict (as depicted in Fig. 3.1).

This chapter investigates these nonlinear dynamics by comparing resonance models with controlled experiments conducted in a narrow rectangular container under harmonic forcing. The emergence of nonlinear effects for larger forcing amplitudes can be precisely captured with this configuration.

Parts of the content presented in this chapter led to the publication¹ of B  uerlein and Avila (2021a)

The content of this chapter is structured as follows:

- Section 3.1: General overview of the theoretical background on liquid sloshing and a review of existing research on sloshing resonances.
- Section 3.2: Main research questions.
- Section 3.3: Experimental methods and characterization of the sloshing motion.
- Section 3.4: Experimental findings of the sloshing resonance and assessment of different resonance model approaches.
- Section 3.5: Summary.

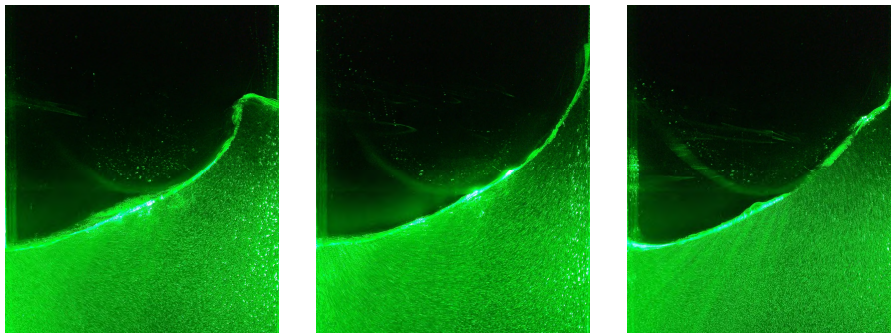


FIGURE 3.1: Raw images of three different sloshing waves near the resonance.

¹B. B  uerlein carried out the measurements, the data analysis, and wrote the first draft of the manuscript.

3.1 Theoretical background

This section provides a theoretical foundation for understanding the sloshing dynamics investigated in this chapter, with a focus on resonance phenomena and their modeling. It begins with the classical description of sloshing in linearized potential flow theory, from which the general wave motions and natural frequency of the sloshing modes can be derived (section 3.1.1). While this linear approach gives a fundamental model of sloshing motion, it fails to describe the nonlinear effects which emerge at larger sloshing amplitudes, for example, during resonances. This includes amplitude dependent shifts of the resonance frequency, coupling of different modes, and more complex wave motions (section 3.1.2). One example of describing the emerging nonlinear interactions is through a weakly nonlinear analysis of potential flow as the multimodal model derived by Faltinsen et al. (2000) (section 3.1.3). In addition to this approach, equivalent mechanical models are presented as a simplified, low-dimensional analogues system often utilized in engineering applications (section 3.1.4). For all descriptions of resonant sloshing, the effect of energy dissipation in a the wave motion has to be considered (section 3.1.5).

3.1.1 Sloshing in potential flow theory

A simple and common description of sloshing is through the potential flow theory, that provides analytical solutions for the natural frequencies and waveforms of the normal modes of the system. Abramson (1966) provides a comprehensive summary of these solutions in the *NASA-SP-106* report for various tank geometries. This work was later revised and updated with new findings by Dodge (2000).

This chapter is focused on a rectangular fluid container under lateral excitation with a width w , a depth l , a filling level h and a surface displacement ζ in the x - y -plane, see Fig. 3.2.

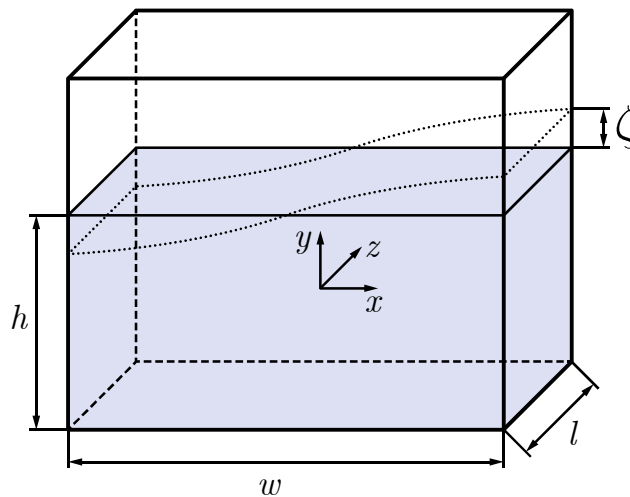


FIGURE 3.2: Sketch of a rectangular container with a width w , depth l and filling level h . Sloshing waves create surface displacements $\zeta(x, z, t)$. The origin of the coordinate x, y, z is set at in the center of the container.

In potential flow several simplifying assumptions are made: the fluid flow is inviscid ($\nu = 0$), irrotational ($\nabla \times \mathbf{u} = 0$) and incompressible ($\rho = \text{const.}$). The fluid velocities are assumed to be small with linear wave motion, and the container is rigid. In potential flow theory, an inviscid and irrotational flow can be described using a velocity potential $\nabla \Phi = \mathbf{u}$. Combining this with the conservation of mass under the condition of incompressibility

($\nabla \mathbf{u} = 0$), the full NS-Eq. (1.1) is simplified to the Laplace equation,

$$\nabla^2 \Phi = 0. \quad (3.1)$$

For sloshing in a rectangular container, the following boundary conditions can be set:

1. No flow through the container walls,

$$\nabla \Phi \cdot \mathbf{n} = 0 \quad \text{on all rigid boundaries.} \quad (3.2)$$

2. Linearized displacement of the wave at the free surface,

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \Phi}{\partial y} \quad \text{at } y = h/2. \quad (3.3)$$

3. Linearized free surface dynamics driven by gravity,

$$\frac{\partial \Phi}{\partial t} + g\zeta = 0 \quad \text{at } y = h/2. \quad (3.4)$$

Assuming that all solutions are harmonic in time, Eq. (3.1) can be solved by separation of variables, where the velocity potential Φ is expressed as the product of eigenfunctions E and a harmonic component,

$$\Phi(x, y, z, t) = E_x(x) E_y(y) E_z(z) e^{i\omega t}. \quad (3.5)$$

By imposing the listed boundary conditions, the eigenvalues are computed from which the natural frequencies of the wave components can be calculated (for more details of the solution see Dodge (2000)). The natural frequency of two-dimensional waves in the xy -plane is,

$$\omega_n^2 = \frac{g\pi}{w} n \tanh \left[\pi n \frac{h}{w} \right]. \quad (3.6)$$

where n is the mode number, $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration, w is the container width, and h is the filling level. The hyperbolic tangent term reflects the effect of the filling level: for larger filling ratios h/w in a deep tank, the frequencies asymptotically approach a frequency limit, and the different modes scale with $\sqrt{n}$. Each natural frequency corresponds to a normal mode of the surface wave elevation. The first six modes are displayed

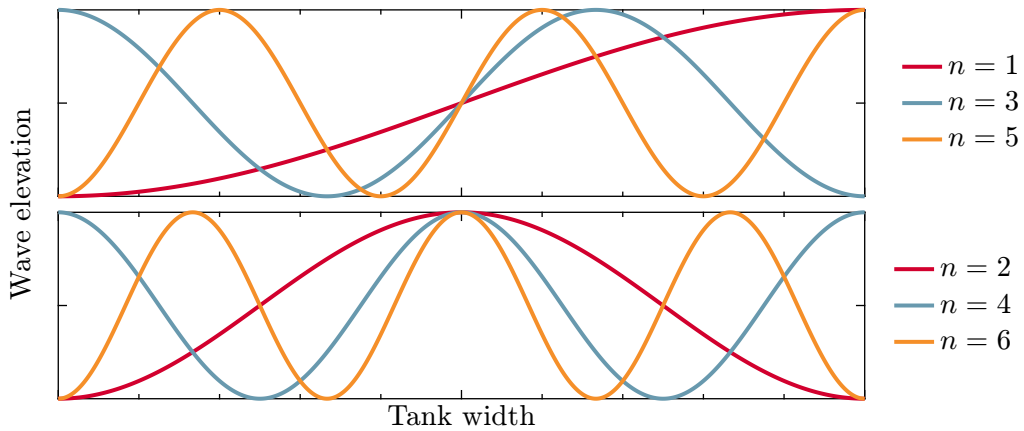


FIGURE 3.3: Dimensionless shape of the two-dimensional sloshing wave for the first three antisymmetric modes (top) and axisymmetric modes (bottom) in a rectangular container.

in Fig. 3.3. In the sloshing literature, a distinction is often made between antisymmetric modes (odd mode numbers n) and axisymmetric modes (even mode numbers n) (Abramson, 1966). Only antisymmetric wave modes will create a horizontal displacement of the liquid's mass and produce lateral sloshing forces on the container. The first mode $n = 1$ with the lowest natural frequency and the strongest sloshing forces is generally considered to be most relevant for engineering applications (Dodge, 2000).

The simplified description of sloshing in potential flow works well for small linear wave amplitudes and negligible viscosity effects. However, in the vicinity of a resonance, nonlinearities might dominate, and viscous damping forces significantly affect the sloshing motion (Faltinsen and Timokha, 2009). This makes the dynamics close to the resonance far more complicated, as I will show in the next section.

3.1.2 Nonlinear sloshing

For small motions, potential flow theory with linearized boundary conditions are sufficient to describe sloshing. However, with larger forcing amplitudes or proximity to a resonance, the amplitudes will grow significantly, and the simplified assumptions of the previous section are no longer valid. The sloshing motion can be split into three regimes (Ibrahim, 2005):

1. *Linear sloshing*, where the oscillation amplitudes are small and the wave remains planar.
2. *Weakly nonlinear sloshing*, where the surface shows non-planar motion, modal interactions, frequency shifts of the resonances, coexisting states and hysteresis.
3. *Strongly nonlinear sloshing* includes more complex surface effects like wave-breaking, hydraulic jumps, or even fully chaotic motion.

Early experimental work on resonant waves by Taylor (1953) showed a shift of the resonance peak, with coexisting states and hysteresis in a wavemaker tank. Similar observations were later made by Abramson, Chu, and Kana (1966) in cylindrical tanks, where resonant sloshing could also trigger rotating swirling waves in addition to standing waves. An analytical study by Tadjbakhsh and Keller (1960) predicted that while a large filling level will shift the resonance towards lower frequencies, a lower filling level will move the resonance to higher frequencies. This was later confirmed in experiments by Fultz (1962). Chester (1968a,b) showed that during the primary resonance of shallow water sloshing several modes compete with complex motions and higher harmonics emerging. The transition region between deep and shallow sloshing was further investigated by Waterhouse (1994). For a review of the nonlinear sloshing resonance at different filling levels, see Ockendon and Ockendon (2001).

One of the main differences between the linear potential flow solution and the experimental observation of larger amplitude sloshing, is due to the restricted linearized boundary conditions of the free surface motion. By utilizing a nonlinear boundary conditions that include higher order terms, a weakly nonlinear analysis of potential flow theory (e.g. by Ockendon and Ockendon (1973) and Tadjbakhsh and Keller (1960)) can derive amplitude equations that predict some of the nonlinear effects. A multimodal model approach was formulated by Faltinsen et al. (2000) that captures the effects of resonance shift and multiple mode interacting. This model will be further explained in the next section 3.1.3.

In general, sloshing models such as the multimodal theory are limited to a weakly nonlinear regime. For the more complex strongly nonlinear sloshing motion, the assumptions of a potential flow approximation are no longer valid (Faltinsen and Timokha, 2009). While some equivalent mechanical models aim to predict effects such as impact loads on the container walls (Farid and Gendelman, 2017), an accurate description is usually only

possible with CFD simulations. An additional effect of complex waves during nonlinear sloshing, is the increase of energy dissipation, which will be discussed in section 3.1.5.

3.1.3 Multimodal model

The state-of-the-art approach to model sloshing is via a multimodal model. Here, the sloshing motion is represented as a (modal) system of nonlinear ordinary differential equations, where the coupled modes can be derived from potential theory. This theory was first conceptualized by Moiseev (1958) and later extended by Ockendon and Ockendon (1973) using asymptotic methods to include weakly nonlinear effects and modal interactions. The multimodal approach became prominent with the work of Faltinsen et al. (2000), who formulated nonlinear sloshing as a finite set of modes, capturing both primary and coupled responses in a rectangular tank.

In the multimodal model, the displacement of the surface wave in a container is given as a Fourier expansion of modes. Faltinsen and Timokha (2009) provide the analytical model equations, derived from potential theory for a two-dimensional flow in rectangular container. The main equations necessary for computing this model are presented in this section, with additional parameters in Appendix B.3. For a harmonic lateral forcing of the tank with the form $A \cos(\omega t)$ (with dimensionless forcing amplitude A and forcing frequency ω), the resulting elevation of the free surface ζ as a function of the horizontal container coordinate $\xi \in [-w/2, w/2]$ and of time t reads,

$$\zeta(\xi, t) = w \sum_{n=1}^{\infty} \beta_n(t) \hat{f}_n(\xi), \quad (3.7)$$

where

$$\hat{f}_n(\xi) = \cos\left(\pi n \left(\xi + \frac{1}{2}w\right) / w\right),$$

is the dimensionless waveform of the surface for mode n (see Fig. 3.3 for the wave shapes) and β_n the function of its temporal evolution. Plugging this approach into the potential flow equations, gives an infinite system of (coupled) nonlinear differential equations for the coefficients β_n . The system gets truncated based on the number of relevant modes. The functions of β_n for the first three modes ($n = 1, 2, 3$) from Faltinsen and Timokha (2009) read,

$$\begin{aligned} \beta_1(t) &= \hat{Y} \cos(\omega t) + \tilde{n}_1 \hat{Y}^3 \cos(3\omega t) + \mathcal{O}(\hat{Y}^5), \\ \beta_2(t) &= \hat{Y}^2 (w_0 + h_0 \cos(2\omega t)) + \mathcal{O}(\hat{Y}^4), \\ \beta_3(t) &= \cos(\omega t) \left[\tilde{N}_1 \hat{Y}^3 - AP_3 / (1 - (\omega_3/\omega)^2) \right] + \tilde{N}_2 \hat{Y}^3 \cos(3\omega t) + \mathcal{O}(\hat{Y}^5), \end{aligned} \quad (3.8)$$

with the dimensionless amplitude parameter $\hat{Y}$ and the natural frequencies of the modes ω_n (see Eq. (3.6)). The additional parameters $\tilde{n}_1, \tilde{N}_1, \tilde{N}_2, w_0, h_0$ can be calculated based on the filling ratio and forcing frequency (see Appendix B.3). These solutions for β_n contain the first three harmonics of the driving frequency ω , where the magnitude of each term scales with different orders of the amplitude parameter $\hat{Y}$. For small amplitudes only the first mode β_1 scaling with $\mathcal{O}(\hat{Y})$ will be dominant, resulting in a singular harmonic oscillation with standing sine wave from Eq. (3.1.3). For an increasing amplitude parameter $\hat{Y}$ the additional modes ($\beta_{2,3}$) with higher harmonics and additional waveforms ($\hat{f}_{2,3}$) will appear.

For a steady state surface oscillation under harmonic forcing the dimensionless amplitude parameter can be expressed as a function of the driving parameters of the system. An analytical equation with an added linear viscous damping term was derived by Faltinsen

and Timokha (2017) and reads,

$$\hat{Y} = \frac{P_1 A}{\sqrt{\left[(\omega_1/\omega)^2 - 1 + M_1 \hat{Y}^2\right] + \gamma^2}}, \quad (3.9)$$

which only depends on the filling level (with P_1 , see appendix B.3), the dimensionless driving amplitude A , the driving frequency ω , the damping ratio γ^2 (see section 3.1.5). The nonlinearity of the amplitude response is dictated by the parameter M_1 , which can be calculated with,

$$M_1 = d_1 \left(-w_0 + \frac{1}{2} h_0 \right) - \frac{1}{2} d_2 - 2d_3 h_0. \quad (3.10)$$

The phase lag between driving and sloshing motion of each steady state can be similarly expressed as,

$$\cos \phi = \frac{\hat{Y} \left[(\omega_1/\omega)^2 - 1 + M_1 \hat{Y}^2 \right]}{P_1 A}. \quad (3.11)$$

Note that these solutions are still derived from inviscid potential flow, the viscous damping is only added afterwards to correct the result for the estimated effect of viscous boundary layer forces.

The important parameter determining nonlinear resonance behavior for both amplitude $\hat{Y}$ and phase ϕ of the oscillation is M_1 from Eq. (3.10). Depending on the filling level h/w , M_1 will change sign. The effect of this is shown in the amplitude responses as a function of the dimensionless driving frequency $\Omega = \omega/\omega_1$ in Fig. 3.4a. With large filling levels (red), the parameter M_1 has a negative sign and the resonance of the system is shifted toward frequencies below the natural frequency ($\Omega < 1$). The shift displayed here is sufficiently large for multiple coexisting stable sloshing states to emerge (solid lines), connected by an unstable solution (dotted line). In the regime of shallow filling levels (blue), the resonance peak shifts to higher frequencies ($\Omega > 1$), with a positive parameter M_1 . The shift toward lower frequencies is called *softening* and toward higher frequencies *hardening*. This terminology is taken from the Duffing (1918) model, which will be further explored in section 3.1.4. Exactly at a threshold filling level h_{crit} (black line), there is no frequency shift and the system behaves exactly like a harmonic oscillator without coexisting or unstable solutions. The transition between hardening and softening resonance in sloshing has been investigated for a long time (Fultz, 1962; Ockendon and Ockendon, 1973; Tadjbakhsh and Keller, 1960; Waterhouse, 1994). The critical filling level for the transition threshold in the Faltinsen et al. (2000) model is $h_{crit} = 0.3374w$. The effect of the resonance shift and coexisting states continuously increases with the driving amplitude A of the system. At small driving the response will be more similar to the harmonic oscillator response of h_{crit} .

The same effect of frequency shift can also be seen in the phase lag response in Fig. 3.4b. For low frequencies, the sloshing oscillation is in phase with the driving, asymptotically approaching $\phi = 0^\circ$. The response above the resonance peak is out of phase with $\phi \rightarrow -180^\circ$. Same as for the amplitude, the phase will show different coexisting and unstable solutions with a strong resonance shift.

After the original conception by Faltinsen et al. (2000), the multimodal model for rectangular tanks was later extended to work with different types of excitation motion Faltinsen and Timokha (2001) and incorporate the shallow tank regime $0.1 \leq h/w \leq 0.24$ Faltinsen and Timokha (2002). In a series of publications the multimodal approach was also formulated for three-dimensional waves in square base rectangular tanks (Faltinsen, Rognbakke, and Timokha, 2003), include effects of higher modes (Faltinsen, Rognbakke, and Timokha, 2005), geometry effects of the tanks (Faltinsen, Rognbakke, and Timokha,

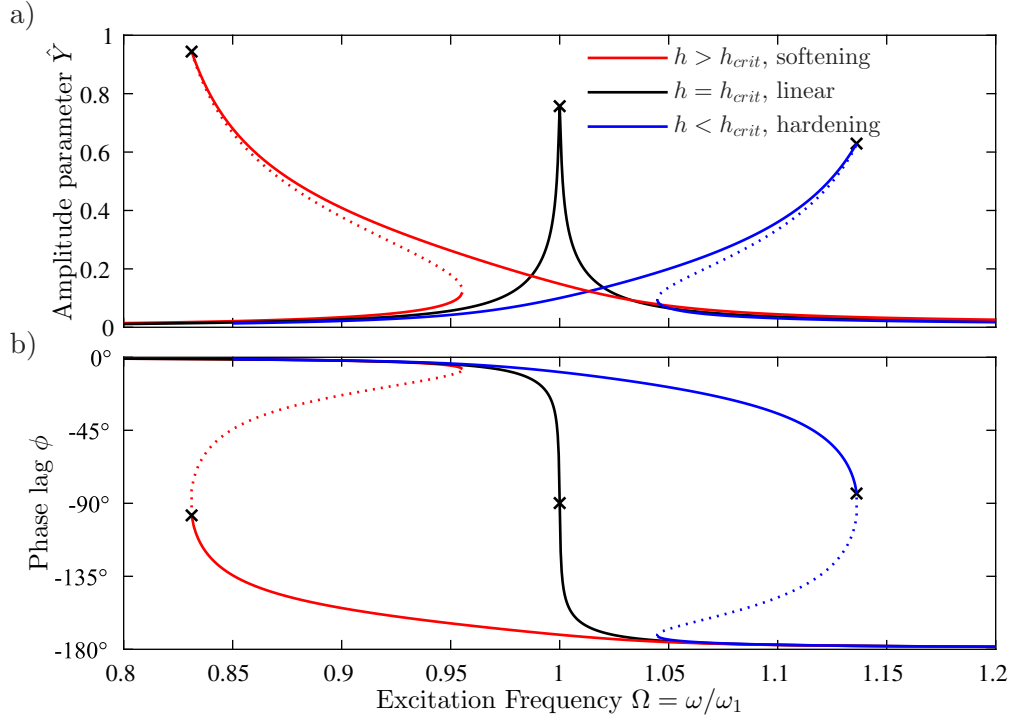


FIGURE 3.4: Frequency response curves of the amplitude parameter a) and phase lag b) obtained from the multimodal model by (Faltinsen and Timokha, 2009) with Eq. (3.9) and Eq. (3.11), for the different filling level. The x marks the maximum amplitude for each filling level. Dotted lines denote unstable solutions.

2006), linear viscous damping (Faltinsen and Timokha, 2017) (as shown in Eq. (3.9)) and non-parametric forcing (Faltinsen, Lagodzinskyi, and Timokha, 2020).

The large advantage of the multimodal approach is that it provides a physics-based solution for sloshing motion, which is far less computationally demanding than a full CFD solution of the Navier-Stokes equations. It can capture nonlinear interactions between the modes and can be applied to different filling levels, forcings, and container geometries. However, unlike the fully analytical solution shown in this section, the solution for more complex scenarios, such as shallow water sloshing (Faltinsen and Timokha, 2002) or three-dimensional tanks (Faltinsen and Timokha, 2017) will require numerical approaches to predict the resonance. A simplified engineering approach of an equivalent mechanical model will be shown in the next section.

3.1.4 Equivalent mechanical models

Although sloshing motion can become highly complex, for many applications it is often represented by simplified equivalent mechanical models (Dodge, 2000). The goal is to use a realistic representation with the same mass and inertia, center of mass, natural modes, forces and moments that act on the container walls due to the sloshing motion (Ibrahim, 2005). The two main approaches are mass-spring-dashpot models (Deng and Tait, 2008; Li, Di, and Gong, 2012; Noorian, Haddadpour, and Ebrahimian, 2016) and pendulum models (Sumner, 1965). In general, the parameters of equivalent models are derived analytically with potential flow assumptions (Graham and Rodriguez, 1952). An overview of model parameters for the most common tank geometries was summarized by Ibrahim (2005). Linear equivalent mechanical models are limited to small oscillations with planar wave motion, far away from resonances.

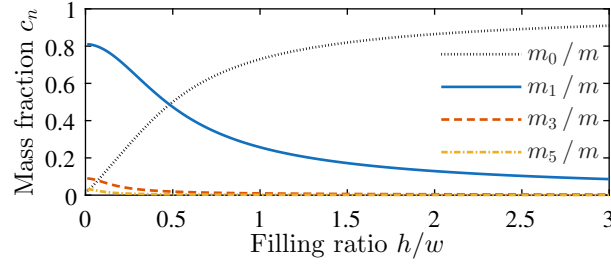


FIGURE 3.5: Mass fraction m_n/m of the first three asymmetric modes $n = [1, 3, 5]$ compared to the resting mass m_0 that does not contribute to sloshing motion.

For an equivalent mechanical model with a mass-spring-dashpot system, the equation of motion for any mode number n is that of a harmonic oscillator,

$$m_n (\ddot{x}_n + 2\delta_1 \dot{x}_n + \omega_n^2 x_n) = m_n F \cos \omega t, \quad (3.12)$$

with m_n as the liquid mass in motion, x_n the horizontal displacement of its center of mass, the viscous damping δ_n and natural frequency ω_n . In this approach, each mode n is treated as a separate harmonic oscillator, with m_n corresponding to the moving mass as a fraction of the total mass m of fluid in the container. The mass fraction of each mode $m_n = c_n m$ can be computed for a rectangular container as a function of the filling level in potential theory (Graham and Rodriguez, 1952),

$$c_n = \frac{8}{\pi^3} \frac{\tanh \left[\pi n \frac{h}{w} \right]}{n^3 \frac{h}{w}}. \quad (3.13)$$

This equation is only valid for odd mode numbers n , as the horizontally symmetric waves of even mode numbers will produce no horizontal displacement of fluid mass. The mass fraction continuously decreases for higher filling levels, with the mass of the first mode always being dominant, see Fig. 3.5. The mass of the higher modes reduces with $1/n^3$.

To capture more complex dynamics of sloshing, a nonlinear equivalent mechanical model can be used. One approach is to implement more degrees of freedom. While a harmonic oscillator only captures a single degree of freedom, a model like a spherical pendulum can describe the effect three-dimensional swirling waves (Kana, 1989). Alternatively, a linear model can be extended with additional higher-order terms to better reflect the dynamics of larger amplitude sloshing. Here, experimental studies by Bauer (1966) show that a third-order nonlinear spring force can be added to a linear oscillator to show the effect of resonance shifts. Similar findings of cubic forcing terms improving mechanical models were made by Farid and Gendelman (2018), Hill (2003), and Sayar and Baumgarten (1981).

When a harmonic oscillator (Eq. (3.12)) is extended with a cubic nonlinear spring force ($\epsilon_n x_n^3$), the resulting equation is known as the Duffing equation,

$$m_n (\ddot{x}_n + 2\delta_1 \dot{x}_n + \omega_n^2 x_n + \epsilon_n x_n^3) = m_n F \cos \omega t. \quad (3.14)$$

This equation was first introduced by the German engineer Duffing (1918). While investigating the vibrations of machines, Duffing observed that (as translated by Kovacic and Brennan (2011)),

... the operation is much calmer above resonance than at the same distance below resonance,

which could not be adequately explained with the existing linear resonance theories. The Duffing equation can capture essential nonlinear effects, such as resonance frequency shifts,

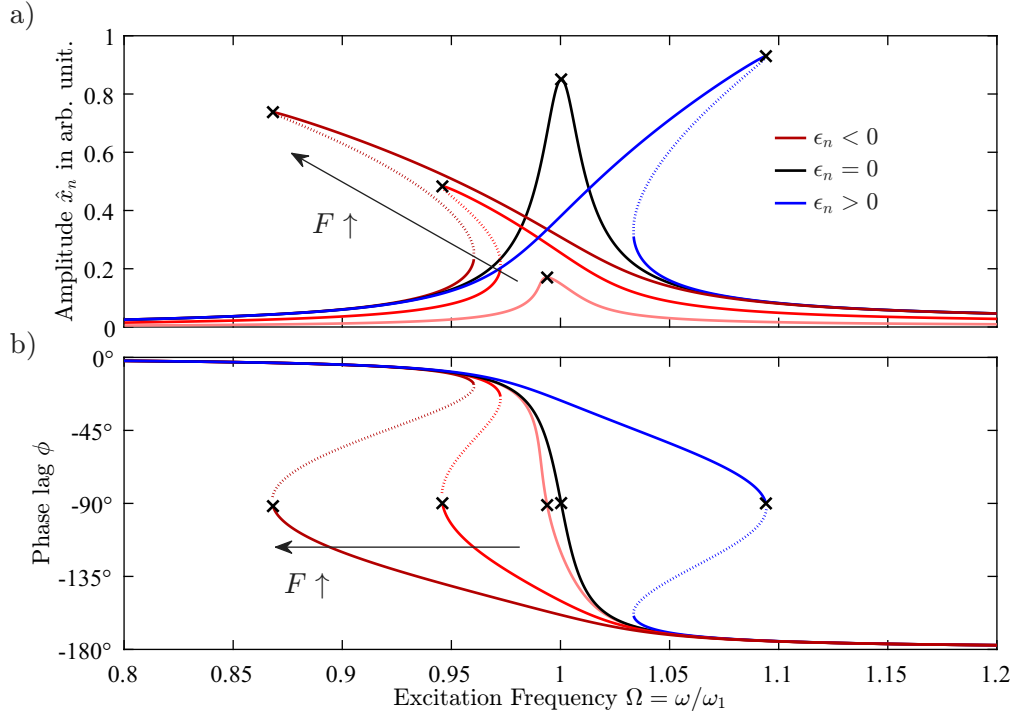


FIGURE 3.6: Frequency response curves of the amplitude a) and phase lag b) from the Duffing oscillator with Eq. (3.16) and Eq. (3.17), for the different nonlinearity parameters ϵ_n . The x marks the maximum amplitude for each filling level. For the softening case $\epsilon_n < 0$ the response is shown for different forcing amplitudes F .

hysteresis, jump phenomena, and even chaotic oscillations, and has become a fundamental example for exploring nonlinear dynamical systems (Kovacic and Brennan, 2011). The steady state solution of a forced Duffing equation (Eq. (3.14)) reads,

$$x_n(t) = \hat{x} \cos(\omega t + \phi), \quad (3.15)$$

where the oscillation amplitude is (derived with the harmonic balance method (Kovacic and Brennan, 2011)),

$$\hat{x} = \frac{F \omega^2}{\sqrt{(\omega^2 - \omega_n^2 - \frac{3}{4} \epsilon_n \hat{x}^2)^2 + (2 \delta_n \omega)^2}}, \quad (3.16)$$

and the phase difference between excitation and oscillation is,

$$\tan \phi = \frac{2 \delta_n \omega}{\omega_n^2 - \omega^2 + \frac{3}{4} \epsilon_n \hat{x}^2}. \quad (3.17)$$

The forced response curves from Eq. (3.16) and Eq. (3.17) are depicted in Fig. 3.6, with a characteristic bend of the response maxima. For a nonlinearity parameter $\epsilon_n > 0$ the restoring spring force is *hardening* and will shift the resonance towards higher frequencies. For $\epsilon_n < 0$ the force is *softening*, resulting in a resonance shifted towards lower frequency. As the nonlinear force scales with x_n^3 , a Duffing oscillator with a very small forcing amplitude F will respond close to a harmonic oscillator. The effect of resonances shift and hysteresis increases with higher forcing F .

The sloshing resonance was first linked to the Duffing model by Chester (1968a), who associated the resonance shift of shallow water sloshing to a *hard spring* solution of a

Duffing equation. Later, the inviscid multimodal model by Ockendon and Ockendon (1973) demonstrated that the predicted resonance, for small amplitude in a rectangular container, behaves exactly like a undamped Duffing oscillator. Additionally, Ockendon and Ockendon (1973) hypothesized that the inclusion of viscous effects from a boundary layer would result in a linearly damped Duffing oscillator. In the sloshing literature, the resonance behavior is often characterized as Duffing-like (Ockendon, Ockendon, and Johnson, 1986; Waterhouse, 1994). The similarity between the sloshing resonance and Duffing model can be seen when comparing its amplitude equation (Eq. (3.16)) to the dimensionless amplitude from the Faltinsen model in the previous section (Eq. (3.9)) and comparing both their response curves (see Figs. 3.6 and 3.4). Despite these prominent similarities, so far there has been no attempt to utilize the full Duffing equation with viscous damping as an equivalent mechanical model. All comparisons remain qualitative in nature and the hypothesize of Ockendon and Ockendon (1973) remains unverified.

For both a harmonic oscillator, Duffing oscillator, or the multimodal theory of the previous chapter, it is necessary to determine the amount of energy dissipation or damping in the system, which will be discussed in the next section.

3.1.5 Damping

Maybe the most important property of the sloshing system for engineering applications is the dissipation rate of energy. Although in the linear sloshing regime the effect of damping is negligible (Ibrahim, 2005), with larger amplitude the dissipation rate will dictate, for example, the maximum amplitude in the sloshing resonance and how fast the wave decays after a perturbation. For a weakly damped system, a resonance will drastically amplify the sloshing motion even for a small forcing amplitude. As a result, internal structures like baffles, meshes, or walls are added into sloshing tanks to increase the amount of energy dissipation and reduce the sloshing magnitude (Dodge, 2000).

While approaches of the multimodal model or equivalent mechanical models can analytically determine the sloshing modes and their contribution to the overall forces, they cannot predict the energy dissipation, as the calculations are based on inviscid potential flow theory. Even if a dissipation term is added to the multimodal model Faltinsen and Timokha (2017), its value has to be derived through other theories or experimental measurements. The same applies to an equivalent mechanical model. An early systematic investigation of the dissipation was performed by Keulegan (1959), who developed an analytical method to estimate the viscous damping rate for sloshing in a rectangular container. This approach is based on the energy dissipation of potential and kinetic energy in a laminar viscous boundary layer at the wetted tank walls and tank floor. The predicted damping ratio for each mode γ_n as the ratio between damping rate δ_n and natural frequency ω_n reads (Faltinsen and Timokha, 2009),

$$\gamma_n = \frac{\delta_n}{\omega_n} = \sqrt{\frac{\nu}{2w^2\omega_n}} \left[\left(\frac{w}{l} \right) \left(1 + 2\pi \frac{n(\frac{1}{2} - \frac{h}{l})}{\sinh[2\pi n \frac{h}{l}]} \right) + 1 \right], \quad (3.18)$$

with the kinematic viscosity of the fluid ν , the container width w , depth l and filling level h . The predicted viscous damping rates of the higher modes are larger compared to the first mode. Higher modes therefore decay more rapidly, which challenges their experimental determination, even if the system is excited at higher mode resonances (Dodge, 2000).

In applications, the dissipation rate will often deviate from the Keulegan (1959) prediction, due to additional damping effects. One example is the effect of surface tension for very small wave amplitudes Viola, Brun, and Gallaire (2018). In the regime of large resonant waves, dissipation will increase via strongly nonlinear mechanisms like hydraulic impacts,

wave breaking or chaotic motion. In these cases small scale turbulent structures will form in the fluid, which cannot be modeled with simple linear viscous damping term (Faltinsen and Timokha, 2009). A more complex model including higher order terms would be necessary in this regime. An accurate description of nonlinear dissipation is an outstanding challenge (Faltinsen and Timokha, 2002; Hill, 2003; Shemer, 1990), which can severely limit the model accuracy during resonance.

3.2 Research questions

The general aim of this chapter is to investigate the nonlinear dynamics of a liquid resonance. Although many individual aspects on nonlinear sloshing and models for describing sloshing dynamics were introduced in the previous section, there is no general understanding of which aspect of a sloshing resonance can and cannot be modeled accurately. In this section, the specific research questions of my thesis – which were already briefly introduced in section 1.4 – are formulated in detail.

RQ1 How does a nonlinear sloshing resonance emerge? Specifically, how does the system change from an quasi-linear response at small amplitudes, towards a nonlinear one with shifted resonance frequencies, coexisting states, hysteresis, and interacting modes?

For small sloshing waves, caused by small excitation amplitudes, the motion can be described by linearized assumptions of potential flow theory. However, as excitations and wave amplitudes increase, nonlinear effects become increasingly important. This leads to shifts of the resonance frequency, hysteresis, and coexisting stable states in a weakly nonlinear regime (Faltinsen et al., 2000). Further growth of the sloshing amplitude leads to effects of nonlinear dissipation, modal interactions, and a strongly nonlinear regime with wave breaking, wave impacts, and even fully chaotic motion (Faltinsen and Timokha, 2009). While these individual effects are known, their gradual emergence is not yet fully understood. This is in part due to the lack of experimental investigations to systematically resolve these different regimes (Hill, 2003). This gap in data makes it difficult to assess the validity of different modeling approaches.

In my investigations, I aim to explore how the sloshing resonance evolves from a linear to weakly nonlinear and strongly nonlinear regime and tracks in which stages the nonlinear dynamics emerge.

RQ2 Which observables of the flow are best suited to describe the dynamics of nonlinear sloshing?

Since sloshing models use different descriptions of the motion, from merely the liquid mass or sloshing forces to a full description of the wave shape, it is necessary to determine observables of the sloshing system, which can be derived from experiments and models. These observables must reliably describe the sloshing dynamics throughout the different regimes of complexity from approximately linear waves, to fully nonlinear wave motion.

One of the main aspects of this chapter is the assessment of how to evaluate the sloshing resonance and to critically examine the description of sloshing resonance from literature.

RQ3 How accurate can different model approaches describe the sloshing resonance? Using different model approaches, from the sloshing literature, mechanical models, and nonlinear dynamical systems theory, which aspects of the resonance can be captured? What are the limitations of existing resonance models?

There is a wide range of existing models that describe the sloshing resonance. Mechanical equivalent models, with linear or nonlinear terms, use a single degree of freedom motion to express the motion of sloshing liquid mass and the resulting forces on the container (Bauer, 1966; Graham and Rodriguez, 1952; Ibrahim, 2005). These theories are highly simplified and cannot resolve effects such as specific wave motion or modal interactions. More sophisticated analytical multimodal approaches derived from potential theory with nonlinear boundary conditions incorporate a full description of the surface wave for two-dimensional and even three-dimensional flows and can capture many aspects of nonlinear interactions (Faltinsen

et al., 2000; Faltinsen, Rognesbakke, and Timokha, 2003). Still, these approaches are limited in their representation of dissipation – being derived from inviscid potential flow – and cannot capture strong nonlinear wave motions (Faltinsen and Timokha, 2002, 2009). Lastly, recent developments in the nonlinear dynamical systems theory, such as the data-driven spectral submanifold theory² (SSM) (Cenedese et al., 2022; Haller and Ponsioen, 2016), offer a mathematical frameworks for general reduced order models to describe nonlinear resonances.

For my investigation, these different model approaches will be assessed for their ability to describe the nonlinear sloshing resonance. Specifically, I aim to determine the limitations of the individual models, and weight their advantages and disadvantages for applications.

Despite the extensive scope of theoretical and numerical work on nonlinear sloshing, the literature has a large gap of modern systematic experimental investigations, particularly in the resonance regime. Many previous studies investigated the forced response curve only for a single excitation amplitude or did not measure the full range of coexisting sloshing state (Feng, 1997; Hill, 2003). To address the research objectives of modeling the nonlinear resonance stated in this section, it is therefore necessary to perform new experiments for my investigations. In the next section, I will introduce the setup and experimental methods that were selected to best capture the nonlinear sloshing resonance.

²Note that the SSM model was not introduced in the theoretical background section 3.1, as the necessary mathematics are extensive and not in the scope of the thesis. A shorter introduction of the SSM model will be given in section 3.4.4. For a more detailed explanation of the underlying nonlinear dynamical systems theory, see Cenedese et al. (2022) and Haller and Ponsioen (2016).

3.3 Methods

To examine how the nonlinear dynamics of a sloshing resonance emerge, it is necessary to investigate the resonance as a function of the two control parameters: forcing amplitude and forcing frequency. In this study, the effects of filling levels and container geometry (which will affect the natural frequency and the amount of resonance softening or hardening (Faltinsen and Timokha, 2001)) or the fluid viscosity (which affects the dissipation rate and resonance amplification (Keulegan, 1959)) will be neglected. The sloshing system for my experiments, as a narrow rectangular container under lateral harmonic forcing, is designed to produce a quasi-two-dimensional wave, which can be fully captured and evaluated through optical measurements. The experimental methods for evaluating the sloshing motion are described in this section.

First, the setup and measurement methods are described (section 3.3.1). For modeling the sloshing resonance it is necessary to define robust observables of the system that can be computed from the model and measured in the experiment. For this purpose, it is first validated (section 3.3.2) that the flow in the experiment is two-dimensional. Afterwards, the center of mass of the sloshing fluid can be extracted as the main observable (section 3.3.3). For the later application of both the Duffing and multimodal model, it is also necessary to determine the damping of the system (section 3.3.4).

3.3.1 Experimental setup

A sketch of the sloshing experiment is shown in Fig. 3.7. In the setup a three-phase motor (i) is placed on a level base plate and harmonically moves an experiment platform in the horizontal plane, through an adjustable eccentric disk and a pushing rod (ii). A rectangular fluid container partially filled with water (iv), a low speed camera for recording the surface (vi) and two high-speed cameras (v) for stereoscopic Particle Image Velocimetry (PIV) are mounted on the platform. The motion of the platform itself is captured using a potentiometer displacement sensor (iii). For PIV measurements the flow is illuminated with a laser light sheet (vii). Two different sets of rectangular tanks are used for the experiments: a glass container ($w = 500$ mm, $h_{max} = 590$ mm) and a larger acrylic glass container ($w = 500$ mm, $h_{max} = 800$ mm) for sloshing with higher wave displacements and better access for the PIV illumination. No significant differences between the sloshing dynamics in both tanks could be observed. The depth of the container $l = 0.1w = 0.05$ m was selected much narrower than the width, to achieve a quasi-two-dimensional flow in the tank, without strong wave motion orthogonal to the forcing direction or three-dimensional mode interactions (Faltinsen, Rognbakke, and Timokha, 2003). The filling level of water in the tank $h = 0.8l = 400$ mm was selected to be sufficiently large to avoid effects from the tank bottom and to prevent strong interactions between the first and higher modes, which are characteristic of shallow filling levels (Chester, 1968a; Faltinsen and Timokha, 2002).

The electric motor in the driving system is a model MDXMA1M 090-32 from Lenze GmbH & Co KG. A LabVIEW control interface allows for the adjustment of the driving frequency of the motor $f = \omega/2\pi$ up to $f \approx 3$ Hz in increments as small as $\Delta f = 0.006$ Hz. The harmonic excitation of the system is generated using a continuously adjustable double eccentric, where the displacement amplitude as the second control parameter of the driving system can be adjusted in a range $x_a = [0 - 25]$ mm. Unlike the frequency, the amplitude is mechanically adjusted and cannot be changed while in motion. Therefore, it is kept constant throughout a series of measurements. A pushing rod connects the eccentric disk to the experiment platform and generates the horizontal quasi-harmonic motion. This type of driving does not result in exact harmonic motion, but rather corresponds to a

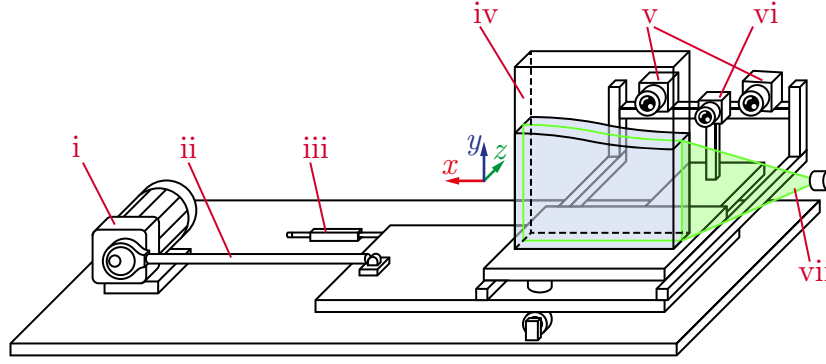


FIGURE 3.7: Experimental setup for sloshing liquid in a horizontally oscillated rectangular tank. A motor (i) drives an eccentric disk that generates quasi-harmonic horizontal oscillations of the platform via a pushing rod (ii). Mounted on the moving platform are the tank (iv), high-speed cameras for PIV measurements (v), and a low-speed camera (vi). A positioning sensor (iii) records the platform motion, and a laser passing through a cylinder lens (vii) provides a light sheet illumination for PIV.

crankshaft-style motion. The resulting motion of the platform reads,

$$x_p(t) = x_a \cos(\omega t) + \left(L_a \sqrt{1 - \left(\frac{x_a}{L_a} \right)^2 \sin^2(\omega t)} - L_a \right), \quad (3.19)$$

with the mechanical amplitude x_a of the eccentric disc, the length of the pushing rod $L_a = 1.1$ m and the driving frequency of the motor rotation ω . Since the investigated amplitudes are much smaller than the length of the rod $x_a \ll L_a$ (see Table 3.2), the second term of Eq. (3.19) can be neglected. Consequently, the harmonic forcing motion can be approximated as,

$$\begin{aligned} x_p(t) &= x_a \cos(\omega t) \quad \text{or} \\ F \cos \omega t &= x_a \omega^2 \cos \omega t. \end{aligned} \quad (3.20)$$

The dimensionless control parameters of this driving setup are the excitation amplitude $A = x_a/w$ and excitation frequency $\Omega = \omega/\omega_1$, where ω_1 is the first natural frequency for a rectangular container as given by Eq. (3.6). For the parameters of my measurements, with gravitational acceleration $g = 9.81 \text{ m s}^{-2}$ and filling ratio of $h/w = 0.8$ the first three natural frequencies are provided in Table 3.1. This research focuses on the resonance of the first mode. Therefore, the time scales throughout this chapter are normalized using the first natural frequency $\omega_1 = 7.800 \text{ s}^{-1}$. A deep sloshing system with a large filling ratio was selected to ensure that natural frequencies remains fairly unaffected by small changes in the filling ratio. Moreover, the large dimension of the container allows for flow measurements at relatively low excitation frequencies ($f = [0.5 - 2] \text{ Hz}$) and small amplitudes $x_a = [0.46 - 3.21] \text{ mm}$, where the wave height does not reach the tank ceiling h_{max} . The main geometric and driving parameters that were used during my measurements are listed in Table 3.2.

The displacement motion of the platform is captured using a potentiometer position sensor (type 8710-100 from Burster Präzisionsmesstechnik GmbH & Co KG) with an accuracy of $\pm 5 \mu\text{m}$. This enables the continuous monitoring of the instantaneous tank position and exact measurements of frequency, amplitude and phase of the driving. A LabVIEW interface was built to control the motor and acquire data from the motor, the sensors, and the low speed camera simultaneously.

Mode number n	Natural frequency ω_n	Natural frequency f_n
1	$\omega_1 = 7.800 \text{ 1/s}$	$f_1 = 1.241 \text{ Hz}$
2	$\omega_2 = 11.102 \text{ 1/s}$	$f_2 = 1.767 \text{ Hz}$
3	$\omega_3 = 13.598 \text{ 1/s}$	$f_3 = 2.164 \text{ Hz}$

TABLE 3.1: Natural frequencies from Eq. (3.6) of the first three modes, for $g = 9.81 \text{ m s}^{-2}$, $w = 0.5 \text{ m}$ and $h = 0.4 \text{ m}$.

Parameter	Dimensional	Dimensionless
Tank geometry	width $w = 500 \text{ mm}$	$w = 1w$
	depth $l = 50 \text{ mm}$	$l = 0.1w$
	height $h_{max} = [590; 800] \text{ mm}$	$h_{max} = [1.18; 1.6]w$
Filling level	$h = 400 \text{ mm}$	$h = 0.8w$
Excitation frequency	$f = \omega/2\pi = [0.5 - 2] \text{ Hz}$	$\Omega = [0.4 - 1.61] \frac{\omega}{\omega_1}$
Excitation amplitude	$x_a = [0.46 - 3.21] \text{ mm}$	$A = [0.09 - 0.64] \% \frac{x_a}{w}$

TABLE 3.2: Parameters of the tank geometry and the excitation parameters used for this project. The values are given as dimensional and dimensionless values normalized by container width w and the natural frequency ω_1 . Note, that two different tanks with different heights h_{max} but otherwise the same geometry were used.

3.3.1.1 Surface displacement measurements

For subsequent analysis of the liquid surface motion, an industrial monochrome USB camera ($1600 \times 1200 \text{ px}$; 30 Hz) type DMK 33UP5000 from The Imaging Source is used. It is directly mounted on the platform and therefore in a comoving frame with the container. The camera records the free surface elevation with a spatial resolution of 0.328 mm/px . A schematic of the optical setup is shown in Fig. 3.8a. To facilitate automated image analysis, it is essential to capture the liquid surface over the entire width of the container and for each moment throughout the oscillation period with high contrast against the surroundings. To achieve this, the camera does not directly capture the liquid itself. Instead, the free surface is projected onto a semi-transparent polyester screen positioned at the front wall of the container, with a light source behind the container. In the recorded images, the shadow of the free surface appears as a dark line, as shown in Fig. 3.8b. The thickness of the line stems from the meniscus at the walls of the container (see Fig. 3.8c). Compared to directly capturing the surface, this technique produces a higher contrast. A 100 W LED light panel is placed opposite to the camera, to create the shadow projection. The high brightness of the light source allows for recordings with a very short exposure time ($1/200 \text{ s}$), minimizing motion blur and creating a sharp interface. The light panel is placed at a distance of approximately 2 m to reduce the effects of beam divergence in the projection and avoid heating of the fluid.

To determine the surface elevation from the recordings, an automated image analysis is performed. In the first step, the fisheye distortion caused by the camera lens is corrected. A series of images of a checkerboard pattern calibration plate are recorded for this purpose, and an image undistortion function is computed using the Camera Calibration application in MATLAB. The undistorted images are cropped to show only the motion inside the container walls, and some pre-processing is performed to homogenize the contrast throughout the image. For the actual detection of the wave, a light intensity threshold is used to identify the water interface. To remove the effects of the menisci rising at the container walls, only

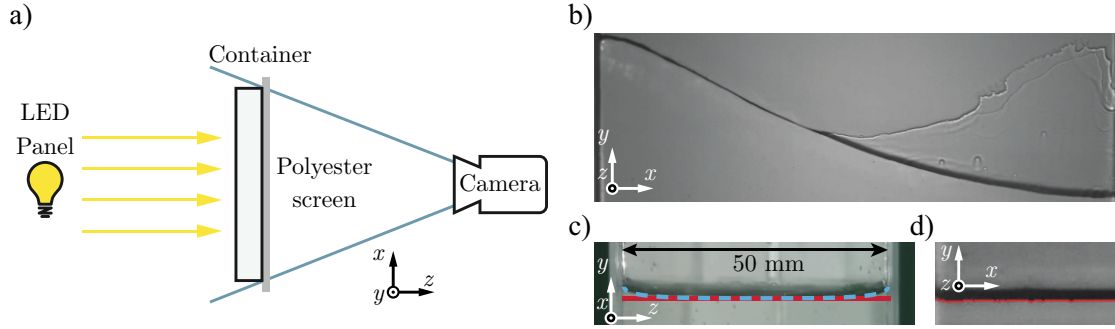


FIGURE 3.8: Method for measurements of the free surface elevation. a) The visualization setup involves a stationary LED panel mounted behind the container, projecting the surface as a shadow on a polyester screen attached to the front wall. A comoving camera records this projection. b) Raw data images show a thick line representing the free surface. c) When viewed from the side the detected surface level (red line) does not include the rising menisci at the wall (blue line), by only using the lowest point the projected shadow in the image analysis, see d).

the lowest bound of the projected shadow is defined as the surface level (see Fig. 3.8c,d). This approach also facilitates the detection of very small surface motion, where the menisci stick to the same level at the tank walls. Additionally, this process is not affected by the visible wetting front on the container walls, as seen in the upper right half of Fig. 3.8b. The identified surface wave is post-processed using a locally weighted scatterplot smoothing. This removes small outliers from e.g. air bubbles or light reflections at the interface. An example of the determined surface with the recorded image can be seen in Fig. 3.11c. For this two-dimensional surface to be representative of the sloshing motion, the flow needs to be quasi-two-dimensional, which will be shown in section 3.3.2. If the surface motion becomes three-dimensional (as for example in the higher frequency regime with more complex modes) the surface detection method may produce incorrect elevations. The accuracy of the surface detection is evaluated in Appendix B.1.

While the surface detection is a reliable tool to measure the sloshing amplitude for large parameter scans, the full characterization of specific flow states requires a more detailed investigation, like stereoscopic PIV measurements.

3.3.1.2 Stereoscopic fluid velocity measurements

The velocity field in the bulk of the fluid is measured using stereoscopic Particle Image Velocimetry (PIV), with a system from LaVision and processing in the Davis 10 software. The main components are two high-speed cameras (type VEO 640L from Phantom, 2560×1600 px; 1.4 kHz) and a pulsed dual-cavity 527 nm Nd:YLF laser for illuminating the fluid (2×50 mJ @ 1 kHz, DM50-527-DH from Photonics). To create a thin laser sheet in the central plane of the container (refer to Fig. 3.7), the laser beam is expanded and directed through a cylindrical lens. The water inside the container is seeded with small tracer particles. Using two stereoscopic viewing angles, all three velocity components can be computed within the two-dimensional light sheet plane, enabling 2D3C (two-dimensional plane, three-velocity-components) velocity analysis.

Both cameras are mounted to the platform and therefore comoving with the container. Due to spatial limitations in the optical setup, only a section of the flow (up to 290×440 mm) can be monitored (see Fig. 3.9). With the assumed reflection symmetry of the standing wave, this is sufficient to capture the dynamics of the flow. The motion of the waves disturbs the illumination, as the interface causes reflections or blocking of the light sheet. The reflections restrict accurate PIV measurements in the vicinity (~ 10 mm) of the free surface.

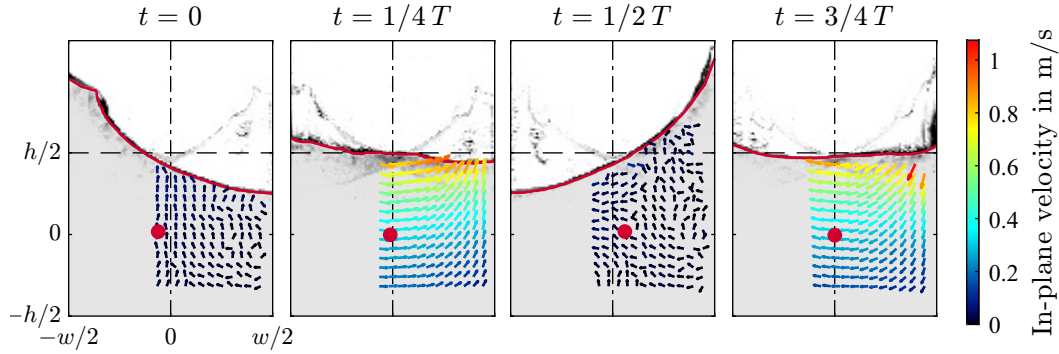


FIGURE 3.9: Example snapshots of sloshing liquid measured with Stereo PIV at a frequency $f = \omega/2\pi = 1.13, \text{Hz}$ and $x_a = 3.17, \text{mm}$, or $A = x_a/w = 0.64\%$. The motion of the surface wave (red line) induces an oscillatory motion of the liquid's center of mass ($\bullet$).

Furthermore, larger amplitude waves and wave-breaking can mix air bubbles into the fluid, which interfere with the laser light. In this regime, some regions of the container walls have to be masked off to avoid reflections and the usable laser intensity is also limited. To optimize the tracer signals, different types of tracers were utilized. In most of the experiments, hollow glass spheres ($9 - 13 \mu\text{m}$ from LaVision) provide sufficient light. In the higher amplitude regime, more reflective silver coated hollow glass spheres ($10 \mu\text{m}$ from Dantec) provide a better tracer intensity signal. Recordings are typically performed with sampling rates of $200 - 500 \text{ Hz}$, based on observed flow velocities in the tank.

The processing is carried out using LaVisons Davis 10 software. To calibrate the setup, a large custom $250 \times 250 \text{ mm}$ calibration plate was placed inside the container and recorded with both cameras. The plate has an even grid dot-pattern spread over two z -planes. In the software, each dot is detected, and a perspective calibration function is computed for both cameras. For processing of the flow measurements, a multi-pass stereoscopic PIV processing is employed, with a subwindow size of $64 \times 64 \text{ px}$ and 75% overlap. This allows for a spatial resolution down to 2.78 mm . The motion of the platform causes a slight vibration in the camera mounting structure of $\sim 12 \text{ Hz}$. This results in high-frequency velocity error in the computed PIV velocity field. This error lies well out of the range of observed sloshing frequencies and their main harmonic, and therefore a low-pass filter is applied when comparing frequency spectra and different surface states in section 3.4.1.2.

An example of a velocity field from PIV for different snapshots throughout an oscillation period is shown in Fig. 3.9. The surface elevation snapshot and determined waves are superimposed on top of the velocity field. Note, that the surface was recorded separately with the same sloshing state as both measurements cannot be performed simultaneously due to the different required optical configuration. The velocity field shows the different phases of an oscillation period, with zero velocity when the elevation is largest and a change in flow direction during back and forth motion of the wave.

The most important result for the PIV method was to test if the flow inside the tank can be treated as quasi-two-dimensional flow based on the fluid velocity inside the bulk of the moving fluid.

3.3.2 Assessing the quasi-two-dimensionality of the flow

In the experiments, a narrow container geometry was selected ($l = 0.1w$) to avoid three-dimensional waves or swirling motion in the tank. This two-dimensionality is necessary to capture the wave motion with a single camera and accurately determine its dynamics (see section 3.3.1.1). Although all waves appear mostly two-dimensional in the investigated

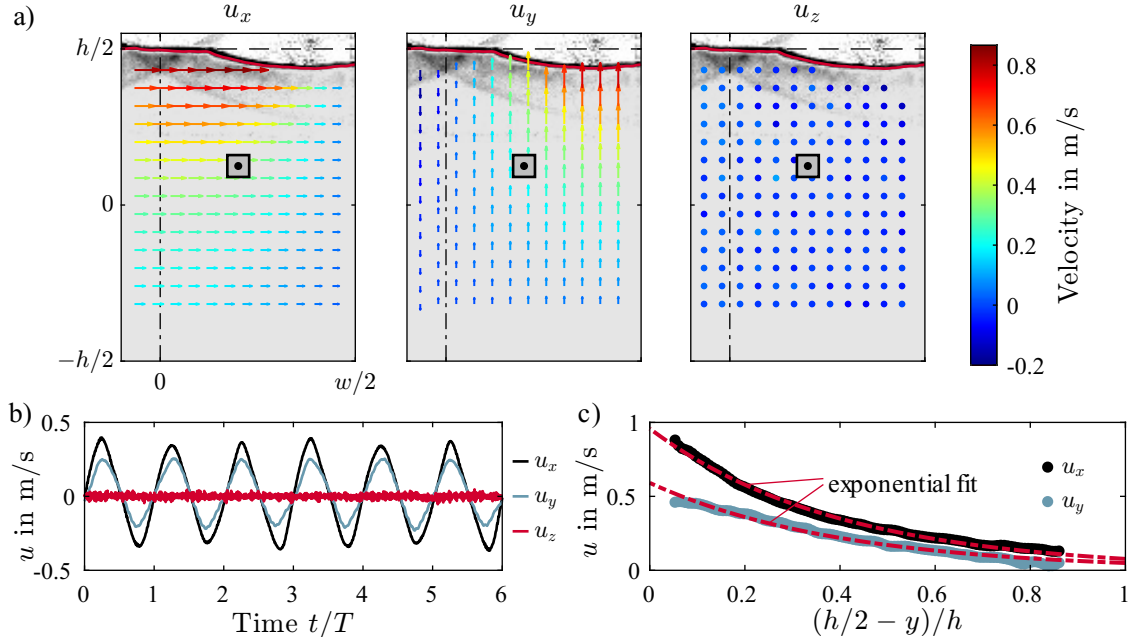


FIGURE 3.10: Stereoscopic PIV measurements are used to check the two-dimensionality of the flow, displayed here for a driving of $A = 0.64\%$ and $\Omega = 0.917$. a) In the recorded velocity field the out-of-plane velocities u_z are small, and overall negligible compared to the in-plane velocities (u_x, u_y). b) A time series of all the velocity components (taken at the black square in a)), show the same trend, with u_z only containing measurement noise. c) The velocity components u_x and u_y exponentially decrease from the free surface towards the tank bottom (see exponential fits – red line). In b) and c) the velocities are spatially averaged over ± 14 mm (size of the square in a)).

range of sloshing states and driving parameters to a visual inspection, stereoscopic PIV measurements were used to validate this assumption. In Fig. 3.10a all three components of the velocity field in the bulk of the fluid are shown. For better visual clarity, only every 8th vector in the fields is depicted. This measurement was taken with a large driving amplitude near the first mode resonance, where the sloshing motion is strongest. In the snapshots in Fig. 3.10a the surface elevation is close to zero resulting in the velocities being at their maximum of the oscillation period. The in-plane components u_x and u_y depict the expected sloshing motion, with the fluid moving left to right as the wave rises in the right half of the container. In contrast, the out-of-plane velocity u_z is approximately zero throughout the entire field of view. This remains the same for the entire oscillation period, as seen in the time series in Fig. 3.10b. The out-of-plane velocity u_z has some scattering over time, as the vibration error of camera mounting was not filtered out here (see section 3.3.1.2). Additionally, a larger out-of-plane uncertainty is typical for stereoscopic PIV, where the magnitude of the out-of-plane error is usually about 3 – 4 times larger than the in-plane errors (Prasad, 2000). In Fig. 3.10c the development of the in-plane flow velocities over the filling height is shown. Both u_x and u_y exponentially decrease with increasing depth below the free surface. This is consistent with the behavior of deep water surface waves (Bremer et al., 2019). The same exponential trend can be seen for all horizontal positions in the tank.

Based on this analysis of the bulk velocity, the flow in this setup can be treated as quasi-two-dimensional for this configuration. This simplification allow is to evaluate the amplitude and force of the sloshing motion solely based on the single camera measurement of the surface wave.

3.3.3 Center of mass displacement as an observable

It is most common to describe the sloshing amplitude via the elevation of the surface wave (Dodge, 2000; Faltinsen, 2017; Ibrahim, 2005). The wave can easily be measured using e.g. wave probe sensors or optical measurement techniques for one or multiple positions throughout the tank (Arndt and Dreyer, 2008; Faltinsen, Rognebakke, and Timokha, 2005; Ikeda et al., 2012; Royon-Lebeaud, Hopfinger, and Cartellier, 2007). Rarely is the full surface wave captured (Tosun et al., 2017; Viola, Gallaire, and Dollet, 2017). In this section, it will be shown how a single point wave elevation might lead to misinterpretations. The liquid's center of mass oscillation, on the other hand, is identified as a robust observable. This motion can be calculated out of the surface displacement wave, detected with image analysis (see section 3.3.1.1).

In Fig. 3.11a the surface elevation amplitude ζ is displayed as a function of the driving frequency $\Omega = \omega/\omega_1$. For each measurement, the system is initially at rest before the forcing is turned on. After the system reaches a steady state, the amplitudes are determined as the maximum displacement in the wave crest and wave trough against the resting surface level ($\zeta = 0$), phase-averaged over multiple oscillation periods. At small elevations far away from the natural frequency ($\Omega = 1$), the standing waves have a sinusoidal shape with a nodal point in the center of the wave and the same displacement in the crests and troughs. As the elevation increases, closer to the resonance, the peaks of the wave become higher and more narrow, the troughs become wide and flat, and the nodal point shifts away from the center (Dodge, 2000; Ibrahim, 2005). This can be observed in Fig. 3.11c. In the response curves in Fig. 3.11a this leads to a distinct amplitude difference between crest and trough. Despite these well known effects, many studies do not clearly define how their wave amplitudes are determined. This ambiguity can hinder direct quantitative comparisons. Additionally, at larger amplitudes different flow states can emerge (as will be shown in section 3.4.1.2), where the wave shape is further distorted and single coordinate elevation can no longer be a suitable descriptor of the dynamics. The ambiguity of the surface elevation is not only a problem when evaluating the amplitude, but also for determining system characteristics, like the viscous damping rate δ (see section 3.13). It is concluded that the surface elevation at a fixed location does not provide sufficient information about the wave motion.

Since early studies on liquid sloshing, engineering applications often utilize an equivalent mechanical model (like a pendulum or mass-spring system) to substitute the sloshing motion (Ardakani and Bridges, 2010; Bauer, 1966; Farid and Gendelman, 2018; Sumner, 1965). Here, the entire fluid dynamics are simplified as the one-dimensional motion of a single point mass. This can be a valid approach as the liquid's center of mass motion will describe e.g. mass unbalance or reactive forces acting on a tank wall. Although the motion of the liquid's center of mass is sometimes evaluated (Faltinsen et al., 2000), it is rarely used to describe the sloshing amplitude in modern literature. Due to the flow in this study being quasi-two-dimensional (see section 3.3.2) the horizontal center of mass displacement can be calculated from the detected surface wave elevation $\zeta(\xi, t)$, via integration over the container width coordinate ξ ,

$$x(t) = \frac{1}{wh} \int_{-w/2}^{w/2} \xi [\zeta(\xi, t) + h] d\xi. \quad (3.21)$$

An example of the center of mass displacement is indicated with a red dot in Fig. 3.11c, and compared to the center of mass at rest $(0, 0)$. Since the calculation is based on the entire surface wave, it is robust against small errors in the surface detection method. Furthermore, the liquid's center of mass can be used to make a quantitative comparison of different modes or flow states. Due to the left-right symmetry of the standing waves in all measurements, the center of mass $x(t)$ uniformly oscillates around zero. In the following analysis, the

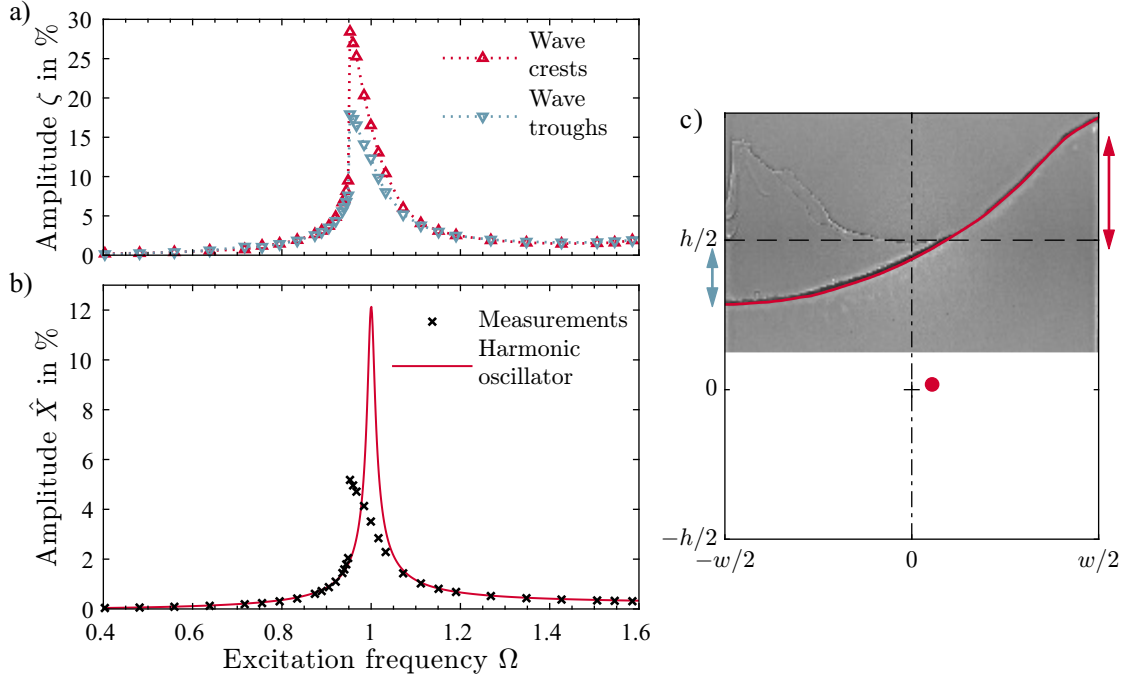


FIGURE 3.11: Frequency response curves when starting each individual measurement from rest at $A = 0.64\%$. a) The surface displacements at the wave crests and troughs lead to different response curves of the sloshing amplitude (ζ). b) By comparison, the amplitude of the liquid's center of mass oscillation ($\hat{X}$) provides a more robust observable for a precise determination of the sloshing amplitude. The solid line (—) represent a linear harmonic oscillator without any fitting parameters. c) A snapshot of a typical surface wave in the vicinity of the resonance (here at $\Omega = 0.953$) demonstrates the typical asymmetric leading to the difference between crest and trough in a).

amplitude of the liquid's center of mass displacement $\hat{x}$ (in dimensionless form $\hat{X} = \hat{x}/w$) is defined as a representative measure of the sloshing amplitude. An example of the advantages of this observable can be seen in Fig. 3.11b. In contrast to the wave elevation in Fig. 3.11a a unique amplitude value per excitation frequency is obtained. In addition, the liquid's center of mass amplitude can be directly compared to equivalent mechanical models, shown here for a harmonic oscillator. Far away from the resonance, this simple linear approach fits the measurements well. The robustness of the center of mass with respect to errors in the surface is discussed in Appendix B.1.

A further advantage of using the center of mass to describe the sloshing amplitude is the determination of the damping rate, described in the next section.

3.3.4 Determining the viscous damping

Both the Duffing oscillator and the multimodal model require the viscous damping rate δ of the sloshing system to calculate the amplitude response. For experimental investigations, the viscous damping rate can be extracted by evaluating the decay of the sloshing amplitude after the excitation is stopped (Dodge, 2000). This is usually measured by using the decay of the free surface elevation. As previously discussed in section 3.3.3, single point surface elevations can provide some ambiguity. The same is true when determining the damping rate. Fig. 3.12a depicts the dissipation of the surface elevation at the outer container wall after the harmonic forcing is stopped at $t = 0$. Two exponential envelope functions are fitted to determine the dissipation rate δ . During the amplitude decay the surface shape changes from an initially distorted wave to a planar sinusoidal wave. This shift in waveform

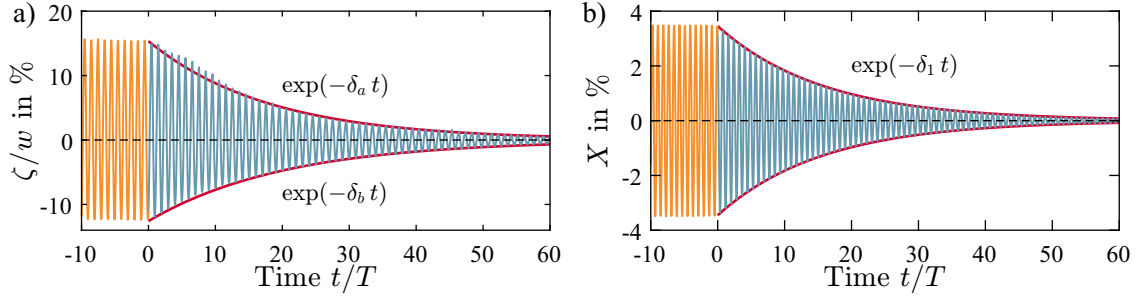


FIGURE 3.12: Observed viscous damping rate based on time series of different observables. The driving of the system ($A = 0.64\%$, $\Omega = 0.999$) was turned off at $t = 0$, leading to an exponential decay of the sloshing amplitude. a) Time series of the surface displacement ζ/w at the left tank wall, where the amplitude is maximal. The same asymmetry as displayed in Fig. 3.11c, leads to a difference in positive and negative displacement. When fitting an exponential decay function this results in different decay rates ($\delta_a = 0.069 \text{ s}^{-1}$, $\delta_b = 0.060 \text{ s}^{-1}$). b) In contrast the lateral motion of the center of mass from the same measurement is left-right symmetric and allows the determination of a single unambiguous damping coefficient ($\delta_1 = 0.064 \text{ s}^{-1}$).

leads to two different dissipation rates for the wave crests and troughs ($\delta_a = 0.069 \text{ s}^{-1}$, $\delta_b = 0.060 \text{ s}^{-1}$). In contrast, the liquid's center of mass amplitude in Fig. 3.12b decays with unique a dissipation rate (here $\delta_1 = 0.064 \text{ s}^{-1}$).

This procedure of determining the viscous damping coefficient is repeated for a range of different initial forcing frequencies (see Fig. 3.13a). Overall, the viscous damping appears to be rather constant with an average of $\delta_1 = [0.065 \pm 0.015] \text{ s}^{-1}$ (corresponding to a dimensionless damping ratio of $\gamma_1 = \delta_1/\omega_1 = 8.4 \cdot 10^{-3}$). For lower excitation frequencies (marked with empty circles in Fig. 3.13a) the initial amplitude of the steady state was very small (surface elevation $< 6 \text{ mm}$ or $\sim 18 \text{ px}$), leading to increasing uncertainties when fitting the exponential decay. Furthermore, when a displacement is of similar magnitude to the meniscus, the surface tension forces will affect both the decay rate and the frequency of surface oscillations (Bauer and Eidel, 1999; Viola, Brun, and Gallaire, 2018). At these small amplitudes the dissipation is not representative of dynamics in the vicinity of the resonance, and these data points were neglected for the evaluation.

In addition to the dissipation rate, the decay can also be used to determine the systems natural frequency. Using a fast Fourier transformation on the decay oscillation (as shown in Fig. 3.13b) reveals the first natural frequency ω_1 to be dominant, independent of the driving parameters in the previous steady state. Repeating this procedure in the same manner as for the determined damping rate, gives an average frequency of $\omega_{1,\text{exp}} = [7.86 \pm 0.10] \text{ s}^{-1}$. This value confirms the analytically calculated natural frequency $\omega_1 = 7.800 \text{ s}^{-1}$.

For comparison with the experimentally derived damping, the damping ratio is also calculated using the analytical model of a viscous boundary layer by Keulegan (1959). With Eq. (3.18) the damping ratio is computed, using the kinematic viscosity of water at room temperature $\nu = 1 \cdot 10^{-6} \text{ m}^2/\text{s}$ and the tank geometry ($h = 0.4 \text{ m}$, $w = 0.5 \text{ m}$, $l = 0.05 \text{ m}$). The predicted dissipation from the first three modes is shown in Table 3.3. As higher modes with larger damping rates decay faster, the relevant damping ratio for this study is that of the first mode $\gamma_1 = 5.57 \cdot 10^{-3}$ ($\delta_1 = 0.0434 \text{ s}^{-1}$). This analytical prediction is approximately 1.5 times smaller than the measured value $\gamma_1 = 8.4 \cdot 10^{-3}$. There are several sources of dissipation that are not considered in Keulegan's estimation, like dissipation in the fluid bulk, turbulent energy dissipation, surface tension effects and energy dissipation from the free surface (e.g. due to wave-breaking) (Faltinsen and Timokha, 2009). Similar differences between the Keulegan damping ratio and experiment were observed by Faltinsen

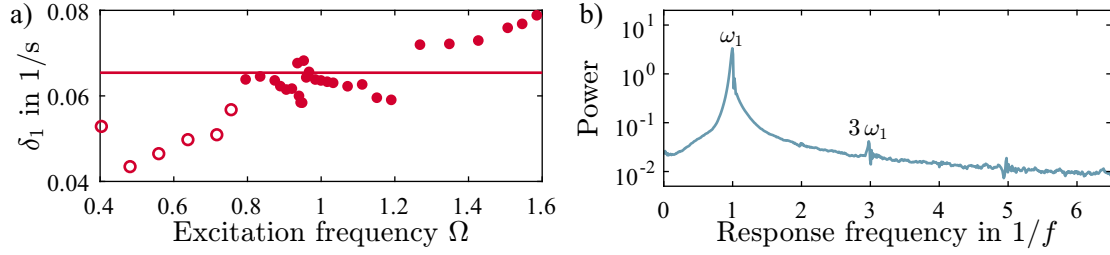


FIGURE 3.13: Experimental method for determining viscous damping rate and natural frequency. a) Determined damping values from exponential fits of the decay (as illustrated in Fig. 3.12b) for different driving frequencies and $A = 0.64\%$. The average $\delta_1 = 0.065 \text{ 1/s}$ is shown as a red line. b) Typical example of the frequency spectrum of decaying sloshing motion of center of mass after the driving is stopped, here for the time series of Fig. 3.12b.

The highest peak at ω_1 indicates that the system decays with the natural frequency.

Mode number n	Damping ratio γ_n	Viscous damping rate δ_n
1	$\gamma_1 = 5.57 \cdot 10^{-3}$	$\delta_1 = 0.0434 \text{ 1/s}$
2	$\gamma_2 = 4.67 \cdot 10^{-3}$	$\delta_2 = 0.0518 \text{ 1/s}$
3	$\gamma_3 = 4.22 \cdot 10^{-3}$	$\delta_3 = 0.0574 \text{ 1/s}$

TABLE 3.3: Keulegan (1959) damping ratio from Eq. (3.18) of the first three modes, for $\nu = 1 \cdot 10^{-6} \text{ m}^2/\text{s}$, $w = 0.5 \text{ m}$, $l = 0.05 \text{ m}$ and $h = 0.4 \text{ m}$.

and Timokha (2017) and Ikeda et al. (2012). Faltinsen and Timokha (2009) state that the Keulegan estimate will only give a 'lower bound' of the actual dissipation. For the range of investigated sloshing amplitudes, the damping is likely to include nonlinear effects, instead of featuring only linear viscous damping. Furthermore, the measured dissipation will also include contributions of higher modes. Since the amplitude and contribution of other modes cannot simply be determined from the decay, no differentiation is possible. For these reasons, the experimentally determined damping ratio of $\gamma_1 = 8.4 \cdot 10^{-3}$ is considered to be more realistic.

So far, the measurement techniques of PIV and surface wave measurements, as well as the main observable of sloshing wave as the center of mass, and damping rate for the system, have been established. In the next section, the analysis of the nonlinear resonance dynamics will be presented.

3.4 Sloshing resonance

The following analysis investigates the nonlinear dynamics associated with the sloshing resonance for quasi-two-dimensional flow in a narrow rectangular container under harmonic forcing. The first part focuses on characterizing the experimentally observed flow regimes of nonlinear sloshing dynamics of the first mode resonance (section 3.4.1). These findings are then systematically compared to three model approaches: the analytical multimodal theory of sloshing waves by Faltinsen et al. (2000) (section 3.4.2), the equivalent mechanical model of a Duffing oscillator (section 3.4.3), and a general reduced-order modeling approach with the spectral submanifold theory (section 3.4.4). In the final section 3.4.5, the quantitative differences between the models and the experiment are evaluated, and the advantages as well as the limitations for the application of sloshing resonances of these models are assessed.

3.4.1 Large amplitude sloshing dynamics

During nonlinear sloshing resonances, a variety of complex dynamics can emerge, which will be described in this section. This includes a hysteresis with coexisting sloshing states (section 3.4.1.1), the development of different flow states with complex wave motion (section 3.4.1.2), and a shift of the resonance with increasing amplitudes (section 3.4.1.3).

3.4.1.1 Hysteresis

For the first measurements of a sloshing amplitude (shown in section 3.3.3 to compare the center of mass to the surface wave, see Fig. 3.11), each frequency response was recorded independently. The container was initially at rest, then the excitation was started until all transients decay and a steady state response could be determined. This procedure is common practice in the sloshing literature (Arndt and Dreyer, 2008; Fultz, 1962; Konopka et al., 2019). The resulting frequency response curve of the liquid's center of mass amplitude $\dot{X}$ (Fig. 3.11b) has a softening dynamics with the maximum amplitude slightly shifted towards lower frequencies $\Omega < 1$, as it would be expected for large filling levels (Faltinsen et al., 2000). The same frequency shift is observed in the wave elevation amplitudes (Fig. 3.11a). In the nonlinear dynamics community, it is well known that starting from a single initial condition alone is insufficient to detect all possible states of a system (Strogatz, 2018). For a nonlinear system, multiple stable states can coexist for the same control parameters. To investigate the sensitivity towards the initial conditions, two quasi-static frequency sweep measurements are performed. After a first steady state is reached the driving frequency is changed in very small increments, first starting at a low frequency and sweeping upwards, and then starting at high frequency and sweeping back down. The two resulting amplitude curves are shown in Fig. 3.14a. During the sweep-up, the amplitude response follows the measurements from rest (black $\times$). Although the smaller frequency steps reveal that at $\Omega \approx 0.95$ the sloshing amplitude suddenly jumps upwards, indicating that the previous state is no longer stable (marked with the blue arrow). During the sweep-down this jump does not occur; instead, lower frequencies result in even larger amplitudes, until a second jump transition at $\Omega \approx 0.92$ (red arrow) down to the small amplitude sloshing state. In between the two jumping transitions, a hysteresis is present. In the following sections, the larger amplitude solutions are referred to as the upper branch, and the smaller amplitude regime is on the lower branch. Such a hysteresis behavior is a typical feature of resonance of nonlinear oscillator dynamics. An example can be seen with both the multimodal sloshing model and the Duffing oscillator response curves (Figs. 3.4 and 3.6), where both stable solutions are connected by an unstable branch. It is evident that the previous method of starting only from rest is insufficient to detect all solutions. Especially the largest response

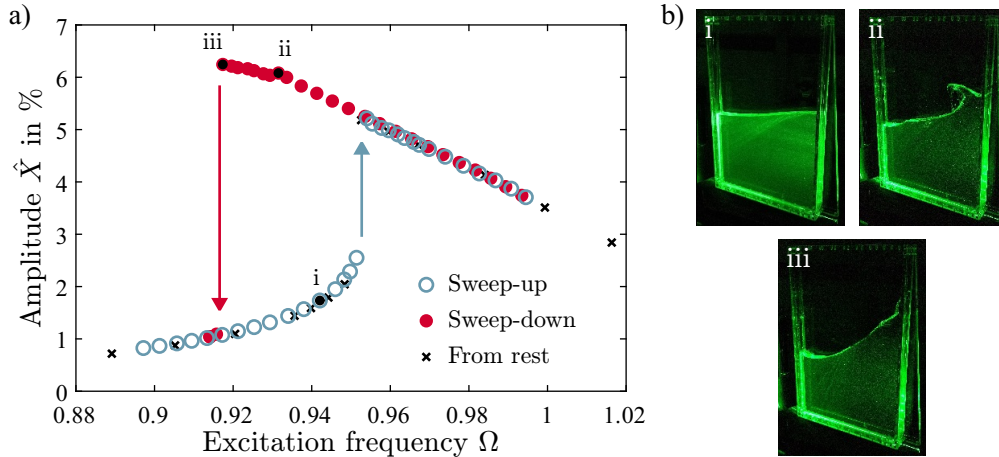


FIGURE 3.14: Amplitude response when performing different quasi-static frequency sweeps at $A = 0.64\%$. a) Sloshing amplitudes when performing a frequency sweep-up ($\circ$) and a frequency sweep-down ($\bullet$) procedure, in contrast to the response curve when recording each measurement from rest ($\times$, measurements from Fig. 3.11b). At the arrows the amplitude jumps up down between the lower (small amplitude) and upper (high amplitude) branches. In between the two jumping transitions the system is hysteretic. b) At larger amplitudes different sloshing states emerge, indicated by the markers in a), i: planar motion, ii: wave-breaking, iii: period-three motion.

amplitudes of the resonance (on the upper branch) – the most critical sloshing motion – could be easily missed, and the sloshing amplitude significantly underestimated. Another observation of this measurement is the existence of different surface states for the different amplitudes, shown in Fig. 3.14b. These will be explored in more detail in the following section.

3.4.1.2 Characterization of different flow state

In the proximity of the sloshing resonance, multiple different sloshing states can coexist. For clear differentiation, these states are further investigated using the surface wave in combination with velocity time series and spectral composition from stereoscopic PIV measurements of each state.

In the regime of smaller excitation frequencies below the resonance $\Omega < 0.9$ and on the lower branch, the wave motion is planar, with a approximately sinusoidal surface elevation (see Fig. 3.15a). The same linear sinusoidal motion can be observed in the velocity time series in Fig. 3.15e and the corresponding frequency spectrum in Fig. 3.15i, consisting only of the driving frequency and the first two harmonics. Similarly, at frequencies above the resonance $\Omega > 1$ only a single state could be seen. In this state, the wave crest is distinctively shifted towards the center of tank, creating a plateau-like wave peak (see Fig. 3.15b). The frequency spectrum (Fig. 3.15j) is similar to the planar state, although the third harmonic is stronger and the second harmonic has almost completely disappeared.

When the frequency is lowered from the plateau state ($\Omega \approx 1$) and the amplitude increases, the sloshing motion diverges into one of two different states. The plateau in the surface wave can become more broad and extend further into the center of the tank. At some point, the wave will become too steep and overturn, resulting in a wave-breaking state. This can be seen in the snapshot in Fig. 3.15c. The velocity time series (Fig. 3.15g) is here more similar to a triangular wave, compared to the smooth oscillation during planar and plateau motion. In the frequency composition (Fig. 3.15k) this results in a much broader spectrum, suggestive of a chaotic state. In the same regime as the wave-breaking state,

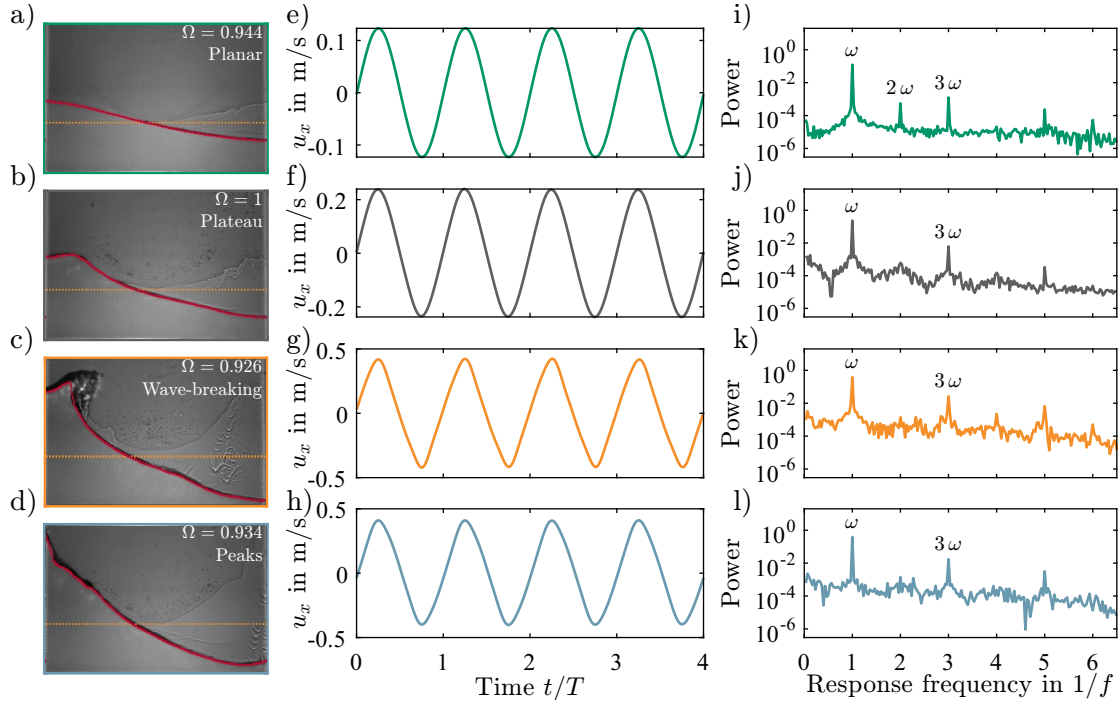


FIGURE 3.15: Characterization of the different observed sloshing states at $A = 0.64\%$. a)-d) Exemplary snapshots of the surface wave state. The detected surface is drawn with — and the resting surface level is indicated with e)-h) Time series of the horizontal velocity u_x in the bulk of the flow obtained with Stereoscopic PIV at a position at horizontal center of tank $x = 0$ and $y = 0.125h$. The velocity is spatially averaged over $\Delta x = 28$ mm and $\Delta y = 28$ mm. i)-l) Frequency power spectra of the time series in e)-h) obtained with a fast Fourier transformation. The investigated flow states are: a),e),i): Planar motion $\Omega = 0.944$, b),f),j): Plateau peaks $\Omega = 1$, c),g),k): Wave-breaking $\Omega = 0.926$, d),h),l): Peaks $\Omega = 0.934$.

the crest of the wave can also shift back towards the wall, creating a narrow and steep peak where the fluid rises the wall (see Fig. 3.15d). This state will be referred to as the peak state. Compared to the wave-breaking state, the frequency spectrum (Fig. 3.15l) is less broad and similar to the plateau motion. Both wave-breaking and peak state, seem to coexist, with one state sometimes transitioning into the other without any change of the driving parameters.

If the frequency is further reduced on the upper branch $\Omega < 0.93$, an intriguing phenomenon is observed: the state always transitions into a period-three motion, in which every third wave peak exhibits a significantly higher elevation at the tank wall. The difference in the three consecutive waves can be seen with the naked eye (see Fig. 3.16a). In the frequency spectrum (Fig. 3.16c) this period-three motions leads to a subharmonic frequency $1/3\omega$ and its linear combinations with ω . The periodicity can be seen best in the phase portrait in Fig. 3.16d. The coloration is only present for better visual clarity of the three different oscillation periods. To my knowledge, this is the first study in which such a period-three state could be measured. Once the frequency is further decreased $\Omega < 0.918$ the amplitude jumps down and reverts back to the planar motion. Such a period-three motion can be an indicator for nonlinear resonant interactions of different modes.

To find the parameter regime where each sloshing state exists, additional sweeping measurements (with the procedure described in section 3.4.1.1) using various initial sloshing states are performed. In Fig. 3.17a the amplitude response curve is displayed as a function of the excitation frequency, with competing sloshing states indicated by different markers and colors. Both the peaks and wave-breaking states coexist for a large regime of different

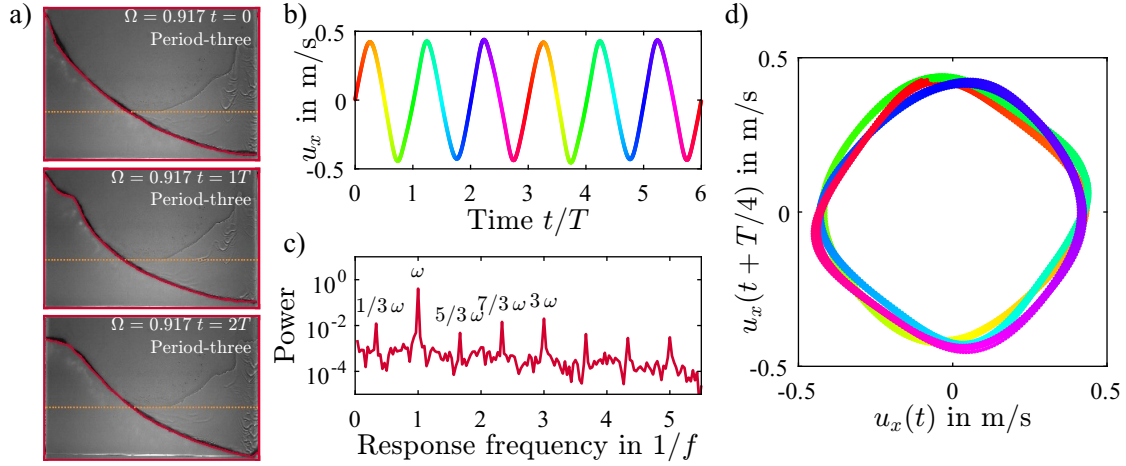


FIGURE 3.16: Period-three flow state at $A = 0.64\%$ and $\Omega = 0.917$. a) Snapshots of the three consecutive repeating peaks. b) Time series of the horizontal velocity u_x (same position and averaging as in Fig. 3.15). c) Spectrum of the time series in b) obtained with a fast Fourier Transform. d) Phase portrait of the dynamics from u_x with delay embedding of $T/4$. The measurements correspond to 30 periods and are plotted as symbols, but appear as a line due to the low scatter in the data. The coloration in b) and d) is only present for better visual clarity.

forcing frequencies. With the states sometimes transitioning from one to another, they seem to be only marginally stable and very sensitive to small perturbations. It is also noteworthy that, despite large difference in the surface waves, the peaks state, wave-breaking, and period-three motion all collapse to a single line when evaluating the center of mass amplitude. This further indicates the robustness of the center of mass as a sloshing amplitude observable. In all measurement procedures, the sloshing on the lower branch always stays planar, and similarly, for higher driving frequencies will always become a plateau motion. For the highest amplitudes around the response maximum, only the period-three state is present. When reducing the driving frequency on the upper branch, this state is reached regardless of whether the wave-breaking or the peaks state was present beforehand.

A further parameter that has not been discussed so far is the phase lag between the sloshing oscillation and the periodic excitation. The phase lag can be a very good indicator of damping in the system, as an inviscid response will always have a phase difference of zero. The corresponding phase lag for each amplitude measurement is shown as a function of the driving frequency in Fig. 3.17b. Note that the phase lag appears mirrored to the amplitudes in Fig. 3.17a as the lower branch planar sloshing is almost in phase with the excitation and the upper branch with larger sloshing amplitude has a larger phase lag. In contrast to the amplitude measurements, the phase lag of the peaks and wave-breaking state can be seen as two coexisting branches. Both branches develop into the period-three state, with the jump-down transition occurring at slightly different frequencies and amplitudes. The presence of further coexisting branches indicates strong nonlinear interactions of different modes leading to bifurcations (Farid and Gendelman, 2018; Ikeda et al., 2012). It can be suspected that there are further stable and unstable states in this regime, which cannot be easily observed in the experiments, due to sensitivity to perturbations or limitations of the control parameters.

It is noteworthy that for both large-amplitude branches, the maximum sloshing amplitude before the jump-down transition has a phase lag of $\sim 90^\circ$, even though the jump occurs at different driving frequencies. This will be further investigated for additional driving amplitudes in the next section.

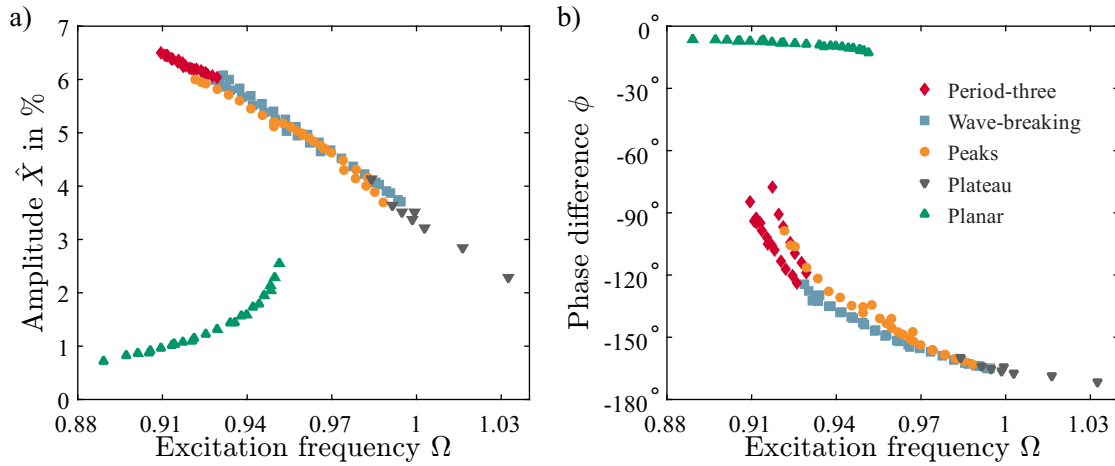


FIGURE 3.17: Response curves with the different competing sloshing states at $A = 0.64\%$. a) Sloshing amplitude and the b) corresponding phase lag between sloshing motion and driving as a function of the driving frequency. The measurement were obtained with various procedures, starting from and using multiple frequency sweeps with different initial states and frequency increments. The symbols indicate the observed flow states from Fig. 3.15 and Fig. 3.16. Note, that the phase lag appears mirrored to the amplitudes as the lower branch planar sloshing is almost in phase with the excitation and the upper branch with larger amplitude has a larger phase lag.

3.4.1.3 Variation of the driving amplitude

As explored in the previous chapters the resonant dynamics for a driving amplitude of $A = 0.64\%$ show multiple effects of nonlinear behavior: The amplitude response is folded with a hysteretic region of two coexisting branches. For larger sloshing amplitudes, multiple sloshing states are coexisting (at least three per driving frequency). At the resonance maximum, a period-three motion with subharmonic oscillations emerges. To investigate the amplitude dependence of these effects, the same frequency sweep measurement procedure is repeated for the resonance at smaller driving amplitudes $A = [0.09, 0.17, 0.32]\%$. The resulting amplitude response curves are shown in Fig. 3.18a. For the smallest forcing $A = 0.09\%$ the system acts similarly to a linear harmonic oscillator. There is no hysteresis and the response maximum is only very marginally shifted with respect to the linear eigenfrequency ($\Omega = 1$), with a peak amplitude (black $\times$) at $\Omega \approx 0.99$. With $A = 0.17\%$ a small hysteresis with a jump transition between the coexisting states first appears. The resonance maximum is shifted further towards lower frequencies. The sloshing state at small driving amplitudes is always a planar wave motion, although at the larger amplitudes for $A = 0.17\%$ the wave crest becomes more pointed. At a driving amplitude of $A = 0.32\%$ the hysteresis grows and is again shifted further towards smaller frequencies. On the upper branch, the waves have sharper crests and flat troughs (compare Fig. 3.11c). It is noteworthy that the amplitude on the upper branch is here of similar magnitude to the wave-breaking and peak state measurement with $A = 0.64\%$, so the emergence of these states is not just a simple amplitude threshold.

When comparing the phase lag for the different driving amplitudes (in Fig. 3.18b), it can be observed that the maximum resonance peaks persistently occur consistently at a phase difference of $\sim 90^\circ$. Here, the sloshing system follows the 90° phase lag or quadrature criterion, which was originally developed for primary resonances of mechanical vibrations (Peeters, Kerschen, and Golinval, 2011). This criterion states that during the maximum response of a system, where the energy input forces and the dissipation forces are in balance, the forced oscillation will have a 90° phase lag. For a nonlinear mechanical system, that

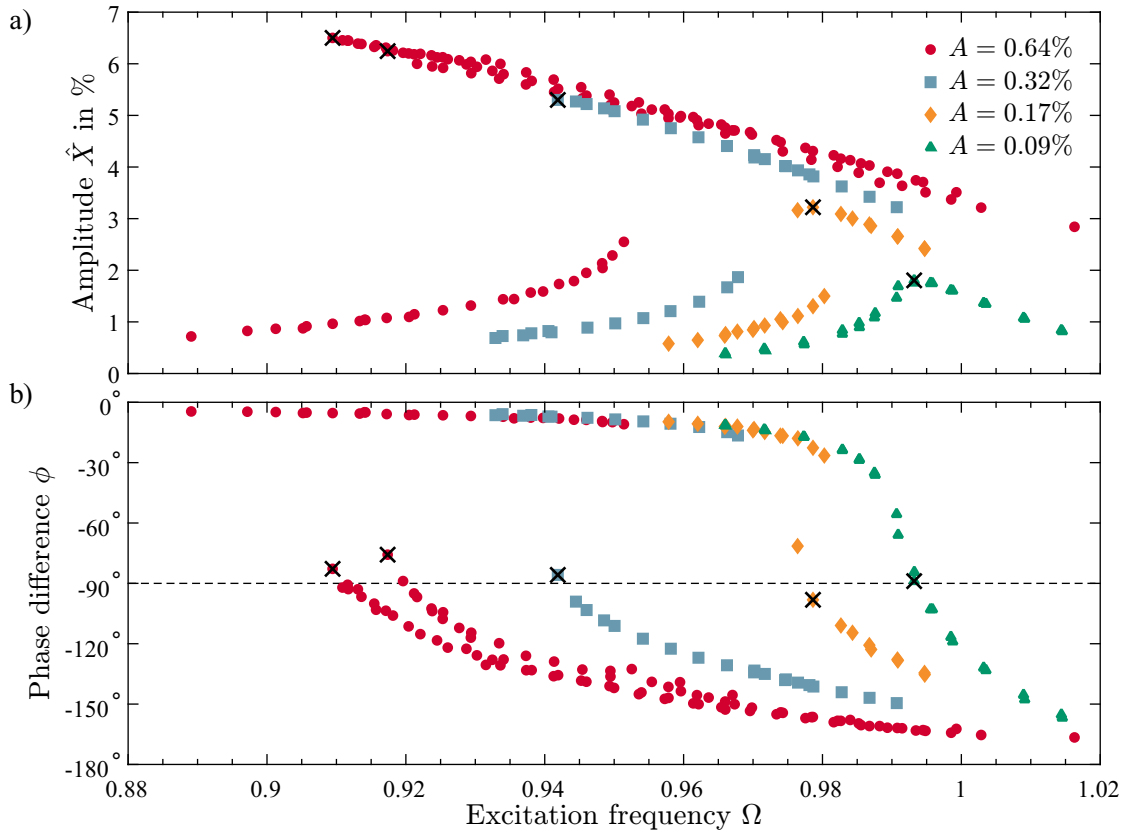


FIGURE 3.18: Response curve of the sloshing experiments for different excitation amplitudes A . The measurements were obtained with a fixed amplitude A and a quasi-static up and down frequency sweep around each resonance peak. For $A = 0.64\%$ multiple sweeps were utilized to capture the competing flow states, see Fig. 3.17. The maximum amplitude during each run is denoted with $\times$. b) phase lag between sloshing motion and the excitation corresponding to the amplitudes in a). The jumping transition from high to low amplitude sloshing always occurs at a phase difference of $\phi \approx -90^\circ$.

indicates that the damping force can be expressed as a polynomial function of velocities and position (Breunung and Haller, 2018). More recently, Cenedese and Haller (2020) could provide a mathematical proof of the 90° phase lag criterion for both primary and subharmonic resonances with arbitrary periodic motion. For the sloshing system, this criterion can act as an important indicator of the maximum amplitude response. From the amplitude alone it is not possible to determine when the system has reached the peak amplitude and a jump transition will occur. The change of the maximum response frequency changes over the amplitude – which is referred to as the backbone curve – is an important property of nonlinear resonances, as it will show how the system responds for a different forcing amplitude.

The sloshing measurements with varying forcing amplitudes highlight the transition from an approximately linear resonance to a strongly nonlinear regime with coexisting states. For comparisons with resonance models in the next sections, it is imperative to investigate how well the models can predict the dynamics throughout the different regimes and which aspect of nonlinear behavior can be captured.

3.4.2 Multimodal model of liquid sloshing

The first approach for modeling the sloshing resonance is the multimodal model by Faltinsen et al. (2000), as an analytically derived solution of a forced sloshing motion which considers multiple interacting modes. With the multimodal approach, both the free surface waves (section 3.4.2.1) and the forced frequency response of the sloshing amplitude and phase lag (section 3.4.2.2) can be analytically predicted and will be compared to the experimental observations.

The sloshing response of the analytical model for a given driving frequency and driving amplitude (see section 3.1.3) is calculated based on the known dimensions of the sloshing container ($w = 0.5$ m, $l = 0.05$ m), the filling level ($h = 0.4$ m), and the experimentally determined viscous damping ratio of the system ($\gamma = 8.4 \cdot 10^{-3}$, see section 3.3.4).

3.4.2.1 Comparison of the predicted surface waves

The multimodal model can be used to calculate the full temporal evolution of the surface wave under harmonic forcing. For each set of driving frequency and driving amplitude, the dimensionless amplitude is computed with Eq. (3.9). From this the time dependent beta function of each mode β_n can be derived with Eq. (3.8). These parameters are then used in Eq. (3.7) to compute the surface as a function of time and the horizontal coordinate inside the tank. For comparison, some predicted surface waves are displayed in Fig. 3.19a,c,e together with the three measured elevations (dashed line) of different magnitudes. To show the contribution of each term, three surface waves are computed for each measurement: first only using the first mode β_1 (dotted line), including the first two modes (dash-dotted line), and with all three modes (solid line). For a small surface displacement with $\Omega = 0.950$ and amplitude parameter $\hat{Y} = 0.084$, the dynamics are dominated by the first mode only and the additional terms have almost no effect. The agreement with the planar wave of the experiment is excellent. Closer to the resonance with $\Omega = 0.962$ (Fig. 3.19c) the larger amplitude parameter $\hat{Y} = 0.233$ results in a more significant contribution of the higher modes. This can be seen with a steeper wave crest, a shifted nodal points, and shallow wave trough. Although in principle all these characterizations are also true in the measurement, the total wave shape is different: The measured rise at the wall is much steeper. In the multimodal model the wave develops a small plateau in the right half of the container, not present in the experiments. For even larger amplitudes (Fig. 3.19e) at $\Omega = 0.909$ and $\hat{Y} = 0.377$ these deviations are intensified, resulting in large qualitative differences with the experiment. The two higher modes, scaling with $\mathcal{O}(\hat{Y}^2)$ and $\mathcal{O}(\hat{Y}^3)$ respectively, have a significant contribution here. In the multimodal model, only a single linear viscous damping term is applied for the first mode in Eq. (3.9). The extreme growth of the $n = 2$ and $n = 3$ modes suggests that higher order damping terms might be necessary for a more accurate prediction. It should be noted that the varying contribution of the higher modes predicted by multimodal model, could also be seen in the measured frequency spectrum of the varying sloshing states in Fig. 3.15. In large amplitude sloshing states the third harmonic (scaling with $\mathcal{O}(\hat{Y}^3)$) was significantly enhanced. The growing contribution of the second harmonic could not be observed, as the spectral analysis was performed for the horizontal velocity in the center of tank. In the multimodal model, the second harmonic is only present in the β_2 term which has an axis symmetric wave, creating no horizontal motion at this location. If the vertical velocity component or a different location is analyzed, the contribution of the second harmonic becomes evident (see Appendix B.4 for additional data).

In existing studies, there are rarely measurements of the full surface wave, which can hinder the qualitative evaluation of models. Typically, only time series of the wave elevation at a single location in the container are used in experimental validations. For this purpose, the predicted time series of the multimodal model are also compared to the experiments in

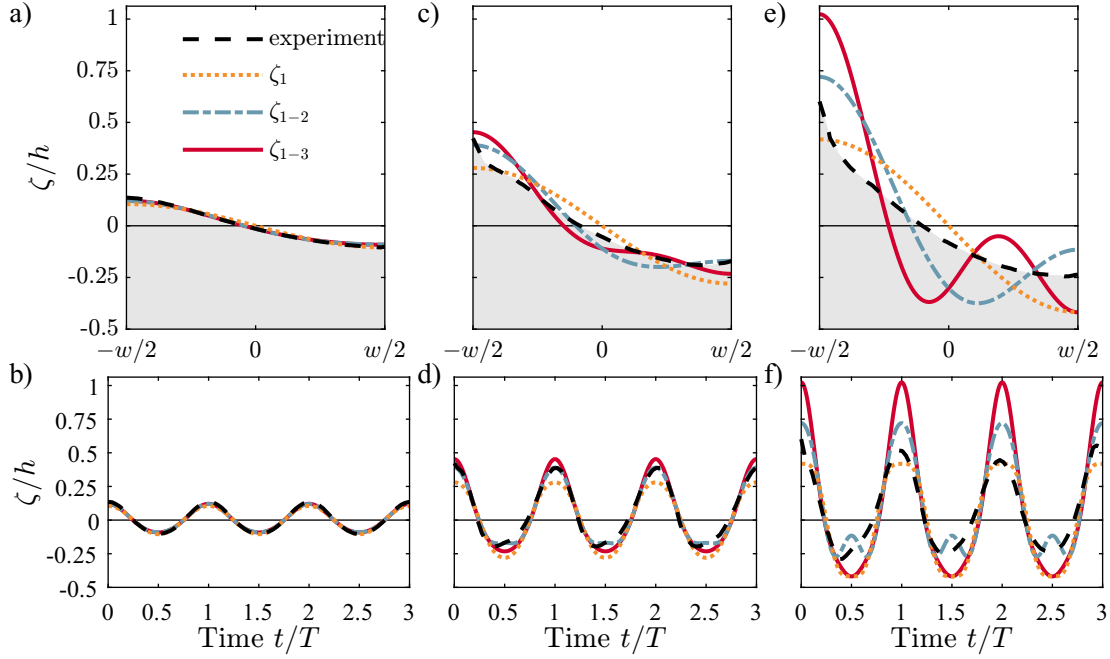


FIGURE 3.19: Comparison of the surface elevation as measurement in the experiments and the predictions of the multimodal model with Eq. (3.7), for three different strength of surface displacement. The the experimentally obtained surfaces are indicated with the black dashed lines. The coloured lines indicate the multimodal model for increasing number of included modes (only the first mode in orange $N = 1$, first two modes in blue $N = 2$, first three modes in red $N = 3$). The snapshots of the surface elevation over the container width a), c), e) correspond to the time series of the displacement at the left wall b), d), f). The phase of experiment data was aligned to fit the model for better visual comparison. The driving parameters are in a)-b) $\Omega = 0.950$, $\hat{Y} = 0.084$, in c)-d) $\Omega = 0.962$, $\hat{Y} = 0.233$ and in e)-f) $\Omega = 0.909$ and $\hat{Y} = 0.377$.

Fig. 3.19b,d,f for the elevation at the wall. With the two lower amplitudes ($\hat{Y} = 0.084$ and $\hat{Y} = 0.233$) the model fits the experiment, despite the discussed differences of the surface wave in Fig. 3.19c. Hence a single point time series is not sufficient to validate the model accuracy. Again, for the largest amplitude in Fig. 3.19f the model significantly overestimates the surface elevation. In conclusion, the multimodal model can deliver excellent predictions for smaller amplitudes, while for larger amplitudes closer to the resonance the wave dynamics differs substantially.

3.4.2.2 Comparison of the multimodal response curves

In the previous chapter, predictions of the multimodal model were compared to measurements from this study, based on the surface wave and its time series. As was determined in section 3.3.3, the liquid's center of mass amplitude is a robust and global observable for the resonant sloshing dynamics. For the multimodal model the center of mass displacement can be easily calculated from the wave elevation function Eq. (3.7) as,

$$X_N(t) = \frac{1}{wh} \int_{-w/2}^{w/2} \xi \left[\sum_{n=1}^N \left(w \beta_n(t) \hat{f}_n(\xi) \right) + h \right] d\xi / w. \quad (3.22)$$

The response curves of the center of mass amplitude and the phase lag (calculated with Eq. (3.11)) are shown together with the experimental measurements in Fig. 3.20.

Far from the resonance peak, the amplitude predictions are in very good agreement

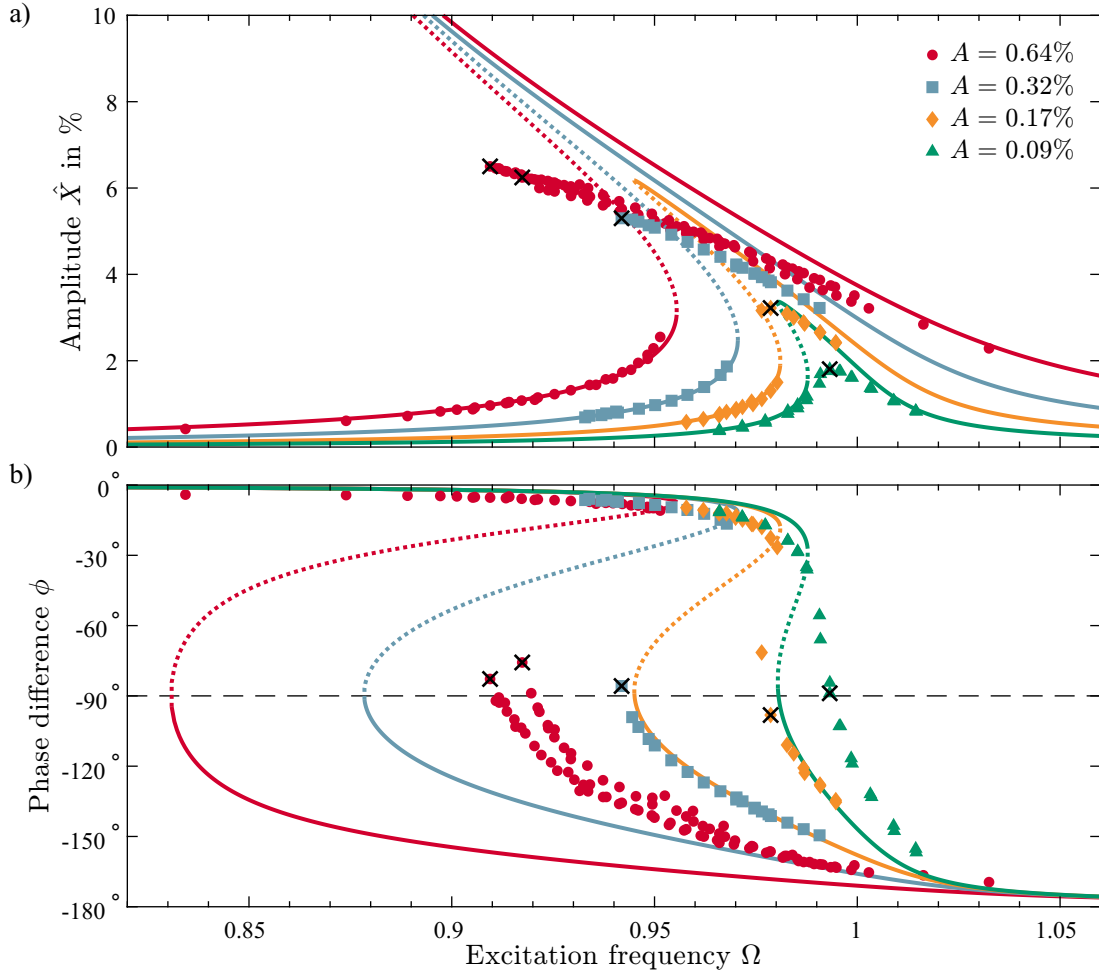


FIGURE 3.20: Response curve of the sloshing experiments (symbols) compared to the multimodal model (lines) for different excitation amplitudes A . a) Response of the center of mass amplitude and b) the phase lag between sloshing motion and excitation as functions of the driving frequency. The measurements are the same as in Fig. 3.18. In the multimodal model the amplitude response is computed using Eq. (3.22) including the first three modes $N = 3$. The amplitude $\hat{X}$ was determined as the maximum displacement over one oscillation period T . The phase lag of the multimodal model is computed from Eq. (3.11).

with the measurements. The same could be seen with the surface waves examples in section 3.4.2.1. But even for the lowest investigated driving amplitude $A = 0.09\%$, the resonance significantly deviates from the experiment. The model already predicts hysteresis and a resonance peak approximately two times larger than the measured maximum. In this amplitude regime, the wave motion at this driving amplitude is still very much linear throughout all frequencies. At higher driving amplitudes, a similar overestimation of the resonance peak continues. The lower branch sloshing amplitude and the frequency of the jump-up transition are well predicted for all forcing amplitudes. However, on the upper branch, the sloshing amplitudes in the experiment are systematically between 8% and 30% smaller than the predicted curves.

In the multimodal model, the damping rate γ is the only parameter not directly calculated, as the underlying potential theory is inviscid. Instead, the measured damping rate from the experiment is used for calculating the amplitude parameter $\hat{Y}$. Artificially tuning this damping rate in the model has very little effect in the frequency regime where the curves match the experiments. By contrast, the damping significantly changes the

magnitude of the resonant amplitudes. Computing the model with a smaller damping (for example, the analytical value from Keulegan's theory $\gamma = 5.57 \cdot 10^{-3}$) amplifies the observed differences to the measurement. But when a larger damping rate is selected (e.g. $\gamma = 15 \cdot 10^{-3}$) the model can be tuned to be more in line with the experiment, especially for the two lowest amplitude curves. At a larger driving, the match remains poor. This suggests that the implementation of the linear viscous damping is the origin of the deviations from the experiment. This is further supported by comparing the phase lag in Fig. 3.20b. Here, the model significantly differs from the measurements even for the regime of very small sloshing wave away from resonance, where the wave and amplitude prediction are excellent.

Overall, the multimodal approach exhibits a systematic error for predicting amplitudes, phase-lag, and waveform in the proximity of the first mode resonance independent of the forcing amplitude. The strength of the multimodal model lies in analytically predicting the contribution of different modes, and the change in dynamics from softening to hardening with lower filling levels (Faltinsen et al., 2000; Faltinsen and Timokha, 2002), which was examined in this chapter. For the investigated parameter range with a deep filling level, the resonance is mainly dominated by a single mode (as shown in section 3.4.1.2). It seems reasonable to examine whether a simpler mechanical model, neglecting multi mode interactions and fully descriptions of the surface wave, can be sufficiently used instead. In the following, the nonlinear Duffing oscillator will be compared with the sloshing motion.

3.4.3 Modeling sloshing with the Duffing equation

As hypothesized by Ockendon and Ockendon (1973) and shown in the previous sections, the amplitude response of the sloshing system resembles Duffing-like dynamics. For sloshing in a rectangular container with a deep filling level, this corresponds to a softening of the response, which becomes more prominent for large amplitudes, as nonlinear forces increase. In this section, the measurements of the forced sloshing response will be directly compared to the Duffing dynamics. First, the Duffing model has to be adapted to the sloshing system of this study (section 3.4.3.1). Afterwards, the amplitude and phase lag from the model are compared with measurements (section 3.4.3.2).

Unlike the multimodal model in the previous section, the model parameters cannot be easily analytically derived. Therefore, the parameters of the Duffing equations are determined through direct measurements from the system (such as the natural frequency and damping), or via fitting the model equations to the measurement response (for the nonlinearity parameter ϵ).

3.4.3.1 Adapting the Duffing equation to sloshing

To use the general Duffing equation (as introduced in section 3.1.4) as an equivalent mechanical model for sloshing motion, the parameters of the equation have to be specified to the sloshing system. Only the first mode $n = 1$ is investigated and will be included in the model. The experimental setup has an external forcing of $m_1 F = m_1 \omega^2 x_a$. Analytical mechanical models predict that only a fraction of the total fluid mass oscillates for any given mode. The same effect could be seen in the velocity field measurements in Fig. 3.10, where the fluid at the bottom of the tank is almost at rest and does not fully contribute to the sloshing motion. The analytical mass fraction for $n = 1$ and $h/w = 0.8$ (with Eq. (3.13) by (Graham and Rodriguez, 1952)) is $c_1 = 0.3183$. In the measurements for the calculation of the center of mass displacement with Eq. (3.21) the full volume of the fluid was considered, instead of an equivalent mass fraction. As the static mass does not move (it is assumed that higher modes are close to zero and can be neglected), the center of mass displacement of the full fluid will be smaller by the factor of c_1 than the first mode mass displacement

x_1 . To directly compare the model with the measurements, I substitute $x = c_1 x_1$. The resulting Duffing equation for the first mode in reads,

$$\ddot{x} + 2\delta_1 \dot{x} + \omega_1^2 x + \epsilon x^3 = c_1 x_a \omega^2 \cos \omega t, \quad (3.23)$$

with $\epsilon = \epsilon_1/c_1^2$. In the resulting steady state solution from harmonic balance $x(t) = \hat{x} \cos(\omega t + \phi)$ the sloshing center of mass amplitude (displacement) is,

$$\hat{x} = \frac{c_1 x_a \omega^2}{\sqrt{(\omega^2 - \omega_1^2 - \frac{3}{4} \epsilon \hat{x}^2)^2 + (2\delta_1 \omega)^2}}, \quad (3.24)$$

and the phase difference between excitation and center of mass motion is,

$$\tan \phi = \frac{2\delta_1 \omega}{\omega_1^2 - \omega^2 + \frac{3}{4} \epsilon \hat{x}^2}. \quad (3.25)$$

Both the amplitude and phase lag response from Eq. (3.24) and Eq. (3.25) as functions of the driving parameters can be directly compared to the frequency response curves of the liquid's center of mass motion.

For applying the Duffing oscillator model to the sloshing experiment data, the natural frequency and the viscous damping rate can be determined from analytical prediction and the decay in the experiment (see section 3.3.4), leaving only the nonlinearity parameter ϵ as a free fitting parameter. A least-square-residual fitting of the function Eq. (3.24) to the center of mass of the amplitude from Fig. 3.18 is performed, resulting in a nonlinearity parameter $\epsilon = -1.44 \cdot 10^4 \text{ m}^{-2} \text{ s}^{-2}$. For more details on the fitting method for the multi-valued amplitude response curves, see Appendix B.2. The fit quality will be discussed in section 3.4.3.2.

To facilitate a better comparison to other future studies, with different system geometries all parameters of the Duffing model are given in a dimensionless form. The dimensionless Duffing equation reads,

$$\ddot{X} + 2\gamma_1 \dot{X} + X + \epsilon X^3 = c_1 A \Omega^2 \cos \Omega t, \quad (3.26)$$

for the dimensionless center of mass displacement $X = x/w$. The time is normalized by the inverse of the natural frequency ω_1 . The dimensionless parameters used in my investigation are the damping ratio $\gamma = \delta_1/\omega_1 = 8.4 \cdot 10^{-3}$ and the dimensionless nonlinearity $\epsilon = (w/\omega_1)^2 \epsilon = -59.2$. All parameters are given in dimensional and dimensionless form in Table 3.4. The dimensionless expression of the response amplitude Eq. (3.24) reads,

$$\hat{X} = \frac{c_1 A \Omega^2}{\sqrt{(\Omega^2 - 1 - \frac{3}{4} \epsilon \hat{X}^2)^2 + (2\gamma_1 \Omega)^2}}. \quad (3.27)$$

The corresponding phase difference Eq. (3.25) with dimensionless parameters reads,

$$\tan \phi = \frac{2\gamma_1 \Omega}{1 - \Omega^2 + \frac{3}{4} \epsilon \hat{X}^2}. \quad (3.28)$$

3.4.3.2 Comparison between Duffing equation and sloshing motion

In this section, the fitted response curves of the Duffing equation are compared to the experimentally obtained response curves throughout the different regimes of nonlinear

Parameter	Dimensional	Dimensionless
Viscous damping	$\delta_1 = 0.065 \text{ s}^{-1}$	$\gamma_1 = 8.4 \cdot 10^{-3}$
Natural frequency	$\omega_1 = 7.800 \text{ s}^{-1}$	1
Nonlinearity constant	$\epsilon = -1.44 \cdot 10^4 \text{ m}^{-2} \text{ s}^{-2}$	$\varepsilon = -59.2$

TABLE 3.4: Experimentally determined parameters of the Duffing equation in dimensional Eq. (3.23) and dimensionless Eq. (3.26) form.

resonances. The response curves of the experiments and Duffing model are displayed in Fig. 3.21 for $A = [0.09, 0.17, 0.32, 0.64]\%$. For the two lowest driving amplitudes $A = [0.09, 0.17]\%$ where the softening effect on the resonance is small, and the waves remain quasi-planar, the Duffing model perfectly captures the experiment (see Fig. 3.21a). Brennan et al. (2008) determined a critical forcing amplitude for the Duffing response to develop hysteresis as $A_c = 0.13\%$ (computed for the parameters of this system). This fits with the first appearance of a very small hysteresis at $A = 0.17\%$. At amplitude $A = 0.32\%$ the model starts to diverge from the experiment. While the amplitude values are still excellent, the response maximum and jump-down transition in the Duffing equation are predicted for lower frequencies. This effect becomes much more pronounced at $A = 0.64\%$, where the hysteresis in the Duffing curve is more than three times as large. The jump-down transition of the Duffing oscillator lies at $\Omega = 0.78$ with an amplitude of $\hat{X} = 9.44$ (approximately 1.5 times larger than the measurement). It should be noted that, for both $A = 0.32\%$ and $A = 0.64\%$ the jump-up transition still perfectly matches the experiments. In the Duffing dynamics, the jump-up frequency is a function of only the driving and nonlinearity parameter, while the jump-down transition and response maximum also depends on the viscous damping (Brennan et al., 2008). Recall that for the largest amplitudes complex sloshing states such as wave-breaking were observed. These states will likely result in much larger nonlinear dissipation that cannot be captured by the linear viscous damping in this approach.

This assumption can be further supported by analyzing the phase lag of the system (see Fig. 3.21b). Note that only the amplitude of the Duffing oscillator was fitted to the measurements. The phase lag is simply calculated based on the system parameters and the amplitude response curves with Eq. (3.25). Again for the two lowest amplitudes the agreement between model and experiment is excellent. For $A = 0.32\%$, where the first deviation in the jump-down transition appeared, the phase lag on the upper branch clearly diverges from the experiment, well before the jump-down transition is reached. The same can be seen for $A = 0.64\%$. Here, the phase lag starts to deviate in the frequency range where the wave-breaking and peak sloshing state are first observed (compare Fig. 3.17). This supports the hypothesized stronger nonlinear-damping for the more complex surface states.

The phase lag proves to be a more sensitive observable of the resonant dynamics than the amplitude. For each driving amplitude, the measured jump-down transition always has a phase lag of $\sim 90^\circ$, providing a robust indicator that the response maximum was reached. While the amplitude could be freely fitted to match the experiments, the phase lag clearly demonstrates where the model dynamics are insufficient to characterize the system. It should also be noted that the subharmonic $\omega/3$ and superharmonic 3ω spectral components (as observed in the period-three state in section 3.4.1.2) are classic secondary frequencies of the Duffing oscillator (Kalmár-Nagy and Balachandran, 2011), stemming from the cubic nonlinearity term.

The Duffing oscillator model seems to be mainly limited by the implementation of linear viscous damping without higher order dissipation. The excellent agreement at

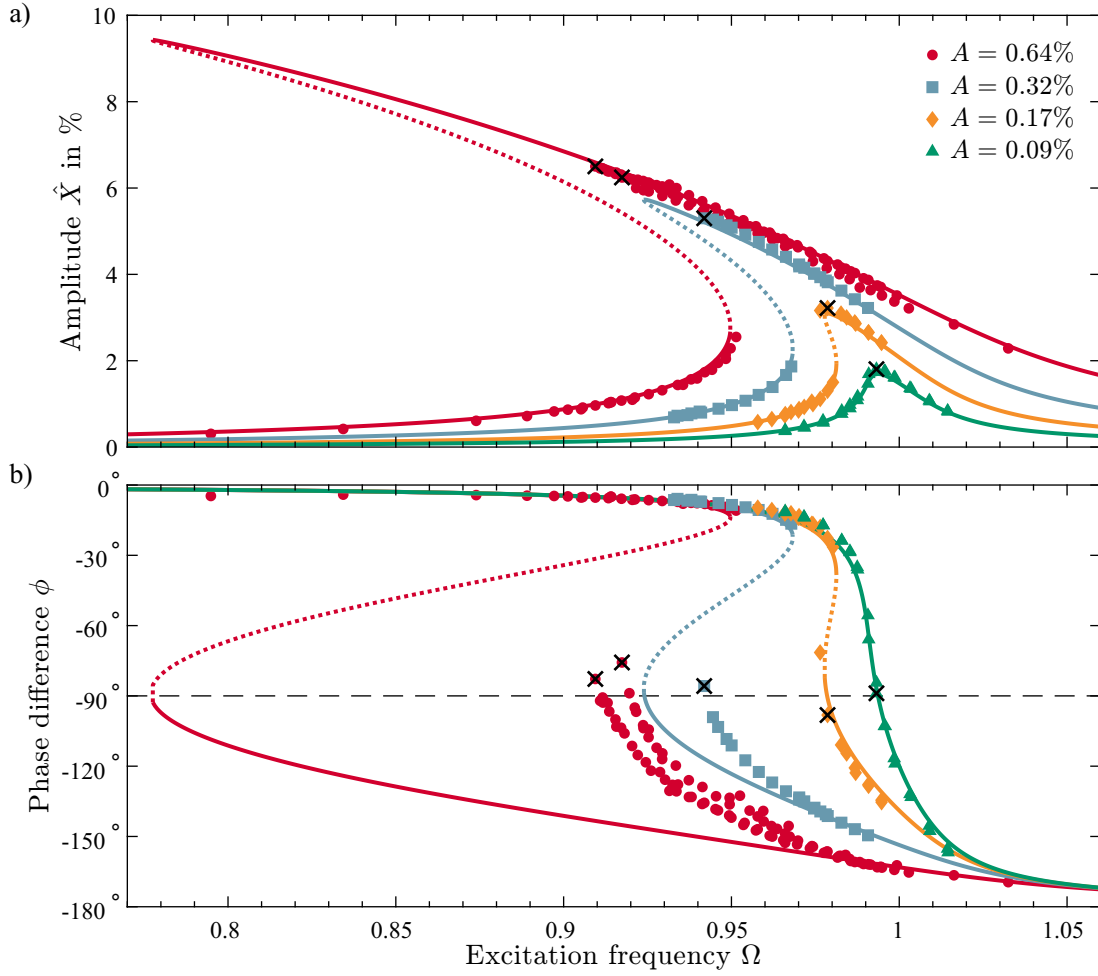


FIGURE 3.21: Response curve of the sloshing experiments (symbols) compared to the Duffing oscillator model (lines), fitted with $\varepsilon = -59.2$ in Eq. (3.26), for different excitation amplitudes A . The dotted lines indicate unstable solutions of the Duffing equation. For the measurement were obtained with a fixed amplitude A and a quasi-static up and down frequency sweep around each resonance peak. For $A = 0.64\%$ multiple sweeps were utilized to capture the competing flow states, see Fig. 3.17. The maximum amplitude during each run is denoted with $\times$. b) phase lag between sloshing motion and the excitation corresponding the amplitudes in a). The jumping transition from high to low amplitude sloshing always occurs at a phase difference of $\phi \approx -90^\circ$.

lower amplitudes indicates that the dynamics could be portrait in a reduced order model on a spectral submanifold (SSM) (Breunung and Haller, 2018), which would allow for implementation of both nonlinear effects on frequency and dissipation of a system. This hypothesis will be further explored in the next section.

3.4.4 Spectral submanifold theory for sloshing motion

In my publication (Bäuerlein and Avila, 2021a) it was hypothesized that the excellent fit of the Duffing oscillator model for lower amplitudes could indicate that the underlying dynamics of the quasi-two-dimensional sloshing can be reduced to a spectral submanifold (SSM). This assumption could be confirmed in a joint publication by Cenedese et al. (2022). The development of the data-driven modeling tool in the publication was performed by the group at ETH Zürich, while we provided experimental data and analysis. In this section only the main findings with the application to the sloshing resonance are presented.

Spectral submanifold theory is a tool for reduced order modeling. The key idea is that for a nonlinear dynamical system with many dimensions, such as a fluid flow governed by the Navier-Stokes equations, the relevant dynamics can be reduced to a much lower dimensional model, thereby drastically reducing the computational effort for solving the system. The linear approach to this would be a projection of the relevant eigenmodes to a spectral subspace. The relevant modes are usually selected as the slowest-decaying or lowest-frequency modes. With a nonlinear or non-linearizable system, a nonlinear continuation of these eigenmodes can be used to create a submanifold. Out of all possible continuations, the spectral submanifold is defined as the invariant, unique, and smoothest continuation without any internal resonances. For a complete mathematical description of this theory see Haller and Ponsioen (2016). The curvature in the spectral submanifold allows the system to account for nonlinearities, while still behaving like a linear spectral subspace. These spectral submanifolds can be computed from a system of equations (like a finite element model (Jain and Haller, 2022)) or using a data-driven approach without prior knowledge of the governing equations (Cenedese et al., 2022). The advantage of this approach is that each mode in the SSM can be represented by an arbitrarily complex polynomial for the nonlinear frequency and dissipation. Thereby, the SSM is not limited to linear viscous damping as the previously investigated multimodal and Duffing oscillator model.

In section 3.4.3.2 it was shown that the sloshing resonance for smaller amplitudes behaves like a Duffing oscillator. This indicates that the relevant dynamics can be reduced to a two-dimensional SSM, making the system an ideal example for this approach. In Cenedese and Haller (2020) a machine-learning based model reduction tool *SSMLearn* was developed, which can fit the SSM geometry based on an input of training trajectories of the autonomous system, without forcing. For the sloshing system, two trajectories from the natural decay of the center of mass motion, after the forcing is stopped (compare Fig. 3.12), are used as input data. Because these trajectories do not contain external forcing, they should be very close to the surface of the SSM. This enables the *SSMLearn* to fit the manifold and compute the reduced order model. From this model, the system dynamics can be predicted, with two examples presented in Fig. 3.22. In Fig. 3.22a a comparison of a measured trajectory of the center of mass decay (not a trajectory used in the previous fitting) and a model reconstructed trajectory, generated from an initial condition of five time-steps, shows very good agreement. With such a decay trajectory, a large range of dynamics at different amplitudes is captured. Initially, the large sloshing amplitude results in stronger nonlinear effects on the dissipation rate and instantaneous decay frequency. During the decay, the dynamics approaches the quasi-linear motion of small amplitudes. From the fitted SSM model, this amplitude dependency on the dynamics can be calculated, as shown with the backbone curves of the change in damping rate Fig. 3.22b and frequency Fig. 3.22c as they are affected by the sloshing amplitude. Compared to the Duffing oscillator, both frequency and damping are nonlinear in the SSM. Note, that for $\hat{X} \rightarrow 0$ the damping approaches $\sim 0.0631/s$ and frequency $\sim 7.81/s$, matching the linear predictions of the system parameters in section 3.3.4, even though no external input or equation for these parameters were given to the model. The backbone of the frequency

in Fig. 3.22c indicates the amount of softening for increasing amplitudes as it is observed in the response curves. The damping backbone curves give a prediction of the increase in dissipation with larger amplitude. For the amplitude range used in the SSM training $X < 4.5\%$ the dissipation already increases by $\sim 15\%$ compared to the linear limit. Such predictions can provide a powerful tool to improve sloshing models, as there are no reliable models that accurately predict the nonlinearity of dissipation analytically or extract it from measurements (Faltinsen and Timokha, 2009; Ibrahim, 2005).

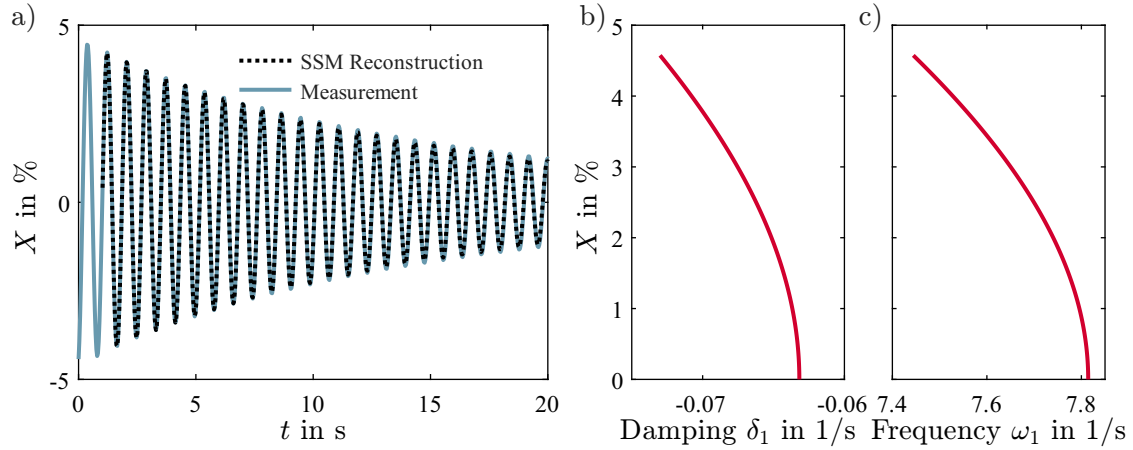


FIGURE 3.22: a) Reconstructed trajectory of the sloshing motion decay compared the measurement with the same initial condition. b) and c) prediction from the SSM of the backbone curve for nonlinear damping rate and resonance frequency for as the amplitude increases. The Figures are created with the SSMLearn tool by Cenedese et al. (2022).

If an external forcing is added to the SSM, it can also be used to predict the forced frequency response curves of amplitude and phase lag, as depicted in Fig. 3.23. The largest driving amplitude $A = 0.64\%$ is not included in this comparison, as the training data only includes amplitudes of $X < 4.5\%$ (compare backbone curve in Fig. 3.23a), limiting the extrapolation range of the model. Furthermore, the largest amplitude curve contains multiple coexisting states and nonlinear resonances in the period-three state. Modeling this would necessitate a higher dimensional SSM and training data containing these dynamics. Despite this, both the amplitude and phase lag response are in excellent agreement with the measurement data. While the Duffing oscillator (in section 3.4.3.2) was able to achieve a similar agreement for the two smallest driving amplitudes, it started to deviate for $A = 0.32\%$, especially in the phase lag. By contrast, the SSM prediction matches for both curves and only required two unforced trajectories to compute the model, compared to multiple response curves, the natural frequency, and damping rate used in the Duffing fit. This makes the SSM approach very promising for cases where knowledge of the system (damping or natural frequencies) or measurement data (highly resolved response curves with multiple driving amplitudes) are limited. A quantitative comparison and more detailed assessment of the different resonance models will follow in the next section.

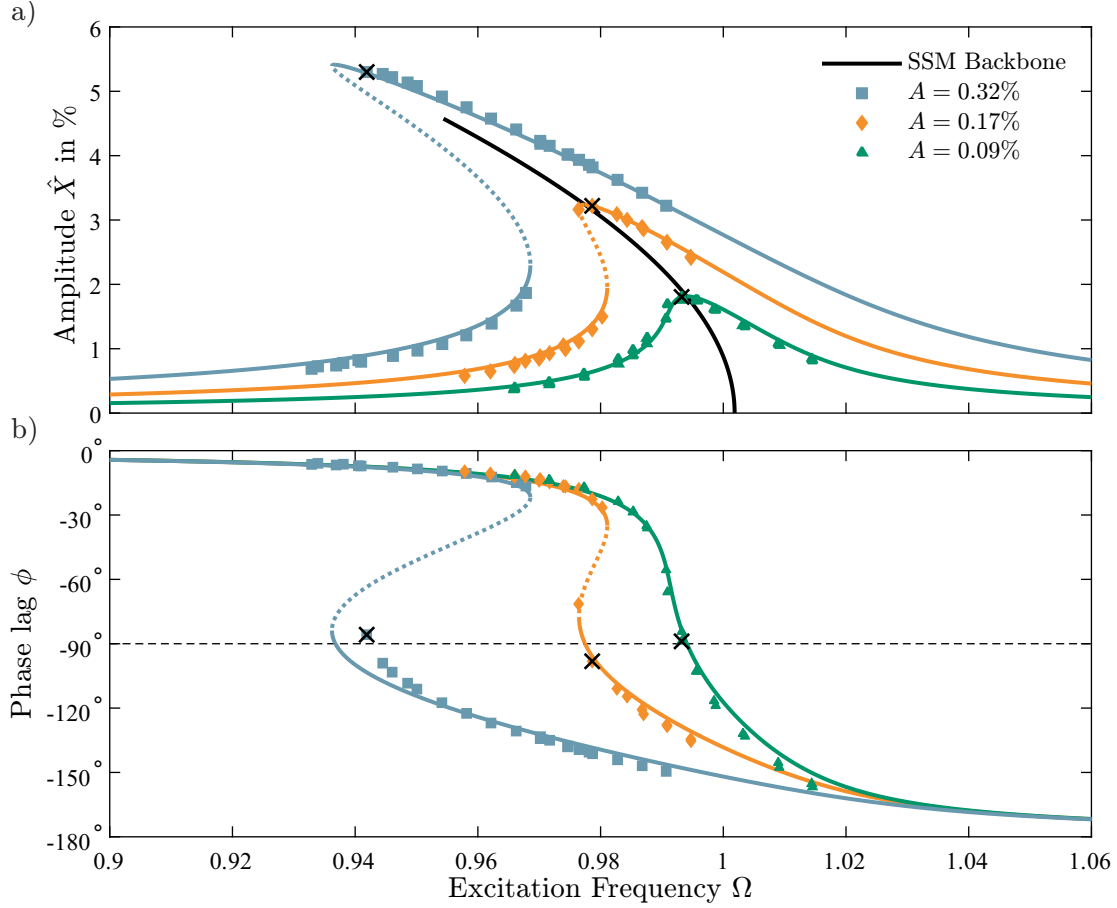


FIGURE 3.23: Response curve of the sloshing experiments (symbols) compared to the spectral submanifold theory (lines) for different excitation amplitudes A . a) Response of the center of mass amplitude and b) the phase lag between sloshing motion and excitation as functions of the driving frequency. The backbone curve in a) how the resonance frequency will shift for increasing amplitudes. The Figures are created with the SSMLearn tool by Cenedese et al. (2022).

3.4.5 Assessment of different sloshing model approaches

In this section, a quantitative analysis of the accuracy between the experiments and model predictions of the multimodal model, the Duffing oscillator, and the SSM is presented and the advantages and disadvantages of each each model are discussed.

3.4.5.1 Detailed analysis of the model accuracies

For a direct comparison of the accuracy of the different resonance models, it is necessary to determine the deviation between measurements and the predicted forced response of both the amplitude and phase lag. To evaluate both responses, the squared residuals between the measurements $(\hat{X}, \phi)$ and the model $(\hat{X}_m, \phi_m)$ and normalized by the squared standard deviation of the measurements,

$$\sigma_{\hat{X}}^2 = \frac{\sum_{i=1}^n (\hat{X}_i - \bar{\hat{X}})^2}{n-1}, \quad \sigma_{\phi}^2 = \frac{\sum_{i=1}^n (\phi_i - \bar{\phi})^2}{n-1}. \quad (3.29)$$

As the sloshing dynamics increases in complexity with higher amplitudes, these normalized residuals are compared with respect to the sloshing amplitude $\hat{X}$. An example of

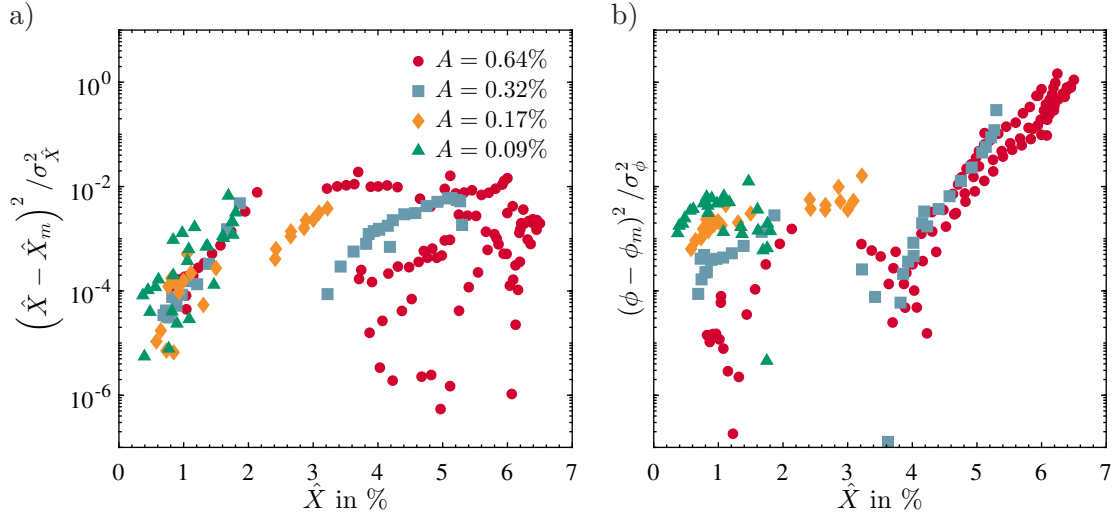


FIGURE 3.24: Normalized residual error between measurements and the Duffing oscillator model for the amplitude and phase lag.

these residuals is shown in Fig. 3.24 for the Duffing oscillator model. As this model was specifically fitted to the amplitude response of the measurements, the amplitude differences remain small and are largely unaffected by the magnitude of the sloshing amplitude $\hat{X}$. A different picture emerges for the phase lag. While the residuals of the low amplitude regime $\hat{X} < 4\%$, are small and of a similar magnitude compared to the amplitude, the residual error strongly increases for larger amplitudes. This is caused by the observed difference in the predicted phase lag around the response maximum on the upper branch (compare Fig. 3.21b). In the presentation of the residual error the boundary between weakly nonlinear dynamics for smaller sloshing amplitude and the higher amplitude regime, where the system diverges from Duffing dynamics, can be identified as $\hat{X} \approx 4\%$.

The same analysis is repeated for the multimodal model and SSM response in Fig. 3.25. As the SSM could only produce responses for forcing up to $A = 0.32\%$, the models are only evaluated on the lowest three driving amplitudes. In comparison to the previously shown Duffing model, the SSM approach yields a comparable residual error for the sloshing amplitudes, with similarly no strong dependency on the amplitude $\hat{X}$. In contrast, the multimodal model exhibits a distinct error profile, as the near resonance amplitude is consistently overpredicted for all forcing amplitudes. Therefore, the error does not systematically scale with $\hat{X}$. Instead, the largest deviations occur at the response maximum of each measurement series (indicated with black $\times$), with residuals increasing in proximity to the resonance frequency. This local divergence significantly increases the mean residual error, despite the model's high accuracy in off-resonance regions.

For the multimodal model, the same locally increased error near the resonance peak, can be observed in the phase lag in Fig. 3.25b. Notably, the phase lag also more clearly differentiates the Duffing oscillator and SSM model. While the Duffing model begins to deviate for $\hat{X} > 4\%$, the residual error between SSM response and the measurement remains small for the entire sloshing amplitude range. These differences can be quantified through the mean residual errors in Table 3.5, where the SSM outperforms both alternative models. Ultimately, these results suggest that the phase lag is the more suitable observable for evaluating resonance model accuracy, as it more sensitively reflects localized model inaccuracies.

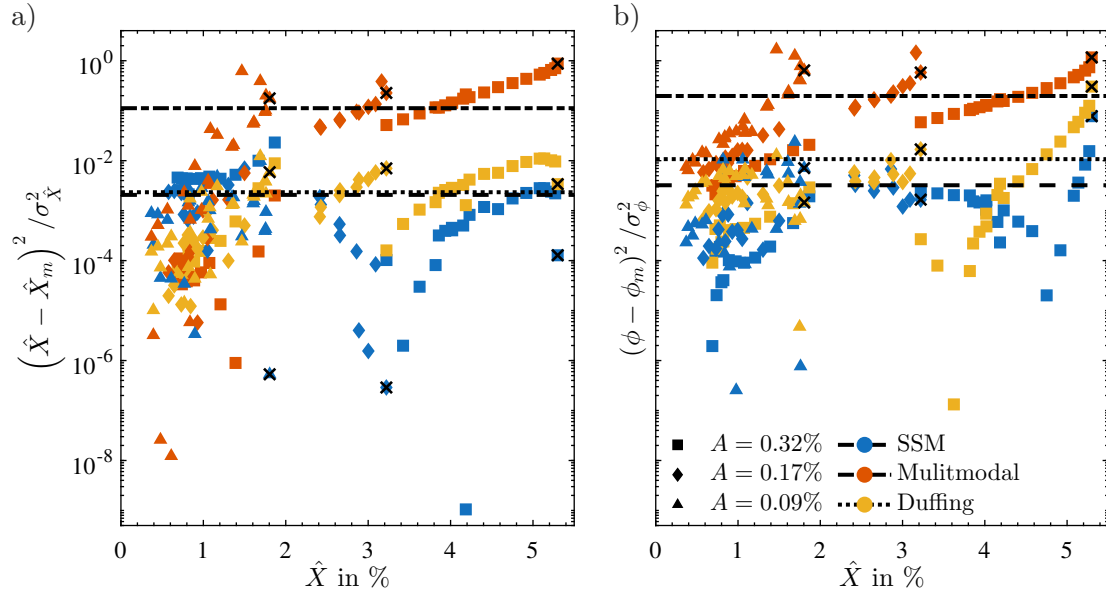


FIGURE 3.25: Normalized residual error between measurements and the three resonance models for the amplitude and phase lag. The color indicates the forcing amplitude A and the symbol the forcing amplitude in each dataset. A black $\times$ marks the maximum sloshing amplitude for that forcing curve. The horizontal lines are the averages residuals (see Table 3.5).

	SSM	Duffing	Multimodal
Amplitude error $(\hat{X} - \hat{X}_m)^2 / \sigma_{\hat{X}}^2$	0.0021	0.0023	0.112
Phase lag error $(\phi - \phi_m)^2 / \sigma_{\phi}^2$	0.0032	0.0110	0.197

TABLE 3.5: Mean normalized residual error between measurements and the three resonance models for the amplitude and phase lag. The errors are evaluated over the lowest three forcing amplitudes $A = [0.09, 0.17, 0.32]\%$.

3.4.5.2 Mechanical mass-spring model: Duffing oscillator

Equivalent mechanical mass-spring models can be an important tool (Ibrahim, 2005), as they are fast to compute and can be used in multiphysics computations of larger system dynamics with little effort. For a linear harmonic oscillator all parameters (natural frequency and damping rate) can easily be determined from a single investigation of the decay. It is important to emphasize, that for small sloshing amplitudes far away from resonance a simple harmonic oscillator is sufficient. In this investigation, the Duffing equation – as most common nonlinear extension of a linear oscillator model – was investigated. This approach has the nonlinearity ϵ as a single free fitting parameter for the system. An accurate fitting requires measurements of the forced response near the resonance, ideally with a large driving amplitude. The Duffing equation predicts an accurate threshold amplitude, where hysteresis first emerges. At lower amplitudes it perfectly fits the liquid's center of mass amplitude and phase lag measurements. With larger forcing, the resonance peak and hysteresis are overestimated, likely due to nonlinear damping effects not implemented in this approach. Despite the good agreement in this study, the model cannot be easily generalized to different filling levels or container geometries. Further parameter studies would be necessary to derive a dependency between the nonlinearity parameter and the

physical parameters of the sloshing system.

3.4.5.3 Multimodal model from potential flow theory

Unlike the Duffing oscillator approach, the multimodal model can be analytical derived without the need for fitting to experimental data. In addition, the multimodal approach contains a full description of the surface wave instead of just a one-dimensional center of mass displacement. As the multimodal analysis is derived with fundamental fluid mechanics equations (potential theory), the parameters have an intrinsic physical meaning. Therefore, the model can be computed for different filling levels (Faltinsen et al., 2000) or include additional terms for three-dimensional modes in different container geometries (Faltinsen and Timokha, 2017). However, as the approach is based on inviscid potential theory, it does not include any damping by itself. Typically, a linear viscous damping term is added for the first mode only. This damping coefficient can be either analytically estimated (with e.g. Keulegan (1959)) or experimentally determined. It was shown, that the experimental damping leads to better agreement with the measured response curves. For the investigated case of quasi-two-dimensional sloshing, the model gives accurate predictions only on the lower branch solution of the sloshing and further away from resonances. The effects of softening and the maximum response amplitude for the resonance are systematically overestimated, also for small driving amplitudes. A systematic difference for all phase lag measurements leads to the assumption that linear viscous damping for the first mode is insufficient for this system.

3.4.5.4 Dynamical system approach: spectral submanifold theory

Compared to the two previous models the spectral submanifold theory is a general approach for reduced order modeling of nonlinear dynamical systems. With the data-driven prediction tool presented by Cenedese et al. (2022) no prior knowledge of the systems dynamics or the governing equations is necessary. Despite this, compared to the other models, the SSM approach was able to achieve the best agreement of the forced response curves with the measurements. In the SSM both frequency and damping rate are nonlinear with respect to the sloshing amplitude, enabling good agreement for the previously difficult phase lag response. While more recent publications were also able to model trajectories containing nonlinear interactions of modes with e.g. period-three motion (Axås et al., 2024), the approach is limited for predicting forced response curves containing multiple interacting modes as they occur for large amplitude sloshing. The excellent agreement with the measurements and minimal training data required make the reduced order SSM model an extremely promising approach for the resonance modeling, especially as it can be universally applied for different sloshing resonances or other dynamical systems.

3.5 Summary

The accurate modeling of sloshing motion and especially its resonance remains important in many engineering applications and presents an outstanding challenge for fluid mechanics over the last 60 years. Throughout the literature, different approaches have been applied to describe sloshing dynamics. In my thesis, the quasi-two-dimensional motion in a narrow rectangular tank with a large filling level was investigated. The goal was to evaluate the capabilities and limitations of different resonance models, via comparison with detailed experimental measurements and utilization of approaches from nonlinear dynamics theory. This summary is structured around answering the research questions formulated at the beginning of this chapter (see section 3.2).

RQ1: Nonlinear sloshing regimes

The sloshing resonance gradually transitions from quasi-linear to nonlinear with increasing forcing amplitude. At the lowest forcing level investigated ($A = 0.09\%$), the wave motion stays planar and the system displays a slight softening, with a maximum amplitude response shifted towards lower frequencies. As the driving amplitude increases, the softening becomes more pronounced, leading to a growing regime of coexisting sloshing states and a hysteresis between both states. At the largest forcing amplitude ($A = 0.64\%$), the planar wave motion gives way to multiple competing flow states, including wave-breaking and period-three motion, around the maximum sloshing amplitude, signaling nonlinear interactions with higher modes.

Crucially, detecting the full range of possible solutions requires extensive experimental studies, with careful tracking of steady states through incremental variations of control parameters (e.g. sweep-up and sweep-down of the excitation frequency in this system). Although such procedures were common in past studies (Abramson, 1966; Abramson, Chu, and Kana, 1966), more recent experimental investigations only contain individual steady state measurements, where the excitation was started with a resting condition. Such a procedure cannot produce hysteresis or the maximum amplitude response if a significant softening or hardening is present. Therefore, the resonant sloshing motion can be significantly underestimated. The sensitivity of a steady state to a change in control parameters is especially critical when considering active feedback mechanisms to counteract sloshing motion (e.g. Konopka et al., 2019). It is also important to stress that multiple response curves with different forcing parameters are essential when investigating resonant dynamics. Due to the nonlinear relationship between forcing and response, a single response curve cannot be used to make predictions for a different forcing, as stated by Breunung and Haller (2018).

RQ2: Evaluation of flow observables

Determining a suitable observable is essential for quantitative analysis of measurement data and comparisons with resonance models. A new approach of utilizing the liquid's center of mass as an observable representing the sloshing amplitude was identified. This can be measured from flow visualizations of the surface wave or calculated from a modeled surface elevation (e.g. with the multimodal model by Faltinsen et al., 2000). Instead of using the surface elevation at a single coordinate in the tank, the center of mass is a robust measure for varying flow states and gives an unambiguous damping rate in the systems natural decay. Furthermore, the center of mass allows for direct qualitative comparison to single degree of freedom mechanical models, like a harmonic oscillator.

Beyond the sloshing amplitude, the phase lag between the driving force and the sloshing oscillation was identified as the most critical indicator of resonance dynamics when comparing

the measurements to a resonance model. The phase lag is directly affected by the damping, and gives an early indicator when the model dynamics are not able to predict the sloshing response. Furthermore, the phase lag is an excellent measure of the response maximum. While sloshing amplitude and the frequency composition can characterize the current sloshing state, they do not indicate the proximity to the maximum response amplitude. Instead, it could be shown that for all driving amplitudes the maximum occurs at a phase lag of 90° . The 90° phase lag criterion suggests damping to be a polynomial function of position and velocity in a nonlinear system (Breunung and Haller, 2018). Therefore, a phase lag measurement can give a directly measurable indicator, if the response maximum is reached. Further studies would need to confirm if this criterion is true for all sloshing resonance (e.g. multiple resonant modes or different container geometries).

RQ3: Comparative analysis of sloshing resonance models

In this chapter, three model approaches were used to describe the sloshing resonance: the analytical multimodal sloshing model by Faltinsen et al. (2000), a nonlinear mechanical Duffing (1918) oscillator, and the spectral submanifold (SSM) theory (Cenedese et al., 2022).

The multimodal model represents a state-of-the-art description of sloshing motion, based on an analytically derived nonlinear solution of potential theory. The model can be generalized to different filling levels and tank geometries while providing a full description of the surface wave. However, because it is based on inviscid theory, it lacks inherent damping. Even when linear viscous damping is manually added, the model systematically overestimates the softening effect and the maximum response amplitude. The consistent mismatch shown in the phase lag measurements suggests that a linear damping term for only the first mode is insufficient for accurately capturing resonant sloshing.

In contrast, the mechanical Duffing oscillator offers a simplified mass-spring approach. By using the nonlinearity parameter of the Duffing equation as a single fitting factor, this model can achieve a perfect fit for the sloshing center of mass amplitude and phase lag at lower driving amplitudes. The model even accurately predicts the threshold where hysteresis first emerges. The match between Duffing dynamics with linear viscous damping and a forced sloshing response was originally postulated by Ockendon and Ockendon (1973), and could be verified in this thesis through the first direct qualitative comparison of the two. However, the reliance on measurement data for the fitting the Duffing oscillator limits its generalizability. Furthermore, at larger forcing amplitudes, the Duffing model overestimates the resonance peak and hysteresis. A mismatch with the phase lag measurements indicates the limitations of the linear viscous damping in the Duffing model. At larger amplitudes, nonlinear damping effects due to the complex wave motion will become dominant.

The most promising results were observed using spectral submanifold (SSM) theory, a general reduced-order modeling approach that is entirely data-driven. Unlike the other two models, the SSM requires no prior knowledge of the governing equations or system parameters and can be reconstructed from a single decay trajectory measurement. The SSM achieved the best agreement with the experimental forced response curves of both amplitude and phase lag, as the approach treats both frequency and damping as nonlinear functions of the sloshing amplitude, overcoming the limitations of the Duffing and multimodal model. While the SSM cannot predict forced response curves where multiple modes interact (such as the observed period-three motions), its minimal training data requirements and high accuracy make it a superior tool for resonance modeling.

Summary of impact

This research emphasizes the qualitative assessment of resonance models, with comparisons to accurate measurement data over a range of sloshing amplitudes. The applicability of any modeling approach will greatly depend on the sloshing regime under investigation. The sloshing resonance will exhibit nonlinear effects of softening and hysteresis, even at minimal forcing amplitudes. With larger sloshing amplitudes, the influence of an increased nonlinear dissipation becomes non negligible, necessitating a shift away from standard linear damping approximations as implement in most sloshing theories. In addition, the prediction of forced response curves which contain complex flow states (such as period-three motion and multimodal interactions) remain an outstanding challenge for current model and highlight a primary objective for future research.

Chapter 4

Conclusions

Many fluid flows in biology, environmental science, and engineering are subjected to periodic excitations of the bulk flow velocity. The resulting periodic nature of the flow can fundamentally characterize the dynamics of the system, such as during the rhythmic pulsation of blood flowing through the cardiovascular network or in the oscillations of fuel in aerospace propellant tanks. Here, the forcing oscillation can induce complex nonlinear effects which are exceedingly difficult to characterize and predict, such as resonance and turbulence. Resonance occurs when an external forcing approaches the natural frequency of the fluid system, leading to drastic amplification of fluid motion and mechanical loads. During turbulence the flow transitions from smooth laminar motion, towards a chaotic, multi-scale flow state. In a pulsatile flow, this transition process is highly dependent on the pulsation cycle, leading to alternating flow stability and intermittent turbulence patches.

In my thesis, these two phenomena were explored through experimental investigations of the resonance of sloshing liquids in an oscillating container and the transition to turbulence in a particle-laden pulsatile pipe flow. The following chapter incorporates the insights gained from both studies to provide a combined perspective on periodically forced fluid systems. Since a substantial part of the thesis involved the conceptualization and building of a novel pulsatile pipe flow experiment and its measurement techniques, the first section of the chapter will summarize the technical advances and the broader impact of the experiment and the methodology developed for both the pipe flow and the sloshing study (section 4.1).

Following the methodological conclusion, the main findings from the sloshing and pipe flow studies will be summarized (sections 4.2 and 4.3). In the introductory chapter, three overarching research objectives for periodically driven flows were formulated (see section 1.2):

- O1** How does a periodic forcing interact with the inherent timescales of the fluid system, expressed through the eigenmodes and the turbulence growth and decay rate?
- O2** Which observables are necessary to describe the nonlinear interactions of a pulsatile or oscillatory fluid flow? What are the operational limits to reliably utilize the observable to model the flow?
- O3** How can selected observables and models be utilized to prediction the development of resonances and turbulence in periodically forced flow? What are the underlying mechanisms of the flow involved in the evolution of these two effects?

The results of both flow systems will be structured around addressing these objectives. Finally, the sections conclude with an outlook for future research, based on the main findings.

4.1 Experimental methods

The study of fluid mechanics often necessitates advanced experimental investigations and methods. Not only to validate theoretical models or numerical results but also to uncover new physical insights and identify the limitations of existing theories. This is especially the case for the dynamics of the periodically forced flows investigated in the thesis. Both the turbulence transition and the development of resonances are dominated by nonlinear interactions, which are exceedingly difficult to predict. In the thesis, specialized experimental setups and measurement techniques were developed and adapted to the unique requirements of this research.

4.1.1 Pipe flow experiment

A newly developed pipe flow experiment was designed to investigate the dynamics of a particle-laden pulsatile flow. Compared to previous pipe flow setups, the experiment is able to include both an adjustable pulsatile flow control and handling of a particle suspension. In particular, the driving system is critical for the investigated transitional pipe flow regime, where even minor inaccuracies in the flow control or structural setup can induce unwanted turbulence, compromising the measurements. In the setup, a servomotor controlled piston can accurately produce pulsations with different frequencies and a variable pulsation waveform. This enables the reproduction of complex pulsations, such as physiological conditions or other unsteady flows. As the particles are mechanically separated from the fluid entering the piston drive, the setup can operate with different particle sizes, materials, shapes, and concentrations without affecting the volume flow rate. With this flexibility, the setup is not only suitable for spherical hard particles in a Newtonian fluid but can also be utilized for more complex flows, mimicking engineering or biological systems.

The implementation of the Shake-The-Box (STB) method for a particle-laden flow represents the most advanced measurement technology in the setup. Resolving the interactions of suspended particles with the surrounding flow has been classically challenging for fluid experiments, as it requires measurements of both phases simultaneously. This setup addresses this challenge, by utilizing the high spatial and temporal resolution of modern commercially available Lagrangian Particle Tracking with STB in combination with refractive index matched hydrogel particles. Using specially developed calibration and evaluation methods, both the three-dimensional fluid velocity field and the particles' translation and rotation are resolved over time. This measurement method offers a promising outlook for future research, as hydrogels can be incorporated with different material properties, shapes, and sizes to investigate complex dynamics and interactions in fluid flows.

Both the experimental setup and measurement technique were designed not only to address the specific research questions presented in the thesis but also with a wide range of future applications in mind. The pipe flow setup is highly adaptable, allowing investigations of different fluids and pipe geometries across various types of pulsatile or unsteady flows. Specifically, the Shake-The-Box measurement technique offers a promising approach for many complex multiphase flows where resolving simultaneous phases is required. These technological achievements, published in Bäuerlein and Avila (2025), serve as a promising basis for future explorations of pipe flow dynamics and complex fluids.

4.1.2 Sloshing methodology

Investigating the sloshing resonance requires characterization of the wave motion. In experiments, this is classically done through the use of wave probe sensors, providing a time resolved wave elevation at fixed coordinates in the sloshing tank. As such measurements cannot resolve the full shape of the wave and its evolution over time, a newly developed

optical measurement technique was implemented for the thesis. Through image processing, the entire surface wave can be captured and evaluated, to clearly distinguish different sloshing states. In combination with careful control of the driving parameter in the experiment, this led to much higher resolution resonance measurement data compared to previous sloshing studies.

These sloshing resonance measurements, published in Bäuerlein and Avila (2021a), already showed an impact in different fields. For the development of reduced order models using spectral submanifold theory, the measurement data and response curves from this study are used to further develop and benchmark new approaches (Axås, and Haller, 2023; Axås, Cenedese, and Haller, 2022). A recent publication by Milyaev and Timokha (2022) uses the sloshing amplitude and phase lag response curve measurements for a machine learning approach to predict the nonlinear dissipation of the system. This shows the advancements of the thesis, paving the way for more accurate future developments of models and new insights into the sloshing resonance.

In the next two sections, the research results of my thesis for both pipe flow and sloshing will be presented.

4.2 Turbulence in particle-laden pulsatile pipe flow

The human cardiovascular system is perhaps the most prominent pulsatile driven flow. Despite this prevalence, many aspects of cardiovascular fluid mechanics remain to be understood. An example being the transition to turbulence in large vessels, which can be linked to the progression of cardiovascular disease. As this system includes many complex features, fundamental investigations of the underlying interactions and mechanisms are necessary. My thesis addresses this need by performing the first fundamental study on the turbulence transition of particle-laden pulsatile flow in a straight rigid pipe. The system is characterized by a complex parameter space, including the Reynolds number (Re), the Womersley number (Wo), and the specific pulsation waveform and amplitude, alongside with particle-specific variables such as diameter and concentration. The investigation specifically focuses on elucidating the perturbation mechanism of rigid, spherical, neutrally buoyant particles and the dynamics of a complex physiological pulsation.

It was shown that the turbulence transition in particle-laden pulsatile flow is defined by the emergence of intermittent turbulent patches during each period, characterized by rapid growth during the velocity peak and relaminarization during the low-velocity phase. In contrast to previous studies on single-phase pulsatile flows with large pulsation amplitudes, where turbulence originates from the modal growth of helical instabilities during deceleration (Morón, Feldmann, and Avila, 2022; Xu et al., 2020), this particle-laden system follows the growth mechanisms of puffs and slugs observed in steady pipe flow. During the larger Reynolds numbers of pulsation peak, vortices forming in the wakes of individual particles act as finite-amplitude perturbations that directly trigger turbulence. This mechanism is dictated by the advective timescale of the driving waveform, with the strength and duration of the developing turbulence patches being directly proportional to the advective duration of the acceleration peak. As a result, a longer pulsation period at lower Womersley numbers will produce the strongest turbulence. The same mechanism could be observed with perturbations of an individual large particle, a suspension of many smaller particles, and a geometric perturbation of a bend pipe segment, and using both a physiological waveform and a harmonic pulsation with the same peak Reynolds number.

O1: Interaction between periodic forcing and inherent timescales

In the case of particle-laden pulsatile flow, this investigation reveals that turbulence development is dominated by the advective timescale rather than growth through phase specific helical eigenmodes. The system essentially behaves like a steady flow subjected to a slow transient in the Reynolds number. When the periodic driving exceeds a critical Reynolds number threshold, turbulent puffs are triggered and grow based solely on the advective duration where the waveform remains above that threshold. When the timescale of the forcing is changed through the Womersley number, the strength and duration of turbulence scale linearly with the period duration, while the effect of the Sexl-Womersley flow profile appears to be negligible for the turbulence development. If the timescale above the critical Reynolds numbers becomes too short for puffs to form, the flow remains laminar.

O2: Observables for characterizing intermittent turbulence in particle-laden flow

The first critical observables can be acquired from the time-resolved pressure difference between laminar and perturbed flow, measured over a segment of the pipe. When turbulence develops, the resulting increase in local friction produces a measurable pressure difference, allowing precise detection of the onset and relaminarization in phase, the duration, and strength of turbulent patches. While these observables enable the characterization of the relationship between driving parameters and some turbulence patch dynamics, they cannot reveal the underlying perturbation mechanism. The dynamics of particle-laden flows are inherently complex, with a three-way coupling of particles affecting the flow field, the flow field affecting particle motion, and the particle affecting other particles. As these interactions are intrinsically three-dimensional and occur phase-specific due to the pulsatile driving, a 3D time-resolved flow field measurement via Lagrangian Particle Tracking using STB was employed. The observed flow field reveals the initial flow perturbation of vortices in the wake of a particle, a breakdown into a patch of turbulent eddies, and the consecutive decay and relaminarization during the low velocity phase. This also reveals a flattening of the Sexl-Womersley flow profile, removing any strong inflection points and therefore inhibiting the growth flow instabilities. Simultaneously the interactions between pulsation and particle motion were also observed. During the deceleration phase, the inertia of the particle rotation decays slower than the velocity, resulting in a strong radial tumbling motion. This could indicate a novel mechanism for inertial particle migration. The findings of the thesis highlight the importance of resolving both fluid and particle dynamics to understand particle-laden pulsatile pipe flows.

O3: Underlying mechanisms and predictability of the flow

My results demonstrate a new regime of turbulence onset with large amplitude physiological waveforms, where the underlying main mechanism of turbulence is characterized by intermittent turbulent puffs and slugs dominated by the advective timescale of the flow. Significantly, this mechanism is not limited to particle-laden flows, but will also occur with geometric perturbations of a sufficiently large amplitude, such as pipe bends. Here, the bend introduces a secondary Dean flow perturbation which breaks down into the turbulent puff. In this large perturbation transition regime, the pulsatile flow dynamics can be predicted on the basis of steady flow observations. With an established critical Reynolds number required for a particle perturbation to trigger puffs, both the duration and the strength of turbulence in a pulsatile flow can be determined using only the bulk Reynolds number, the Womersley number, and the waveform. This represents a significant simplification of the turbulence transition problem, as a complex determination of the

phase-specific stability of the flow profile or the specific growth rates of a perturbations is no longer required to characterize the transition. With the stronger flow perturbations present in the cardiovascular system (such as the aortic arch), this observed regime can likely be relevant for the turbulence onset in more complex physiological flows.

Outlook

This thesis provides the first fundamental study of particle-laden pulsatile pipe flow. It could be shown that pulsatile flow is very susceptible to turbulence from particle perturbations. As this system is largely unexplored and has a range of different parameters affecting the flow, there are still many aspects to be studied and understood:

1. In the scope of this study the focusing mechanism of particles in pulsatile flow has not yet been tackled. This presents some methodological difficulties, as the migration mechanisms are very sensitive to buoyancy effects (Matas, Morris, and Guazzelli, 2004). Due to the large pipe and particle diameter of this setup, these are difficult to avoid, and will especially appear during longer phases with low velocities in a pulsation. Some preliminary experiments not presented in the thesis showed high variance for different Womersley numbers. In a slower pulsation, the particles would migrate and form particle trains in the acceleration phase, which would then break up as the flow decelerates. By contrast, the particles in a higher Womersley flow appeared almost unaffected by the pulsation, performing the same axial and radial focusing as in steady flow. This is phenomenologically similar to the turbulence transition in Womersley flow, where the high Wo flow transitions like a steady flow (Xu and Avila, 2018). Furthermore, this study could show a strong radial migration driven by the inertial rotation of the particles, as the flow decelerates. This could indicate further migration mechanisms specific to pulsatile flows.
2. For the dynamics of particle migration, there are also many aspects that are not yet understood in steady pipe flow. So far, there are few studies on the effects of the particle concentration on the mechanisms of inertial focusing (Aouane et al., 2022). It can be suspected that, above some critical concentration, interactions like collisions become dominant, and will suppress the formation of stable annuli or axial particle trains. The experiment designed in this thesis would be ideally suited to study these aspects, as it can measure both the radial and axial particle positions and their three-dimensional interactions with the fluid flow.
3. The effects of particle concentration on the turbulence onset was explored in a limited range ($\phi_p \leq 3\%$). Studies of the turbulence transition in steady flow indicate that an increase in particle-particle interactions at higher concentrations leads to a different turbulence state of global fluctuations instead of localized puff structures (Agrawal, Choueiri, and Hof, 2019; Hogendoorn, Chandra, and Poelma, 2022). This global fluctuation state could also be relevant in physiological flows where highly concentrated red blood cells interact with the flow.
4. This study focused on the turbulence transition with a physiological waveform pulsation, featuring a sharp acceleration peak followed by a longer period of low velocity. With the investigated types of perturbations and flow parameters, this resulted in temporally intermittent turbulent patches, that decay within a single pulsation period. It is likely, that for lower amplitude in combination with higher Womersley numbers (as for studies by e.g. Xu and Avila (2018)) or a shorter low velocity phase, the particle-induced turbulence will be sustain through multiple periods and could show effects like puffs splitting or decaying similar to steady flow dynamics.

4.3 Nonlinear sloshing resonance

Liquid sloshing resonance occurs when a tank containing a free-surface liquid is subjected to external oscillations. When the external forcing frequency approaches the systems natural frequency, the sloshing becomes resonant, leading to an amplification of the wave motion and sloshing forces. This phenomenon is especially critical in aerospace engineering, where large amplitude resonant sloshing in propellant tanks poses a significant risk to the structural integrity of the vehicle and can interfere with the guidance and control systems during flight. The inherently nonlinear nature of these dynamics makes resonance notoriously difficult to predict. In my thesis, the nonlinear resonance dynamics of a quasi-two-dimensional sloshing motion in a rectangular tank subjected to harmonic forcing was experimentally investigated and described using various modeling approaches. The research focused on the forced response for multiple driving amplitudes surrounding the first sloshing mode, as this mode carries the highest energy and is the most critical for engineering applications.

The experimental findings were evaluated against several theoretical models (a multimodal sloshing model, a nonlinear Duffing oscillator, and the spectral submanifold theory) to determine their accuracy for predicting resonant sloshing. The first model evaluated was the multimodal analytical solution by Faltinsen et al. (2000) based on potential flow theory, which is capable of predicting forced wave motion for various filling levels and tank geometries. Despite the prevalence of such multimodal theories in literature, this investigation shows a systematic overprediction of the response at the resonance peak, even for small amplitude planar waves. By contrast a Duffing oscillator model provides a much better fit for small amplitude sloshing. However, for large forcing amplitudes the Duffing model similarly overpredicts the maximum sloshing response. In this regime, the larger sloshing amplitudes in combination with complex flow states (such as breaking waves or period-three motion) lead to a nonlinear increase in the dissipation rate. The divergence from Duffing dynamics can be best observed in the phase lag of the sloshing motion, which closely coupled to the systems dissipation. The most successful results were achieved using the SSM theory, which consistently outperformed the other models. The SSM used a novel data-driven approach for reducing the complex fluid dynamics into a low-dimensional manifold, which can accurately capture the sloshing response with an amplitude dependent nonlinear increase in dissipation, using only minimal training data.

O1: Interaction between periodic forcing and inherent timescales

In the linear regime, the sloshing response is dictated by two inherent timescales: the natural frequency of the eigenmode and the damping rate of the mode. For a given set of forcing parameters, the response can be simply calculated using these static values. However, as the periodic forcing frequency approaches the eigenmode and the sloshing amplitude grows, nonlinear effects introduce a shift in the natural frequency, leading to softening or hardening behavior. While existing analytical theories, such as Faltinsen's multimodal model, aim to predict these frequency shifts, my results demonstrate that accurately describing the sloshing response equally requires accounting for the nonlinear effects of dissipation at larger amplitudes. As the sloshing motion increases, complex wave motion and the interactions of multiple modes significantly increase energy dissipation. Existing analytical models, which are largely derived from inviscid potential flow and simply add a linear viscous dissipation term to the first mode only, cannot capture this effect, leading to systematic inaccuracies and severe limitations for large amplitudes.

O2: Observables for characterizing the nonlinear resonance

In this study, multiple observables were utilized to characterize the sloshing dynamics and compare the resonance models with experimental data. The center of mass of the fluid was identified as a robust observable for the sloshing amplitude. In the experiments, this center of mass was determined from optical flow visualization measurements of the surface wave. Compared with localized surface displacement measurements, the center of mass is far less sensitive to measurement inaccuracies and provides an equally representative measure for different sloshing states. When evaluating resonance models, the center of mass motion can be directly compared to mechanical equivalent models such as a harmonic oscillator or Duffing oscillator.

To characterize the resonance response itself, the phase lag between the forcing and the sloshing response was identified as a critical observable. In the proximity of a resonance peak, the phase lag serves as an early indicator of inaccuracies in a model, where the response amplitude can still match with the measurements. The phase lag is inherently more sensitive to the effect of dissipation in the system, and therefore reveal the regime in which effects of nonlinear dissipation become non-negligible. It was also demonstrated that the softening sloshing resonance follows the 90° phase lag criterion for its maximum amplitude response. Therefore, the phase lag will indicate proximity to the maximum response. Utilizing these attributes, the phase lag can act as a key observable for evaluating sloshing resonance in future studies.

O3: Underlying mechanisms and predictability of the flow

My findings demonstrate the inherent limitations of conventional models that rely on linear viscous damping, such as the multimodal theory and the Duffing oscillator model. While these approaches capture frequency shifts, they fail to account for nonlinear dissipation, which was identified as a key mechanism for accurate modeling of the larger sloshing amplitude regime. Nonlinear dissipation remains notoriously challenging to describe from both analytical and experimental perspectives, as it encompasses effects from multiple modes, complex wave interactions, local turbulence, hydraulic impacts, wave breaking, and chaotic fluid motion.

In light of these challenges, data-driven approaches, such as the SSM, serve as a valuable alternative. In the SSM tool utilized in the thesis, the manifold is fitted to trajectories of the natural decay. By using a large initial amplitude, the decay timeseries captures the full system dynamics as they evolve from the nonlinear regime down to the linear resting state. Consequently, the manifold can encompass nonlinear effects on both frequency and dissipation without requiring any prior knowledge of the underlying fluid mechanics of the system.

In the model comparisons performed throughout the thesis, the SSM approach consistently yielded the most accurate results. This success indicates that data-driven manifolds are a powerful tool for future resonance modeling. Furthermore, they can provide a robust benchmarking standard for evaluating analytical theories of nonlinear sloshing dynamics.

Outlook

Some remaining challenges and open questions for future sloshing and resonance modeling that arise from this study are listed here:

1. Maybe the most promising result of this study is the modeling of resonant sloshing dynamics with the spectral submanifold theory. Since this approach only requires as single trajectory of the systems natural decay, it is an extremely powerful tool not

only for modeling sloshing but also other nonlinear dynamics (Cenedese et al., 2022). In the work presented here, this approach is not yet able to predict the resonance under the largest forcing amplitude investigated, where multiple coexisting states and most notably the period-three sloshing motion emerges. This type of behavior can be attributed to a nonlinear coupling of multiple modes (Faltinsen et al., 2000). In a recent update to the *SSMLearn* tool, Axås, Cenedese, and Haller (2022) achieved much faster computation times, enabling the use of higher dimensionality training data and larger order manifolds. With this method, it is possible to include the sloshing wave shape in the training data, and predict the full surface displacement over time. More recently Axås et al. (2024) were able to use a higher dimensional SSM containing multiple modes, to predict individual trajectories of period-three sloshing. The prediction of the forced response containing multiple coexisting sloshing states, as observed in the experiment for the largest amplitude, is still an outstanding challenge.

2. In the thesis, the investigation was limited to a single filling level with a softening resonance. As a result, the nonlinearity parameter of the Duffing equations could only be fitted for this single case, and no direct connection between its value and the system parameters could be established. With lower filling levels, the sloshing system will transition from a softening to a hardening resonance (Faltinsen and Timokha, 2009). Extending this study and the Duffing fit to lower filling levels could help in determining a scaling law of the nonlinearity parameter, comparable to the multimodal model by Faltinsen et al. (2000).
3. Lastly, the thesis was focused on the quasi two-dimensional sloshing motion in a rectangular container. Increasing the depth of the container, or using a cylindrical container, will introduce three-dimensional wave motion and swirling waves (Arndt and Dreyer, 2008; Faltinsen and Timokha, 2017). In an experiment, investigating this type of flow will require more advanced measurement techniques to capture the full displacement of the waves. Additionally, models containing only a single dominant mode, like the Duffing oscillator, are less likely to accurately predict the resonant dynamics. For this regime a higher order SSM containing multiple modes could provide a promising modeling approach.

Appendix A

Pipe flow

A.1 Inertial particle-focusing in steady flow

There are still many aspects of inertial particle migration in steady pipe flow that are not fully explored, such as the required development length of particles to focus on their stable annulus positions, the emergence of axial particle trains, and the transition from the outer Segre-Silberberg annulus towards the inner annulus. Some preliminary results from the newly built pipe experiment are shown in this section. These measurements were also used to validate the newly developed particle localization method from section 2.3.5.

For the measurements presented here, small particles $D_p = 0.1D$ with a volume concentration of $\phi_p \approx 1\%$ are seeded into the steady pipe flow with different Reynolds numbers. The three-dimensional particle measurement setup is located $330D$ downstream from the pipe inlet. With this setup the three-dimensional position of each passing particle can be determined. To investigate the effect of radial focusing, firstly only the radial position is considered. Fig. A.1 shows the statistics of the radial position for two different Re after the flow is fully developed $t > 330D/U$. For small Reynolds numbers most of the particles focus on the single stable radial position the outer Segre-Silberberg annulus, here with $Re = 700$ at $r = 0.87R$. As the Reynolds number exceeds $Re = 800$ the second stable radial position emerges, closer to the center. Initially, at $Re = 800$ the distribution between both annuli is approximately equal. When Re is increased, the fraction on the inner annulus grows, but even for Reynolds numbers up to $Re = 1300$ (depicted in Fig. A.1b) there is still a measurable fraction of $\sim 20\%$ of the particles on the outer annulus. Compared to the narrow radial positions of the Segre-Silberberg the distribution of the inner annulus is much broader, similar to the observations by Matas, Morris, and Guazzelli (2004) and Morita, Itano, and Sugihara-Seki (2017).

The detected stable radial positions for different Reynolds numbers are shown in

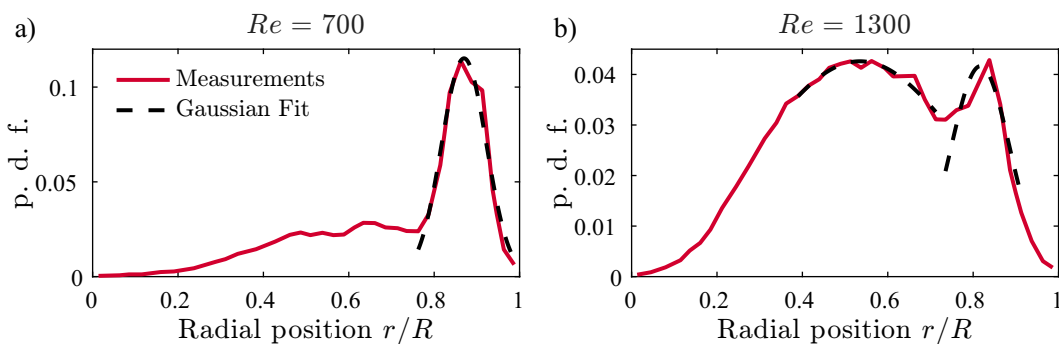


FIGURE A.1: Probability density function of the radial particle position measured at an axial position of $330D$ for $Re = 700$ a) and $Re = 1300$ b) with a particle suspension of $D_p = 0.1D$, $\phi_p \approx 1\%$. The black lines indicate a fitted Gaussian distribution through each peak.

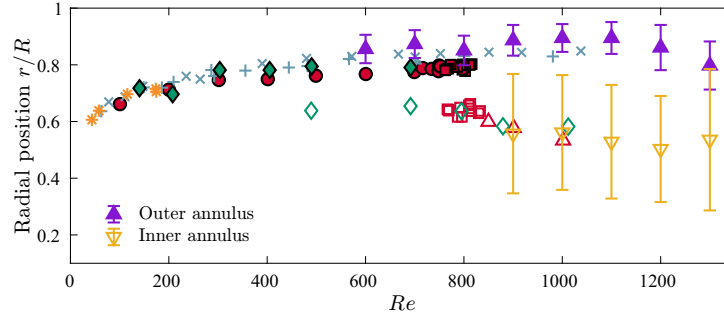


FIGURE A.2: Radii of the particle focusing annuli as a function of Re . Open symbols correspond to the inner annulus and solid symbols to outer Segre-Silberberg annulus. The yellow and purple triangles indicate new measurements from the thesis ($D_p/D = 0.1$; $L/D = 330$), with errorbars corresponding to standard deviation from the fitted Gaussian distribution. Further experimental data by Nakayama et al. (2019) (red circles, squares, and triangles, $D_p/D = 0.1$; $L/D = 1000$), Matas, Morris, and Guazzelli (2004) (blue crosses and plus signs, $D_p/D = [0.095; 0.11]$; $L/D = 313$), Morita, Itano, and Sugihara-Seki (2017) (green diamonds, $D_p/D = 0.11$; $L/D = 250$) and Segré and Silberberg (1962) (orange stars, $D_p/D = 0.11$; $L/D = 214$).

Fig. A.2, combined with data from previous studies (Matas, Morris, and Guazzelli, 2004; Morita, Itano, and Sugihara-Seki, 2017; Nakayama et al., 2019; Segré and Silberberg, 1962). In the new experiments the lowest Re that could be investigated is $Re = 600$, for smaller Reynolds numbers the effects of buoyancy from slight density mismatching cannot be neglected. The reason for this is the large diameter of this pipe leading to larger particles and much higher buoyancy forces (scaling with D_p^3), compared to other experiments. Looking at the outer annulus, the radial positions from this study tend to be closer to the wall this can be explained by the increased hydrodynamic interactions with the higher particle concentration compared to other studies. A similar trend could be seen in simulations by Aouane et al. (2022). Unlike the measurements of Nakayama et al. (2019), where the outer annulus is no longer present for $Re > 900$, this study shows some outer annulus focusing for all investigated Re . There are multiple explanations for this. Firstly, the relative pipe of length of Nakayama et al. (2019) is much longer then the realizable length in this setup ($1000D$ which would correspond to 28 m). Additionally, the flow of Nakayama et al. (2019) is much more dilute $\phi_p = 0.001\%$ which prevents particle-particle interactions. Recent simulations by Aouane et al. (2022) indicate and increased focusing length with higher particle concentrations. Observations of the flow show very frequent particle-particle interactions, such as collisions, knocking particles from their stable position. This suggests that even for a much longer pipe, the particles would not fully focus for this larger concentration.

A.2 Additional flow measurement

A.2.1 Large particle at $Wo = 4$

Fig. A.3 shows the deceleration phase and relaminarization of the flow at $Re_m = 1277$, $Wo = 4$ with a large hydrogel particle, using both STB snapshots and profiles of the turbulent kinetic energy. Similar to the observation of section 2.4.2.4, the flow is stronger perturbed in the upstream wake of the hydrogel sphere. As the flow approaches the low velocity phase the turbulent kinetic energy decays in similar manner to the dynamics at $Wo = 8$. Again there is also some remaining particle inertia while the flow velocity is close to zero.

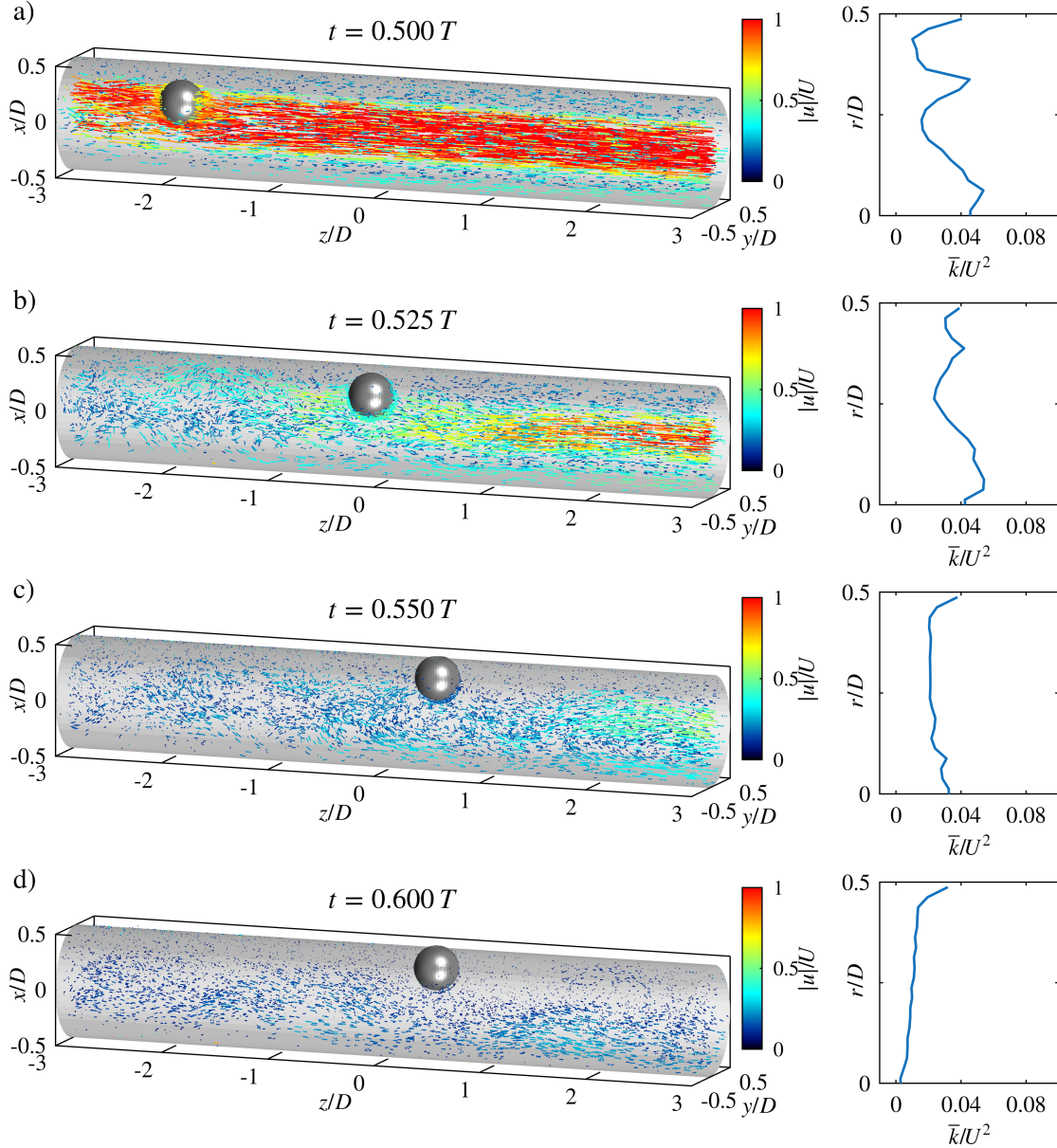


FIGURE A.3: a)-d) Snapshots of flow trajectories from STB measurements for a physiological pulsation with $Re = 1277$; $Wo = 4$ with the perturbation of large hydrogel particle $D_p = 0.41D$, $\phi_p \approx 0.5\%$ at $t = [0.5; 0.525; 0.55; 0.6]/T$. Next to the snapshots are corresponding profiles of the turbulent kinetic energy k . The Profiles turbulent kinetic energy are averaged of the axial field of view and azimuthally averaged over the entire pipe cross section.

A.2.2 Particle suspension at $Wo = 4$

If the Womersley number is reduced, the turbulence patches are triggered earlier in the pulsation phase. Fig. A.4 shows the onset at $Re_m = 1277, Wo = 4$ with a suspension of small particles using both STB snapshots and profiles of the turbulent kinetic energy. Compared to the measurements at $Wo = 8$, discussed in section 2.4.2.5, the onset appears during the acceleration peak, where the higher instantaneous Reynolds number allows for a more rapid increase and higher magnitude in the turbulent kinetic energy. This coincides with the longer duration and larger strength observed in the pressure difference measurements.

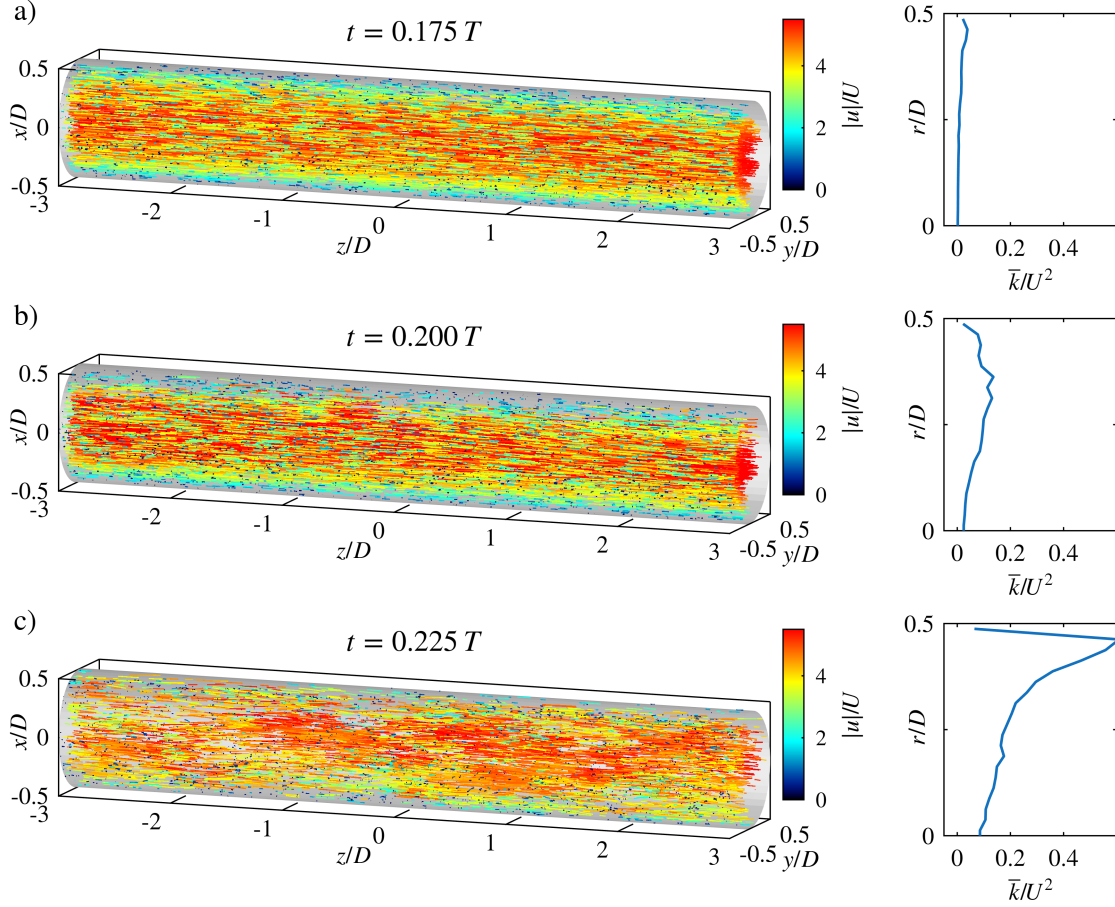


FIGURE A.4: a)-c) Snapshots of flow trajectories from STB measurements for a physiological pulsation with $Re = 1277; Wo = 4$ with the perturbation in a suspension of small particles $D_p = 0.1D, \phi_p \approx 0.5\%$ at $t = [0.175; 0.2; 0.225]/T$. Next to the snapshots are corresponding profiles of the turbulent kinetic energy k . The profiles of turbulent kinetic energy are averaged of the axial field of view and azimuthally averaged over the entire pipe cross section.

A.2.3 Effect of the bending radius

For the results presented in section 2.4.3, the effect of the bending radius on the resulting turbulence development was mostly negligible. The analyzed flow measurements only show a bending ratio $D/D_{bend} = 0.034$. To contrast these observations, the STB measurements are repeated with a smaller bend of $D/D_{bend} = 0.017$. The effect on the flow profiles and turbulent kinetic energy is shown in Fig. A.5. Compared to the stronger bend, presented in Fig. 2.60, the asymmetry of flow profile is initially less pronounced. This can be attributed to a reduced Dean flow for the bending. A similar effect is visible in the turbulence kinetic energy, where the small bending show a delayed development of the turbulence patch. For $D/D_{bend} = 0.034$ the strongest fluctuations were observed at $t = 0.5T$, while here the fluctuations peak at $t = 0.6T$. This also results in a delayed relaminarization and flattening of the velocity profile.

While this effect could not be clearly observed in the pressure differences, the high resolution STB results reveal a measurable difference between two different bending ratios. The detected delay with a smaller bend, is similar to observed effect of the particle concentration. A smaller perturbation magnitude, either through fewer particles in the fluid or reduced Dean flow, is expected to have a higher Reynolds threshold for puff to be triggered and will therefore show a delayed turbulence onset in the pulsation phase.

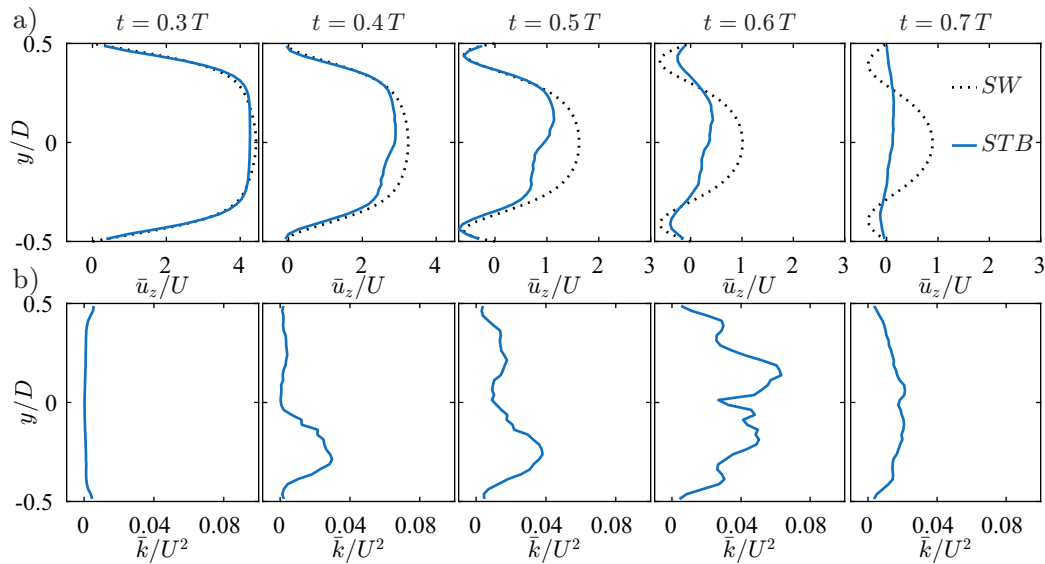


FIGURE A.5: a) Profiles of the streamwise velocity u_z averaged over the entire axial field of view ($\sim 6D$), compared the flow profile measured without a perturbation (black). b) Profiles of the turbulent kinetic energy k . Both the profiles of velocity and turbulent kinetic energy are axially averaged over $2D$ in the three colored sections and azimuthally averaged for cross-section half facing the bend ($y < 0$) and the half facing away from the bend ($y > 0$).

Appendix B

Sloshing

B.1 Accuracy of surface detection and center of mass

The sloshing evaluation uses a novel image processing method to detect the surface wave motion (described in section 3.3.1.1). In this section, the accuracy of these surface measurements and resulting error on the determined sloshing amplitude are discussed. With increasing wave amplitudes the surface becomes more complex, as can be seen in the snapshots in Fig. 3.15. During the larger waves, the meniscus in the recorded flow visualizations will be much thicker. As the evaluation method determines the surface height as the lower boundary of the meniscus, this leads to a systematic deviation for large surface displacement. An example of this is shown in Fig. B.1 with a timeseries of the surface displacement at the container wall compared to the displacement spatially averaged over the container width with

$$\bar{\zeta} = \frac{1}{w} \int_{-w/2}^{w/2} \zeta(\xi, t) d\xi. \quad (\text{B.1})$$

If the surface detection had no bias, the averaged displacement should be constant at zero throughout the oscillation, as the amount of fluid in the container does not change. Instead, the averaged surface height falls below zero, with the strongest deviations when the surface displacement is highest in the oscillation period.

To systematically evaluate this effect, the averaged surface height was evaluated for different surface wave measurements. Fig. B.2 shows the averaged surface height as a function of the maximum surface displacement. This reveals a clear trend of the evaluation method identifying a lower surface level with larger waves. As discussed in section 3.3.3, the surface displacement has some weaknesses when it is evaluated to describe sloshing. For this reason, this thesis utilizes the center of mass as the sloshing amplitude observable.

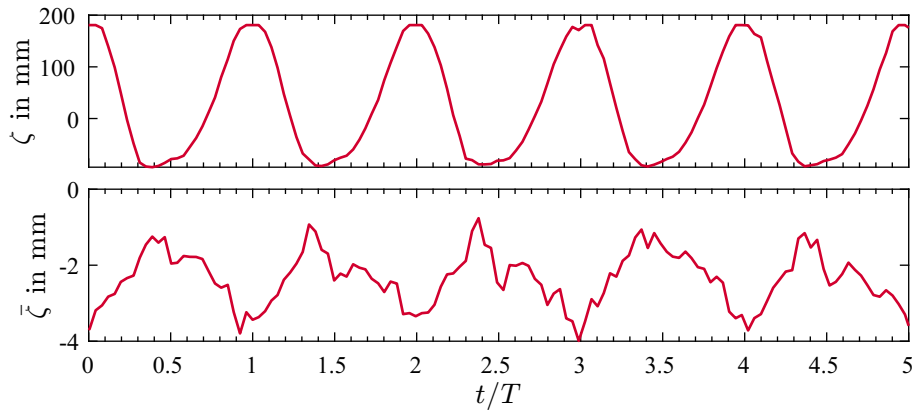


FIGURE B.1: Timeseries of the local surface displacement the wall (top) compared to spatially averaged surface level over the container width (bottom) for $\Omega = 0.934$, $A = 0.64\%$.

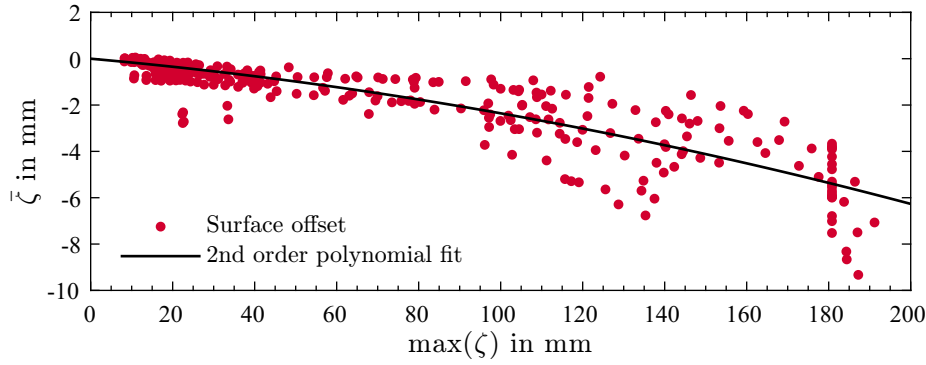


FIGURE B.2: Spatially averaged surface height over the container width $\bar{\zeta}$ as a function of the maximum surface elevation, determined from different steady state waveforms with different forcing frequencies and amplitudes.

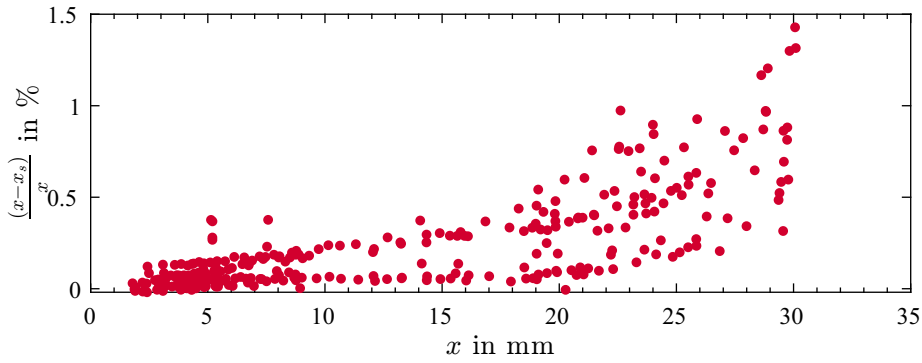


FIGURE B.3: Difference between the center of mass calculated from synthetic surface waves x_s with zero averaged surface bias and the center of mass of measured surface waves x , as a function of the center of mass. The synthetic wave were calculated for same surface states shown in Fig. B.2.

The effect of the demonstrated bias of the surface detection on the center of mass will be evaluated next.

To determine how inaccuracies in the surface wave affect the center of mass amplitude, synthetic surface data are created and compared to the measurement. For the synthetic data, it is assumed that the identified surface detection bias will approximately have an equal over the entire container width. Therefore, the synthetic surface data contain the actual identified surface waves with a subtracted bias, to create a surface wave with a spatially averaged surface level of zero. Afterwards, the center of mass amplitude is calculated for the original measured surface wave (x) and the synthetic surface (x_s). The resulting difference between both amplitudes is shown in Fig. B.3. Compared to the measured wave, the synthetic data will produce slightly smaller center of mass amplitudes, with an increasing difference for higher sloshing amplitudes. Even for the largest recorded sloshing waves, the difference between synthetic amplitudes and measured amplitudes remains very small below 1.5%. For this reason, the center of mass amplitude can be assumed to be sufficiently robust against errors in the surface wave detection.

The same comparison to was repeated with synthetic surface data where the bias is not subtracted evenly across the entire tank width, but instead with a subtracted offset proportional to the local wave elevation. In this case, the difference in the center of mass will be even smaller. However, for both synthetic datasets, the difference in surface height will have a much stronger effect on the vertical center of mass position.

B.2 Fitting the Duffing model

As described in section 3.4.3.1, the only parameter that was not analytically calculated or derived from measurements when using the Duffing oscillator model is the nonlinearity ε . This parameter can be fitted by comparison of the forced response curve from the Duffing model to the measurements. Due to shape of the response curves, a specific least-square curve fitting method had to be developed. The response of a nonlinear oscillator can be multi-valued, with a coexisting upper and lower branch solution. For these curves it is important to compare the measurements only to the correct branch of the model. This can cause errors if the model cannot produce a solution on the correct branch, as shown in the example in Fig. B.4. If the jump-up frequency is lower in the model compared to measurements, some lower branch solutions cannot be correctly correlated to the model (seen in larger amplitude curve). The same error appears for the jump-down frequency on the upper branch (seen in smaller amplitude curve). For the least-square fit of the nonlinearity parameter, these data points have to be identified and omitted. As the overall number of data points is large, the effects of removing a few individual samples has no strong bias on the fit.

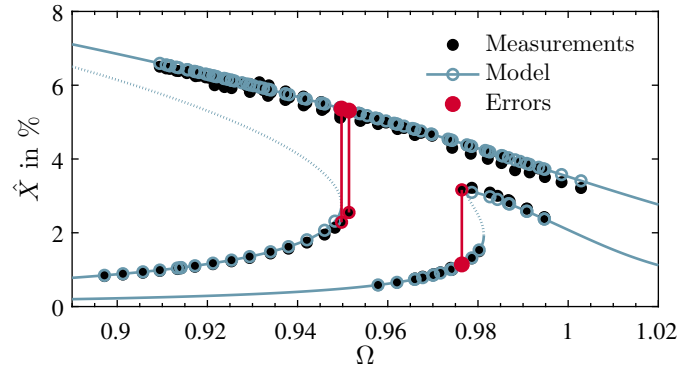


FIGURE B.4: Measurements of sloshing amplitude response compared to the Duffing oscillator model. The blue circles mark points on the Duffing curve where the difference between model and measurement can be calculated. The red circles do not have a correct match on the curves and are omitted.

B.3 Multimodal model parameters

In this section the remaining parameters that need to be computed to use the multimodal model by Faltinsen and Timokha (2009) from section 3.1.3 are listed. The additional parameters are,

$$\begin{aligned}\tilde{n}_1 &= -\frac{d_2 + h_0(3d_1 + 4d_3)}{2(9 - (\omega_1/\omega)^2)}, \\ \tilde{N}_1 &= \frac{-3q_2 + q_4 + 2h_0(-q_1 - 4q_3 + 2q_5) - 4q_1w_0}{4(1 - (\omega_3/\omega)^2)}, \\ \tilde{N}_2 &= -\frac{q_2 + q_4 + 2h_0(q_1 + 4q_3 + 2q_5)}{4(9 - (\omega_3/\omega)^2)},\end{aligned}\tag{B.2}$$

and,

$$w_0 = \frac{d_4 - d_5}{2(\omega_2/\omega)^2}, \quad h_0 = \frac{d_5 + d_4}{2((\omega_2/\omega)^2 - 4)}.\tag{B.3}$$

The parameters d_i and q_i are dependent on the filling ratio h/w in the tank, and are defined as,

$$\begin{aligned}d_1 &= 2\frac{E_0}{E_1} + E_1, \quad d_2 = 2E_0 \left(-1 + \frac{4E_0}{E_1E_2} \right), \\ d_3 &= -2\frac{E_0}{E_2} + E_1, \quad d_4 = -4\frac{E_0}{E_1} + 2E_2, \\ d_5 &= E_2 - 2\frac{E_0E_2}{E_1^2} - 4\frac{E_0}{E_1},\end{aligned}\tag{B.4}$$

and,

$$\begin{aligned}q_1 &= 3E_3 - 6\frac{E_0}{E_1}, \quad q_2 = -3E_0 - 9\frac{E_0E_3}{E_1} + 24\frac{E_0^2}{E_1E_2}, \\ q_3 &= 3E_3 - 6\frac{E_0}{E_2}, \quad q_4 = -6E_0 - 24\frac{E_0E_3}{E_1} + 48\frac{E_0^2}{E_1E_2} + 24\frac{E_0^2E_3}{E_1^2E_2}, \\ q_5 &= 6 \left(\frac{1}{2}E_3 - \frac{E_0}{E_1} - \frac{E_0E_3}{E_1E_2} - \frac{E_0}{E_2} \right),\end{aligned}\tag{B.5}$$

respectively, with

$$\begin{aligned}E_0 &= \frac{1}{8}\pi^2, \quad E_n = \frac{1}{2}\pi \tanh \left[\pi n \frac{h}{w} \right], \quad \text{for } n \geq 1 \quad \text{and,} \\ P_n &= \frac{2}{n\pi} \tanh \left[\pi n \frac{h}{w} \right] ((-1)^n - 1).\end{aligned}\tag{B.6}$$

B.4 Timeseries of sloshing states

The magnitude and frequency composition of the flow velocity will depend on both the location in the tank and the velocity component which is evaluated. For the analysis in section 3.4.1.2, only the horizontal velocity u_x in the horizontal center of the tank is evaluated. In this section, both the horizontal velocity u_x and the vertical velocity u_y and their frequency compositions are shown in Fig. B.5 in the horizontal center of the tank $x = 0$ and in Fig. B.6 in the right half of the tank $x = 0.2h$, for different sloshing states. In center of the tank the vertical velocity is consistently close to zero for all sloshing states and the frequency spectrum of horizontal velocity shows a very small component of the 2nd harmonic 2ω , due to the axis-symmetric shape of the $n = 2$ mode. By contrast, in right half of the tank both velocity components are of similar amplitude and the frequency spectrum of both velocities contains a much stronger contributions of the 2nd harmonic.

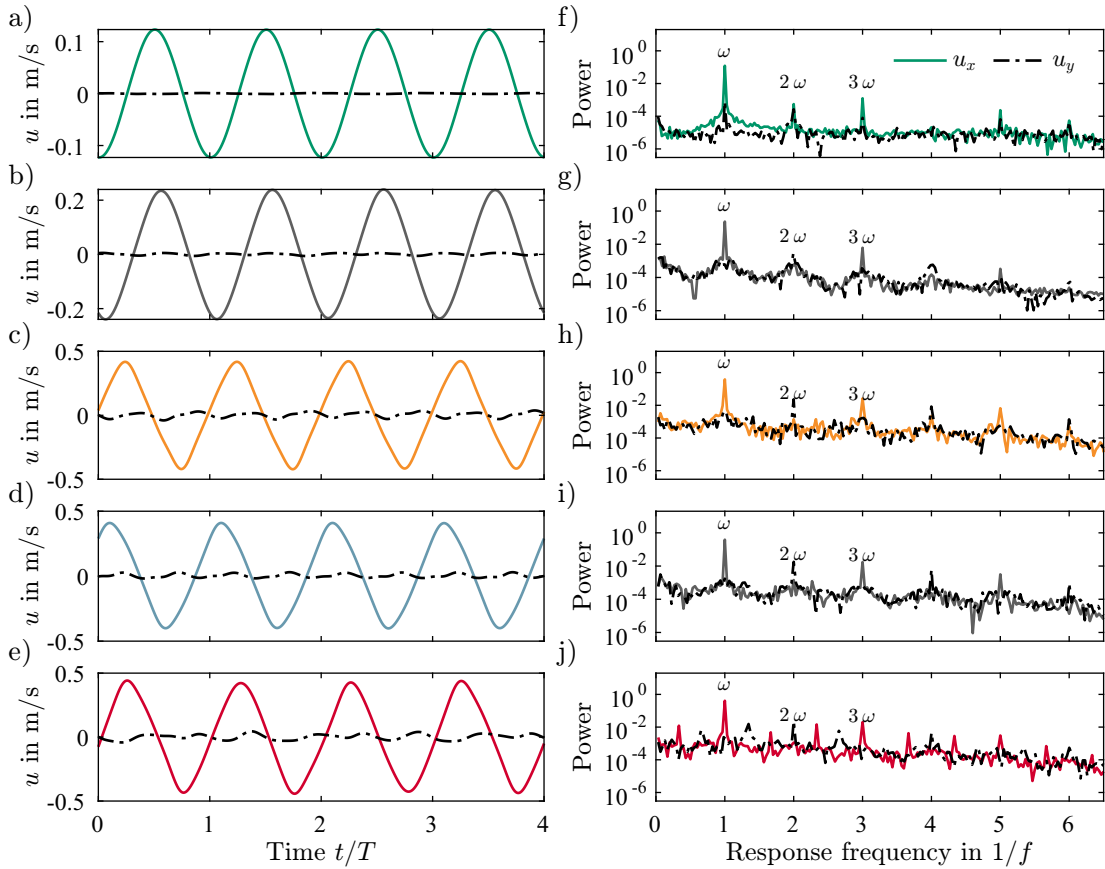


FIGURE B.5: a)-e) Time series of the velocity components u_x (solid line) and u_y (dotted line) in the bulk of the flow obtained with Stereoscopic PIV at a position at horizontal center of tank $x = 0$ and $y = 0.125h$. The velocity is spatially averaged over $\Delta x = 28\text{ mm}$ and $\Delta y = 28\text{ mm}$. f)-j) Frequency power spectra of the time series in a)-e) obtained with a fast Fourier transformation. The investigated flow states are: a),f): Planar motion $\Omega = 0.944$, b),g): Plateau peaks $\Omega = 1$, c),h): Wave-breaking $\Omega = 0.926$, d),i): Peaks $\Omega = 0.934$, e),j): Period-three motion $\Omega = 0.917$.

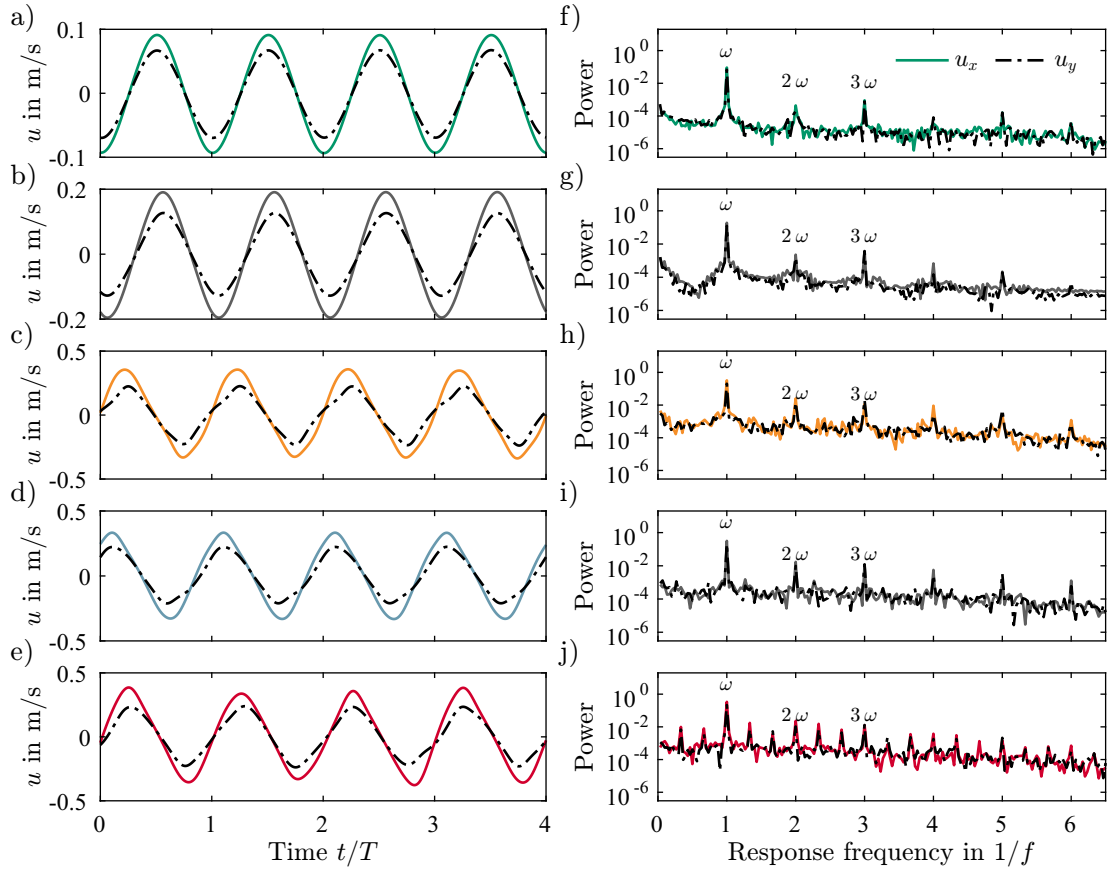


FIGURE B.6: a)-e) Time series of the velocity components u_x (solid line) and u_y (dotted line) in the bulk of the flow obtained with Stereoscopic PIV at a position to right of center of tank $x = 0.2h$ and $y = 0.125h$. The velocity is spatially averaged over $\Delta x = 28$ mm and $\Delta y = 28$ mm. f)-j) Frequency power spectra of the time series in a)-e) obtained with a fast Fourier transformation. The investigated flow states are: a),f): Planar motion $\Omega = 0.944$, b),g): Plateau peaks $\Omega = 1$, c),h): Wave-breaking $\Omega = 0.926$, d),i): Peaks $\Omega = 0.934$, e),j): Period-three motion $\Omega = 0.917$.

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- Bäuerlein, B. and Avila, K. (2021b). “Three-dimensional particle tracking in pipe flows from distorted images”. In: *28. Fachtagung "Experimentelle Strömungsmechanik"*. Bremen, Germany: Deutsche Gesellschaft für Laser-Anemometrie GALA e.V. ISBN: 9783981676471.
- Bäuerlein, B., Schanz, D., Schröder, A., and Avila, K. (2022). “Lagrangian particle tracking and measurements of particle rotation elucidate dynamics of pulsatile particle-laden pipe flow”. In: *29. Fachtagung "Experimentelle Strömungsmechanik"*. Ilmenau, Germany: Deutsche Gesellschaft für Laser-Anemometrie GALA e.V. ISBN: 9783981676488.
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Oldenburg, 10.03.2026

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(Bastian Bäuerlein)