Transport and biological activity in chaotic open flows: a numerical study in idealized oceanic flows

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Abstract

The interplay between hydrodynamic motion and nutrient availability influences the distribution of plankton that makes up the basis of the aquatic food web. Vertical transport of nutrients controlls the primary production in particular in and around upwelling areas. Horizontal stirring by mesoscale structures like vortices and wakes redistributes nutrients and enhances primary production, and can also initiate plankton blooms and affect the competition and coexistence of different plankton species. Vertical upwelling in connection with and Ekman flow and strong mesoscale activity occurs in the region the Canary Islands in the Atlantic ocean. The nutrient rich waters from the upwelling interact with the wake of the Canary Islands, giving rise to filaments and mesoscale structures of increased biological productivity and complex dynamics. An increased primary production is also observed in the area far away from the upwelling zone. The effects of mesoscale eddies on the horizontal transport of nutrients, the primary production and the initiation of plankton blooms are investigated in a time dependent two-dimensional flow around an obstacle and with a upwelling region.

Both the hydrodynamic and the biological model used in this thesis are simplified models. They seek to reproduce the main features of the system, with the least possible degree of conceptual complexity. Although concepts about the processes involved are kept as simple as possible they are intended to give a rather realistic representation of the processes one wishes to model. They can therefore be complex from a mathematical point of view. Nevertheless the advantage of simple process models is that they allow direct investigation of the role played by each generic process. Such processes may be difficult to diagnose in simulations of a particular area. The advantage of taking a broader view is that it may allow the recognition of features common to several different systems. Despite the apparent simplicity of the models they can provide a deeper understanding of the most relevant processes.

The horizontal transport across the wake is studied under different conditions with a periodic, non-periodic and turbulent flow. Primary production in the region is enhanced due to vorticity redistributing the upwelled nutrients. Under certain conditions the interplay between wake structures and the biological activity leads to plankton blooms inside the mesoscale vortices. The residence time of the plankton species in the structures in the wake is important. It is determined by the chaotic saddle, the vorticity and other nonlinear effects in the flow. The long residence times in the vicinity of the island and the hydrodynamic forcing and confinement of plankton in the vortices initiate localized plankton blooms.

Zusammenfassung

Das Zusammenspiel von hydrodynamischer Strömung und die Verfügbarkeit von Nährstoffen beeinflussen die Verteilung des Planktons, das die Grundlage der aquatischen Nahrungskette bildet. In der Nähe von Aufquellgebieten bestimmt der vertikale Transport von Nährstoffen die Primärproduktion. Horizontales Mischen durch meskoskalige Strukturen wie zum Beispiel Wirbel und Fronten verteilt die Nährstoffe und steigert die Primärproduktion und kann auch Algenblüten verstärken oder die Konkurrenz oder die Koexistenz verschiedener Planktonspezies beeinflussen. Vertikales Aufquellen in Verbindung mit einem Ekman-Strom und starke mesoskalige Aktivität findet in dem Gebiet bei den Kanarischen Inseln im Atlantischen Ozean statt. Die nährstoffreichen Wassermassen aus dem Aufquellgebiet wechselwirken mit der Wirbelstraße bei den Kanarischen Inseln, wobei Filamente und mesoskalige Strukturen mit hoher Primärproduktion und komplexer Dynamik entstehen. Selbst weit von dem Aufquellgebiet entfernt, wird eine erhöhte Primärproduktion beobachtet. Die Effekte der meskoskaligen Wirbeln auf den horizontalen Transport von Nährstoffen, der Primärproduktion und bei der Auslösung von Algenblüten werden in einer zweidimensionalen zeitabhängigen Strömung um eine Insel mit einer Aufquellzone untersucht.

Sowohl das hydrodynamische als auch das biologische Modell, die in dieser Dissertation verwendet wurden, sind vereinfachte Modelle. Sie geben die wesentlichen Eigenschaften des Systems wieder, wobei konzeptionelle Komplexität möglichst vermieden wird. Die Konzepte über die zu Grunde liegenden Prozesse werden dabei so einfach wie möglich gehalten, obgleich in der Absicht diese Prozesse möglichst realistisch wiederzugeben. Die Modelle können deswegen aus mathematischer Sicht sehr komplex sein. Der Vorteil einfacher Modelle liegt darin, daß in ihnen die Rolle jedes einzelnen Prozesses untersucht werden kann. Die Rolle einzelner Prozesse lässt sich in Simulationen für ein bestimmtes Gebiet schwieriger erkennen. Der Vorteil einer allgemeineren, umfassenderen Betrachtungsweise liegt darin, daß in ihr allgemeingültige Eigenschaften erkennbar werden, die mehreren Systemen gemeinsam sind. Trotz der scheinbaren Einfachheit der Modelle, können sie zu einem tieferen Verständnis der relevantesten Prozesse beitragen.

Der horizontale Transport auf die andere Seite der Wirbelstraße wird in drei verscheidenen Strömungen mit periodischem, nicht-periodischem und turbulenten Eigenschaften untersucht. Die Primärproduktion in der Region wird unter dem Einfluss der Wirbel bei der Umverteilung der Nährstoffe erhöht. Unter bestimmten Bedingungen führt das Interagieren der Strukturen der Wirbelstraße mit den vertikal transportierten Nährstoffen zu Algenblüten innerhalb der mesoskaligen Wirbel. Die Aufenthaltszeiten der Phytoplanktonspezies in den verschiedenen Strukturen der Wirbelstraße ist von Bedeutung. Sie wird durch den chaotischen Sattel, der Wirbelstärke und andere nichtlineare Effekte in der Strömung bestimmt. Die langen Aufenthaltszeiten der Nährstoffe und des Planktons in der Nähe der Insel und das Eingeschlossensein des Planktons innerhalb der Wirbel führen zu lokalen Algenblüten.

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Chapter 1

Introduction

The image of the Earth taken by the astronauts of the moon-landing mission Apollo 11 as their spacecraft on its lunar orbit had just passed the dark side of the moon is one of the most circulated images of the Earth. The fascinating view of the entire planet taken from an angle with the sun illuminating the entire surface shows the Earth mostly in tones of blue and white. Eversince fluid water existed on the planet, the colors of the light reflected by the atmosphere and oceans formed complex patterns of high artistic value, making our planet so distinct from an extraterestrial perspective. This symphony in blue and white is perhaps the most spectacular evidence of the two systems where life has developed in: the oceans and the atmosphere. The exact processes and circumstances that led to the appearance of the first life forms is the subject of ongoing research, but it is generally accepted that the medium where the evolution from the first simple organisms to complex species took place was a fluid one. The worlds oceans were the cradle of life, where the molecular compounds that formed the first species on the planet reacted with each other. Reacting substances and self-organization under the influence of external forcing are very hot research topics in physics and the life-sciences.

Still today in nature and in engineering systems chemical or biological active substances are often transported in flows. If the distribution of the carried substances in the flow is not homogeneous, the biological or chemical activity is affected by the dynamics of the underlying flow. In the past years, scientific effort was directed towards a better understanding of the complex processes taking place in the flow. The outcome of the reactions in these systems can be slightly different from the same reaction taking place in a well mixed environment (Tél et al., 2005). These interactions have been studied in the context of geophysical phenomena such as the formation of the ozone hole in the stratosphere (Edouard et al., 1996; Kiss et al., 2003; Paireau &

Tabeling, 1997) and chemistry (Menzinger & Jankowski, 1986; Menzinger & Dutt, 1990; Epstein, 1995; Metcalfe & Ottino, 1994; Ali & Menzinger, 1997). The transport of reacting substances is relevant in various other fields, such as plankton population dynamics (Scheuring et al., 2000; Konopka et al., 2004; Scheuring et al., 2003) and geophysical sciences in general (Haynes, 1996; Tan et al., 1998; Schmalzl et al., 1996), microfluidics (Strook et al., 2002; Bottausci et al., 2004; Benettin et al., 1986) and combustion (Williams, 1985; Kiss et al., 2003).

All these systems have in common that the interacting particles are transported by the motion of the fluid, and they change their properties due to the chemical or biological interactions. It can be therefore said that these particles are *active*, in the sense that they are not just passively advected by the flow but they follow dynamical processes of their own. For example when plankton 'particles' (cells) reproduce and die, their number changes. Flows that carry active substances are often referred to as *active flows* in literature. An unusual feature of many of these systems is that reaction take place along filamentary spatial structures in time dependent flows. The distribution of the reactants corresponds to fractal patterns. Examples of active flows are the atmospheric transport of pollutants and chemical agents, such as ozone or fluor-chlorides, or the transport of plankton and nutrients in the ocean currents.

Small organisms like plankton species living in the marine environment are a descriptive example of biological species transported in a flow. Different species may interact with each other in a food web while they are under the influence of the flow. Satelite images of the concentration of microscopic marine species floating on the ocean surface usually reveal the fractal patterns in the distribution of phytoplankton. These patterns are more common in areas with strong mesoscale activity where fronts and vortices are present (see Fig. 1.1). The visible non-stationary structures on the surface are formed by the coupling between the internal and external forcing on the ocean and the biological activity. The spatial and temporal dynamics of marine phytoplankton in a time dependent current is a classical example of a biologically active flow.

Quantifying the influence of the hydrodynamic forcing on the growth, production and distribution of plankton and marine microorganisms is a complex task. The interest in quantifying these effects is motivated by the fundamental role that microorganisms and plankton in particular play in the marine food web. Plankton communities form the basis of marine food webs throughout the worlds oceans. These freely-floating and weakly-swimming microorganisms can be found in marine or freshwater-habitats. Plankton species can be broadly divided into two groups, phytoplankton and zooplankton. Phytoplankton are unicellular microorganisms that perform photosynthesis. Zooplankton is a microorganism that grazes on phytoplankton, a predator animal. Zooplankton itself is eaten up by larger organisms. Therefore phytoplankton and zooplankton form the basis of the marine food web supporting all other forms of life. Their distribution on the ocean surface determines the abundance of fish. A subject currently under discussion is whether phytoplankton also plays a role in the context of planetary climate change. Phytoplankton performs photosynthesis on the basis of carbon that diffuses from the atmosphere into the water. Plankton transforms CO_2 into monosaccharides and energy in several stages by means of chlorophyll. Nutrients and the products of this reaction form the basis for the production of proteins in the cells. This complex process that leads to the augmentation of the total mass of the organisms is referred to as *primary production*. As the planets surface is covered by two thirds with oceans, plankton distribution affects the CO_2 -balance in the atmosphere. The study of the factors that control the abundance of phytoplankton is therefore a subject of great scientific interest, not only for biologists but also for the climate research community.

The growth of phytoplankton depends on the availability of light and nutrients. The uppermost layer of the world's oceans, the euphotic zone is bathed in sunlight during the daytime. In this surface mixed layer, plankton species are active. A balance between light and nutrient availability provides favorable conditions for plankton. As light is rapidly absorbed in the water column, life conditions are less favorable in deeper layers, although nurient concentrations may be higher there. Nutrients can be supplied to the upper ocean layers by various physical transport mechanisms. Coastal and open ocean upwelling, episodic mixing up of deep water by winter convection, continuous diapycnal diffusion and eddy induced vertical motions can all contribute to fueling biological production in the surface ocean. Physical processes also govern the depth of the turbulent surface mixed layer. In deep winter mixed layers in midlatitude and high altitude regions are turbulent, and phytoplankton may not remain in the euphotic zone long enough for the growth to take place. Stabilization of the surface layer in spring can then give rise to sudden blooms of algae.

Plankton species in the open ocean are thus immediately dependent on a variety of physical processes. According to previous studies these processes influence the growth rate, and the spatial and temporal variability of the oceanic primary production, as shall be explained in the following.

Fronts are observed at the contact surface of water masses with different physical properties, such as salinity or temperature. Stochastic perturbations in the forcing of the water bodies can lead to instability in a front. Examples



Figure 1.1: Filamental structures on an satellite image of the Northern Atlantic close to the coast of Norway (image by NASA SEAWIFS) http://visibleearth.nasa.gov/cgi-bin/viewrecord?5278

of instabilities in the vertical are internal waves, and for instabilities in the horizontal plane meandering jets. From an ecological point of view one of the key process in an unstable frontal region is vertical transport. It leads to an increased flux of nutrients, mainly nitrates to surface waters, increasing primary production. Vertical fluxes can be associated with increased biological activity (Denman & Gargett, 1995). Some mechanisms that become active in connection with vertical processes have been identified in previous studies. They take into account the physical transport of both plankton and nutrients to the surface and also consider light absorption.

A more refined analysis of the different species of phytoplankton reveals that marine phytoplankton species have evolved to fill a variety of ecological niches. Differences in physiology-related processes between species reflect the diversity in adaptation to different conditions. For open ocean phytoplankton, the most important difference are the varying abilities to cope with low-light and low-nutrient conditions. In the ocean adsorption of light and fluxes of nutrients are factors controlled by the physical environment that the phytoplankton experiences (Martin et al., 2001a). They do affect the phytoplankton community structure. Vertical transport can influence the community structure for example by inducing a local inhomogeneity in the nutrient concentration. A common assumption in the oceanographic community is that the equilibrium composition of a plankton population is homogeneous horizontally, but varies with depth. The mixed layer is considered to be *oligotrophic* if the nitrate concentration levels are limiting phytoplankton growth at the surface. The nutrient limitation in the mixed layer can be removed in upwelling regions, if deep-water nitrate levels are high enough. When deep waters are brought to the surface, the nutrient limitation is locally removed. In this regions we speak of *non-oligotrophic* conditions. Upwelling of nutrient rich waters is observed east of the Canary Islands in what would otherwise be an oligotrophic region. If during upwelling a population is transported to the surface too fast for it to track the changing equilibrium, the composition of upwelled phytoplankton populations will be atypical for this surroundings (Martin et al., 2001a). This heterogeneity of transient nature can be exacerbated by the populations response to the new surroundings. Light intensity decreases with depth whereas nitrate concentration generally increases. Therefore deep nutrient-loaded waters may experience a phytoplankton bloom as they rise to the surface and receive more light. Because of the afore named differences in physiology-related processes, different species will respond to the environmental changes at different rates. Thus one can expect significant changes in the local community structure if vertical transport is initiating local algae blooms (Martin et al., 2001a). Typical vertical velocities for the stratified open ocean are very small, of the order of a few centimeters per day as the gradients are small. In the strong geostrophic circulations associated with the instabilities on a front, velocities of up to 10 m per day were observed due to the large gradients of salinity or temperature. The vertical flux of nutrient can have a substantial effect on local plankton populations.

In the homogeneous environment in the mixed layer different plankton species compete directly for the same resources. According to the principle of the 'competitive exclusion' one could assume that only the best adapted species would survive, while the other species eventually die out. This apparent contradiction is known as the *paradox of phytoplankton* (Hutchinson, 1961). Plankton species in the ocean live in a physically inhomogeneous environment. Wind forcing on the oceans surface and thermohaline convection lead to the formation of mesoscale structures such as vortices, fronts and currents. Mesoscale structures can maintain barriers to transport in the physical environment, that separate different areas in the flow, so called shelters. These can separate competing species and indirectly promote the *coexistence* of species (Martin et al., 2001b).

Spatial heterogeneity or *patchiness* of marine phytoplankton populations is one of the oldest and most robust observations of open ocean oceanography (Bainbridge, 1957). Structure is found in phytoplankton distributions at scales ranging from meters to the scale of basins (Mann & Lazier, 1991). Initially these non-random spatial distributions of plankton were viewed as collections of isolated coherent patches whose distribution gave rise to the heterogenous distributions observed by (Bainbridge, 1957). Early theories about the origins of patchiness focused on understanding the dynamics of isolated circular phytoplankton patches. One of the earliest models for phytoplankton patch formation was presented by (Kierstead & Slobodkin, 1953) and (Skellam, 1951). It is known as the KiSS-model, and stipulates that the dynamics and size of a circular plankton patch is controlled to two adverted processes: biological growth that increases the population and dispersion by currents that causes a loss of phytoplankton from every patch.

The filaments visible in Fig. 1.1 demonstrate that spatial heterogeneity in phytoplankton populations is not in fact composed of simple circular structures. The filamental structures exist at a continuum of scales, continually interacting and being modified by the flow. In the last years it was recognized that oceanic turbulence is in fact strongly anisotropic (McWilliams et al., 1994). The surface of the oceans is populated with coherent structures such as eddies and fronts that are capable of strong directional deformation of phytoplankton patches. It is therefore natural to study the effects of these hydrodynamic structures on the biological activity.

Spatial heterogeneity or patchiness in phytoplankton distributions is a very common oceanographic observation. The strongly localized patches of colored water associated with augmentend phytoplankton concentrations are reported also in the area that is studied in this thesis, the Canary Islands region. According to (Martin, 2003) there is generally still little consensus on the causes and consequences of this ubiquitous phenomenon. Many theories are present in the literature and specific cases have been investigated, but there is still no general theory available for this complex phenomenon, the patchiness of plankton. Previous studies confirm that spatial heterogeneity can strongly influence ecosystem stability (Steele, 1974), diversity (Bracco et al., 2000), dynamics (Brentnall et al., 2003) and regional productivity (Martin et al., 2002). An understanding of patchiness is vital to understanding of the marine ecosystem as a whole. Large scale fish population and climate models are currently using different simplified techniques to resolve biological activity on subgrid scales. Patchiness is typically a subgrid phenomenon and the high resolution of the models used in this thesis make it possible to study it.

One can seek to learn more about the effects of the hydrodynamic motion and the dynamics of plankton by directly studying the spatial distribution of plankton in the ocean. Data about the distribution can be extracted from aerial and satellite images of the chlorophyll concentration. Recently automated mobile laboratory stations in boxes mounted on the front of ships are also used to collect information about the distribution of plankton. The data analysis technique that has received the most interest is multifractal analysis. This approach is increasingly used in geophysical applications including the study of phytoplankton distributions. It allows the description and extraction of information on the structure and spatial distribution. This method for extracting information from plankton distributions provide evidence that marine phytoplankton distribution exhibit multifractal structure (Seuront et al., 1996, 1999; Hernández-García et al., 2002).

In tracer release experiments it is possible to follow the trajectories of pointlike tracers transported by the flow of over a certain time. Phytoplankton can be considered to behave like a point-like tracer that is avected with the same velocity as the flow. In time dependent flows the trajectories that are initially adjacent separate at an exponential rate. This feature of the horizontal motion in most time varying flows (Ottino, 1989; Waseda & Mitsudera, 2002). The multifractal distributions of tracers found can be linked to the structures in the underlying flow (Toroczkai et al., 1998; Károlyi et al., 1999; Neufeld et al., 2000). A classical example of a time variable flow is the flow past an obstacle. At low Reynolds-numbers vortices detach periodically behind the obstacle and travel with the flow for a distance until the energy contained in them is dissipated. This general feature, the von Karmanvortex-street has been observed in geophysical flows in the oceans and the atmosphere. The biological activity in a flow past an island was studied numerically with simple biological models in the works by (Toroczkai et al., 1998; Károlyi et al., 1999; Neufeld et al., 2000).

The fractal distribution of active tracers in chaotic flows has been advanced as a solution to the paradox of plankton (Scheuring et al., 2000; Károlyi et al., 2000; Bracco et al., 2000; Scheuring et al., 2003). Recent theoretical studies also showed that the underlying multifractal geometry of a flow can induce multifractal distributions of both passive and active tracers within it (Sommerer et al., 1996). These structures result in an increase of the primary production of the system (Toroczkai et al., 1998; Károlyi et al., 1999; Neufeld et al., 2000, 2002), as nutrients and phytoplankton are efficiently mixed by the flow. The results of these studies were a motivation to study these effects using more refined biological models for the biological activity.

The time variability of the oceanic flows is a result of temporal variability of the physical forcing that drives the currents. The time variability can be observed in the structures present in the flow, such as jets and vortices. Mesoscale structures have a size of about 1-300 km and are often observed in the spatial distribution of phytoplankton. These structures are often associated with the aforementioned hydrodynamical structures such as fronts and currents (Strass, 1992; Venrick, 1990; Falkowski et al., 1991). More recently it has become apparent that fronts (Pollard & Regier, 1992) and eddies (Martin & Richards, 2001) facilitate strong upwelling of nutrients in small regions of 5-10 km diameter. Phytoplankton in the upper layers of the ocean consumes nutrients leading to extremely low nutrient levels. Lower layers are usually nutrient rich, as dead biological material sinks due to gravity. Strong upwelling regions are therefore substantial sources of nutrients for the surface waters. As a consequence phytoplankton production and biological activity is strongly enhanced locally in the upwelling areas. At these mesoscales the heterogenous physical forcing leads to *patchiness* of both the plankton and primary production.

Global carbon cycle models, GCCM should be designed to take into account this local phenomenon. The poor resolution of these global models does not always permit to resolve eddies and fronts. State of the art GCCM barely 'permit' mesoscale eddies and fronts (Oschlies & Garcon, 1998; Mahadevan & Archer, 2000). Due to low resolution, they are not able to reproduce accurately the internal structure. There is evidence for an enhancement of the biological activity at mesoscale and below, in form of the filamental structures observed on satelite images of the phytoplankton distribution. The visible concentration gradients due to the filaments are reflected by the multifractal distribution of plankton, that is related to the structures in the flow. It appears to be important to estimate the effect of this forcing on the primary production at scales that cannot be resolved by large scale models. The aim is to find estimates for the errors occurring in GCCMs due to the lack of resolution of these processes. Oceanographic models incorporated in by GCCMs have a gridlength in the order of 5-10 km. For comparison, the kinematic flow model used in this work resolves structures of 500 m, well within the resolution required to observe mesoscale structures.

Several authors have studied primary production in attempts to quantify the impact of mesoscale forcing of phytoplankton in specific cases. A large number of studies were concerned with the effects of vertical transport in unstable fronts. An increase in the primary production in a frontal region due to a front becoming unstable was found by (Flierl & Davis, 1993; Spall & Richards, 2000; Martin et al., 2001a). These modeling studies showed that primary production may increase locally by up to 100% by an unstable front. (Oschlies & Garçon, 1998) studied the role of vertical transport in many models of regions with strong mesoscale activity, such as meandering jets. The biological model used is similar to the model used in this thesis. It is coupled to a three dimensional hydrodynamic model in order to study the primary productivity in a model of the North Atlantic Ocean. Other studies of the effects of vertical transport on phytoplankton were performed with more complex models. In (Martin et al., 2001a) a model with two classes for phytoplankton is coupled to a three dimensional physical model. The two classes of phytoplankton represented in this model are characteristic of large and small species of both phytoplankton and zooplankton.

Sharp increase in phytoplankton concentrations observed in the cores of the vortices were studied previously in models without seasonal forcing. These localized blooms normally appear as the result of nutrient enrichment with nutrients (Edwards & Brindley, 1996). A more specific mechanism triggering plankton blooms appears in so called *excitable systems*. These class of plankton models have a specific functional form. A certain triggering perturbation in the plankton concentration can lead to a temporary bloom, before the system returns to the normal state. The biological models used in this thesis don't exhibit *excitable dynamics*. The long-term behavior for the parameterization used is a stationary point. The observed sharp increase in concentration is therefore a transient phenomenon. Excitable dynamics can lead to sustainable plankton blooms in open flows despite the transient dynamics (Hernández-García & López, 2004).

More recently the influence of horizontal forcing on primary production was the subject of interest. Several studies have shown that ocean jets can behave as barriers to transport across them, or can act as mixing enhances depending on parameter regime (Bower et al., 1985). Eddies are often formed in oceanic currents such as the Gulf Stream or the Canary Current. The water body carried within the eddy may have biogeochemical properties that are different from those of the surrounding waters, and it does not mix with the outer water for long times. In this view, eddies are horizontal carriers, independent of their role in vertical nutrient transport. The horizontal velocity field induced by the eddies has also been suggested to play an important role in determining the spatial distribution of phyto- and zooplankton (Abraham, 1998; Mahadevan & Campbell, 2002).

Eddies have a dual nature when it comes to dispersion. In addition to the strongly dispersive strain regions that form between eddies they have the ability to shield the waters within their core from dispersion. As a result tracers released in the core will be distributed homogeneously across the core but may take considerable time to be dispersed from the core itself (Martin et al., 2001b) The presence of such oases free of strong mixing in an otherwise dispersive flow is one instance of heterogeneity or intermittent turbulence. In the presence of eddies mixing can be imperfect, and strong concentration gradients may appear. The horizontal transport and mixing contributes to the formation of plankton patchiness and ultimately to the primary production enhancement. The influence of horizontal transport, mixing and stirring has been investigated in several numerical studies ((Abraham, 1998; López et al., 2001a; Hernández-García et al., 2002, 2003; Martin, 2003)). The study of ocean jets and horizontal transport, is particularly relevant to ocean modeling in general. (Bower, 1991; Samelson, 1992b; Meyers, 1994; Cencini et al., 1999; Rogerson et al., 1999).

There is a strong mesoscale activity in the wake of the Canary Islands. Winds parallel to the African coast drive the main current along the coast and promote the upwelling of deep water close to the coast. This physical forcing leads to current at the surface transversal to the main flow direction, an *Ekman flow*. As a consequence nutrient rich waters are pumped to the surface close to the coast. This upwelling zone close to the African coast is one of the most active upwelling areas on the planet. There are strong upwelling activities and mesoscale structures such as eddies and fronts present throughout the year in this region (Arístegui et al., 1997; Barton et al., 1998, 2004; Pelegrí et al., 2005). Wind stress in the lee of the islands plays also a role in the generation of mesoscale eddies (Arístegui et al., 1997; Barton et al., 1998). This strong mesoscale activity resulting from detaching vortices, the vortex street in the wake of the islands seem to have significant importance for the biological activity in the region. Significantly higher concentrations of plankton are observed in the wake of the islands compared to other areas with the same latitude. Tropical oceans usually have considerably lower primary production.

The intense mesoscale activity in the wake of the Canary Islands entrains filaments of nutrient rich waters giving rise to filaments of great biological production and complex dynamics. These filaments are clearly visible on satellite images. In this thesis the mechanisms controlling the formation of the filaments and the entrainment of nutrients from the upwelling zone are investigated.

Both the hydrodynamic and the biological model used in this thesis are simplified models. They seek to reproduce the main features of the system, with the least possible degree of conceptional complexity. Although concepts about the processes involved are kept as simple as possible they must give a rather realistic representation of the processes one wishes to model. They can therefore be complex from a mathematical point of view. Nevertheless the advantage of simple process models is that they allow direct investigation of the role played by each generic processes. Such processes may be difficult to diagnose in simulations for a specific area. This is a central guideline for the work presented here. The advantage of taking a broader view is that it may allow the recognition of features common to different specific systems. Despite the apparent simplicity of the models used, they can provide a deeper understanding of the most relevant processes.

A simplified two dimensional hydrodynamical model reproduces these main characteristics of the flow in the Canary region is coupled to a three component biological model for nutrients, phytoplankton and zooplankton. The effects of mesoscale eddies on the horizontal transport of nutrients, the primary production and the initiation of plankton blooms are investigated in a time dependent two-dimensional flow around an island representing the island of Gran Canaria. This *generic ocean flow* is time dependent and opens the possibility for *chaotic advection*. The spatial resolution used for the simulations is high in order to resolve filamental structures resulting from the chaotic properties of the flow.

Primary production in the wake is enhanced due to the role of the vorticity redistributing the upwelled nutrients and influencing phytoplankton growth. Under certain conditions the interplay between wake structures and the biological activity leads to plankton blooms inside the mesoscale vortices. The timescales of these processes are important. The residence times of the plankton species in different structures in the wake is determined by the chaotic saddle and other nonlinear effects in the flow. The long residence times of nutrients and plankton in the vicinity of the island and the confinement of plankton within vortices lead to localized plankton blooms.

This thesis is organized in two main parts: in the first part, chapter 2 basic concepts in the theory of passive and active flows are presented. In section 2.1, the topological structures present in open chaotic flows, such as manifolds and non-attracting saddles are reviewed. The interplay between these general features of time dependent hydrodynamic flows and the biological species is presented from a theoretical perspective. Some of the mechanisms presented were found to be relevant also for the biological activity in our hydrodynamic flow.

In section 2.2 some concepts of mathematical modeling of plankton dynamics are presented. Earlier plankton models that form the basis for the particular model used in this thesis are presented. In this section we focus on the history and development of the particular (N,P,Z) model used in this thesis. The formation of the model equations and its dynamic behavior are discussed.

In section 2.3 the numerical algorithm used to solve the advection-

reaction-diffusion equations that describe the time evolution of the coupled model is discussed. The so called semi-lagrangian algorithm is a numerical method to solve the equations on a Eulerian grid.

The following three chapters of the thesis contain the interpretation of the results of the numerical simulations. Chapter 3 is derived from the first published article, (Sandulescu et al., 2006). It is a numerical study of the horizontal transport of nutrients across the wake of the Canary Islands. The study was performed with passive tracers that are continuously launched in the vicinity of a von Karman vortex street. An analytically derived generic flow model for the Canary island wake is introduced. This hydrodynamic model described in section 3.3 is used with minor modifications in all the studies in this thesis. The parameters of the model are derived from observational data, as described in section 3.4. The main mechanisms controlling transport across the Canary Island wake are identified and transport is studied under different assumptions regarding the prevailing conditions in the flow and the wake. The horizontal transport from the upwelling zone across the wake is studied under different conditions, with a periodic, non-periodic and turbulent flow model.

The studies presented in the following two chapters of this thesis, were performeded on a coupled advection-reaction-diffusion system. The model for flow in the Canary Island region is coupled to a model for biological activity. That is a three component trophic-chain model consisting of nutrients, phytoplankton and zooplankton.

Chapter 4 is derived from the article (Sandulescu et al., 2007a), that was submitted to Journal of Theoretical Biology. We simulate the effects of the afore mentioned horizontal transport mechanisms of fluid across the Canary Island wake, on the spatial distribution of the nutrients and the primary production of plankton on the in the wake of the Canary Islands. The simulations are performed under two sets of initial conditions for the biological system, coresponding to nutrient-rich and nutrient-poor conditions in the Canary current. The spatial and temporal evolution of the phytoplankton distribution in the mesoscale structures is different under these initial conditions. The localized increase of the primary production in the wake and in the mesoscale structures is analyzed in subsection 4.4.1 and 4.4.2. A new mechanism that promotes local plankton blooms in the wake is discussed here.

Chapter 5 was derived from the article (Sandulescu et al., 2007b) submitted to the journal Nonlinear processes in Geophysics. The numerical studies were aimed to better understand the interplay between the timescales of the biological system and the hydrodynamic system in the wake. The mechanisms of the emergence of localized plankton blooms are studied in section 5.4. In section 5.4.1 the timescales of the bloom is studied depending on the initial conditions in the flow. The typical residence times in the mesoscale structures and in the vicinity of the island are studied in subsection 5.4.2. The existence of areas with relatively long residence time in the flow has a positive effect on the primary production. The similar timescales of the biological and hydrodynamic processes can promote local blooms as shown in subsection 5.4.3. Transport and biological activity between the different areas of the model are controlled by the underlying structures such as the chaotic saddle and the vortices in the wake of the island. The emergence of filamental structures in the wake and the contribution of the transport of nutrients from the upwelling zone on the primary production in the area due to these structure is discussed in subsections 5.4.4 and 5.4.5.

In chapter 6 the results of the thesis are reviewed in a summary.

Chapter 2

The models

2.1 Passive advection dynamics in open flows

Particles in seemingly simple flows may have chaotic advection dynamics, characterized by an extreme sensitivity of the motion of these particles to the initial conditions. This type of behavior, *Lagrangian chaos* is a very general feature, found in most real flows (Aref, 1984; Crisanti et al., 1991, 1992; Jung et al., 1993; Ziemniak et al., 1994; Waseda & Mitsudera, 2002; Rybka et al., 1992). Numerical experiments with reactive substances in open chaotic flows are a relatively new but interesting subject of study due to the large number of applications (Tél et al., 2005). But what exactly are open chaotic flows?

Flows can be grouped into two main classes: closed and open ones. A flow is said to be closed if its motion is confined to a bounded domain, that does not allow any exchange of material with the exterior. The trajectories of tracer particles in this flow remain inside the bounded domain forever. In the asymptotic state reached as $t \to \infty$, the distribution of the biological active substances in this type of flow is homogeneous. When the substances are distributed homogeneously in the flow, no enhancement of activity can be observed, unless the system is disturbed out of this equilibrium.

Open flows are more common in nature, as large scale atmospheric ore oceanic flows are not delimited. A flow is said to be open if it is not bounded and there is a net current flowing through the region of observation. A typical example is the flow around an obstacle, if we consider only the region in the vicinity of the obstacle. In Fig. 2.1 a flow around a circular obstacle is sketched. The direction of the flow is from the left to the right in the *observation region*, the vicinity of the obstacles. This flow can be considered open as it passes through the observation area. Most of the particles released in



Figure 2.1: A time dependent flow around a cylindrical obstacle. Two vortices travel with the open flow from the left to the right in the mixing region.

open flows will therefore leave the observation area after a finite time, as their trajectories are unbounded, but some particles can stay in the observation area for longer times. Even flows that are actually closed can in many cases be considered as open, if the time it takes for the single tracer to leave the observation area is much longer than the relevant time scale for observation. For example the ocean is of course a closed fluid system, but if we are looking at a relatively small region surrounding an island, the average return time might be of the order of thousands of years. The flow of a current around and in the vicinity of the island, as the one we study in this thesis, can be considered open.

The question of the interplay between chaotic advection of particles in a flow and activity was first addressed by (Metcalfe & Ottino, 1994) in the context of closed flows. This paper was the first in a series. Studies in open flows followed, eg the studies of (Toroczkai et al., 1998; Károlyi et al., 1999; Tél et al., 2000). These studies were performed with active or passive tracers launched in open flows. This approach based upon chaotic advection of tracers has the advantage that it facilitates the study of the hydrodynamic structures in the flow. The trajectories of the tracers can be tracked for a long time and different methods from dynamics theory can be applied to study the properties of the advection. If tracers interact with each other, the effects of the advection upon the interaction between the tracers can be studied. In this thesis we study the biological activity of plankton species represented by concentration fields in open chaotic flows. The dynamical properties of the advection in chaotic flows are important in this context, and we start by considering the advection of passive tracers. Simulating the advection of concentration fields adds complexity to this study as mixing and diffusion effects have to be considered too.

In the simplest approximation, the particles released in the flow are assumed to be non-inertial and pointlike tracers. Inertial effects of the tracers can be neglected and it can be assumed that at each timestep the velocity of the advected particle is the same as that of the fluid at the same position. This forcing by the flow is referred to as *passive advection*. A pointlike particle moving in the flow with the velocity of the flowfield is referred to as *passive tracer*. The equation of motion of a tracer is then

$$\dot{r}(t) = \frac{dr}{dt} = u(r(t), t),$$
(2.1)

where r(t) is the position vector of the advected particle and u is the velocity of the flow, that can be dependent on time. u may be the solution of the Navier-Stokes equation with appropriate boundary conditions. Here we are primarily interested in the dynamics of advected particles for a given flowfield, and therefore simply assume that u(r, t) is known.

Oceans water masses are incompressible and stratified, since temperature and salinity are normally homogeneous in the depth layers. The horizontal motions that can be induced by temperature and salinity gradients are normally much slower than horizontal motions that are induced by horizontal forcing, such as winds. Ocean flows can therefore in many situations be considered to be two-dimensional systems, if one chooses describe the motion in one single depth layer. In the present thesis the hydrodynamic model describes the motion of a current at the surface of the ocean. A two dimensional incompressible flow is free of sources, eg divergence free: div $u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0$. In this case a specific analytical function, a stream function $\Psi(x, y, t)$ can be defined that describes the velocity field. The derivatives in x and y direction describe the velocity components:

$$u_x(x,y,t) = -\frac{\partial \Psi(x,y,t)}{\partial y} \quad , \quad u_y(x,y,t) = \frac{\partial \Psi(x,y,t)}{\partial x} \tag{2.2}$$

One observes that the equations above have the same structure as the Hamiltonian equations. The variable x then plays the role of the position, y plays the role of the conjugate momentum, and the stream function $\Psi(x, y, t)$ is the Hamiltonian. Then the dynamics of a particle can be described as its position in configuration space at a time that is a multiple of an interval δt . In this description the dynamics of a passively advected particle in a planar incompressible flow is analogous with a one-degree-of-freedom Hamiltonian system. The phase space of this Hamiltonian system coincides with the physical plane of which the flow occurs. The chaotic trajectories in the phase-space of this dynamical system are directly visible, since the phase-space variables of the associated Hamiltonian system correspond to the coordinates of the advected particle in configuration space.

If the flow is stationary, the streamfunction (or Hamiltonian) Ψ does not depend on time and the particle trajectories correspond to the level curves of the stream-function Ψ . These lines are called *streamlines* in fluid mechanics. From a dynamical point of view it is the equivalent of a one-degree-of-freedom time-independent Hamiltonian system, which is always integrable. In most realistic situations, however, the flow is non-stationary, and depends on time explicitly. In this case, one has a one-degree-of-freedom Hamiltonian system with a *time-dependent* Hamiltonian. It is well-known that such driven systems typically exhibit non-integrable dynamics, or *chaos*. This means that an advected particle moves unpredictably, and displays a great sensitivity to initial conditions. A consequence of these features is that two tracer released in the flow with an infinitesimal separation in space, may evolve alog very different trajectories. Their trajectories are then chaotic and will separate at an exponential rate. Lagrangian chaos in the advection dynamics was first studied by (Aref, 1984). It is also sometimes referred to as Lagrangian tur*bulence.* This definition of turbulence is distinct from what is usually called turbulence in fluid dynamics (*Eulerian turbulence*), which implies a very complicated time and space dependence for Ψ . Lagrangian turbulence may be different from turbulence in the Eulerian sense and even though flows are laminar in space, chaotic advection in these simple time-periodic flows is an instance of Lagrangian turbulence, (Aref, 1984; Ottino, 1989; Crisanti et al., 1991; Wiggins, 1992; Rothstein et al., 1999). Flows generating Lagrangian chaos or turbulence will simply be referred to as *chaotic flows* in this thesis.

In this thesis the biologcal activity in time dependent open flows very similar to the flow illustrated in Fig. 2.1 is studied. Such flows are characterized by the existence of a net current. In other words, the fluid is moving from an *upstream* region towards a *downstream* region. In this flow as in most other flows of interest in geophysics, the time-dependent part of the dynamics is restricted to a finite region of space, called the *mixing region*. Let us take the example presented here, the flow with an obstacle placed in the middle in Fig. 2.1. In both the upstream and the downstream region, the flow is asymptotically stationary. The time dependence, and hence the chaotic part of the dynamics, is restricted to the mixing region in the wake of the obstacle.

From the point of view of the theory of dynamical systems, advection in open flows is a *scattering* process: there is an asymptotic region where the dynamics is simple, and a bounded region where the dynamics is nontrivial. This latter is called the *interaction region*, which corresponds to the mixing region for fluids. The advected particles typically come from the asymptotically simple region, stay in the interaction region for a while, and then escape again to the asymptotic region. In scattering systems, the accessible phase-space is unbounded, whereas for a confined system (such as a flow within a container) the accessible phase-space has finite volume. When a scattering system displays chaos, *chaotic scattering* is said to occur (Tél, 1990, 1996). Chaotic open flows are instances of chaotic scattering.

We assume that the two-dimensional flow is time-periodic, with some period T: $\Psi(x, y, t + T) = \Psi(x, y, t)$. The advection dynamics can then be described by a stroboscopic map M, which connects the position (x, y) of the advected particle at time $nT + t_0$ to that at time $(n + 1)T + t_0$:

$$(x_{n+1}, y_{n+1}) = M_{t_0}(x_n, y_n).$$
(2.3)

The map M_{t_0} is area-preserving, since the flow is incompressible.

Chaotic scattering systems are characterized by the existence of a complicated set of non-escaping orbits in the interaction region (that is, the mixing region in case of open flows). These are orbits that never escape to the asymptotic regions, either in the past $(t \to -\infty)$ or in the future $(t \to \infty)$. The simplest of these orbits are the periodic ones with periods that are integers of the flows period, T. All the nonescaping orbits are highly unstable and possess a strictly positive local Lyapunov exponent. These orbits make up a set which has a fractal structure in phase-space, and appear as a fractal cloud of points sprinkled on the 2D space of the fluid in a stroboscopic map. This invariant set of non-escaping orbits is called the *chaotic saddle*, and it is responsible for all the main features of chaotic scattering (Tél, 1990, 1996).

Typical tracer trajectories that lay close to the nonescaping orbits are influenced by them. They follow some of the periodic orbits for a while and later can turn to follow an other one. This wandering among periodic or nonescaping orbits results in the *chaotic motion* of the passive tracers. The union of all nonescaping orbits, the chaotic saddle has a unique fractal dimension that is independent of the instant of time at which the snapshot of the stroboscopic map was taken (Tél et al., 2000).

An important invariant set associated to the chaotic saddle is its *stable* manifold. It is defined as the set of initial conditions (points) in phase space such that their corresponding orbits approach the chaotic saddle asymptotically, as the discrete time $n \to +\infty$. Particles moving along the stable manifold enter the mixing region and never leave it: they are 'trapped' there. However, the stable manifold has in general zero area (more precisely, zero

Lesbegue measure), which means that the probability that a randomly chosen point in phase space belongs to the stable manifold is zero. Thus, almost all orbits will leave the interaction region some finite time after entering it, and only a set of measure zero corresponds to orbits which do not leave. In spite of this, the stable manifold has a great influence on the global dynamics of the system. Particles starting from points close to it will spend a long time in the interaction region before escaping, and these long-lived trajectories are responsible for the sensitivity of the dynamics to initial conditions.

Another invariant set related to the chaotic saddle is the unstable manifold. This is the set of phase-space points whose orbits approach the chaotic saddle asymptotically as $n \to -\infty$. The unstable manifold is the set of points along which points lying infinitesimally close to the chaotic saddle will eventually escape in the course of time. It has a fractal dimension, and due to the invariance under time-reversal of the Hamiltonian system 2.2, its fractal dimension D, is the same as that of the stable manifold.

The unstable manifold can be directly observed in open flows (Sommerer et al., 1996). By releasing a droplet of particles, corresponding to a set of initial conditions which overlap the stable manifold, as time goes on, the particles get advected to the mixing region. Those particles which last a long time there without escaping fall very close to the chaotic saddle, and when they finally leave, they trace out the unstable manifold. In short, once the bulk of the particles has escaped, the remaining ones, with longer lifetimes in the mixing region, are concentrated around a fractal set, namely the unstable manifold. This fact is of fundamental importance for all the results to be described in this work. One particular consequence of this is that classical flow visualization techniques based on dye evaporation or streaklines trace out curves which are different from streamlines or any other Eulerian characteristic of the velocity field. They are in fact the unstable manifolds of chaotic saddles.

The stroboscopic map defined in Eq. [2.3] depends on the parameter t_0 , which is proportional to the phase of the oscillation at the instant the map is taken. For each choice of t_0 , the chaotic saddle, as well as its stable and unstable manifolds, has a different shape. However, all the dynamical invariants, such as the fractal dimension, Lyapunov coefficients, etc., are all *independent* of t_0 . In the original time-continuous system 2.2, these manifolds are fractal filaments with shapes changing in time, but recurring with a period T.

Residence times of tracers in a flow around an obstacle were studied previously by (Jung et al., 1993). The complicated form of the trajectories implies a long time spent in the mixing region. In other words tracers can be temporarily trapped in the mixing region. Scattering systems are either hyperbolic or non-hyperbolic, depending to the stability of the orbits in their chaotic saddles. In hyperbolic systems, all orbits in the chaotic saddle are strictly unstable. One of the consequences of hyperbolicity is that the chaotic saddle has zero measure: almost all initial conditions lead to orbits that eventually escape the interaction region. Another feature of hyperbolic systems is that, if we initially have a large number of particles uniformly distributed on an observation region intersecting the interaction region, the number N(t) of particles that have not escaped the region up to time t decays exponentially, $N(t) = e^{-\kappa t}$. The coefficient κ is the escape rate of the process. It satisfies $\kappa < \overline{\lambda}$, where $\overline{\lambda}$ is the chaotic saddles Lyapunov exponent, calculated as an average of the largest eigenvalues of all orbits in the saddle, weighted by the natural measure of the saddle. Physically, the Lyapunov exponent gives the exponential rate of separation of nearby advected particles in the mixing region.

Another possibility is that the scattering dynamics is non-hyperbolic. In this case, there are marginally stable orbits in the chaotic saddle. These orbits are surrounded by stable regions, from which fluid does not escape. A source for non-hyperbolic decay is the surface of the cylinder or obstacle in the flow. It acts as a union of parabolic orbits, and hence as a smooth torus, which is sticky. Close to the surface, in the boundary layer, this stickiness leads to an immediate power law decay (Jung et al., 1993; Tél et al., 2000). In this case the number N(t) of particles that have not escaped up to time t follow a power law $N(t) \sim t^{-\sigma}, \sigma > 0$, as opposed to the exponential law satisfied by hyperbolic systems.

2.2 The three component phytoplankton model

The mathematical formulation of the temporal evolution of the biological, chemical and physical structures is a challenging task. Modeling the life cycle of phytoplankton is perhaps the most complex part of it, because of the multitude of processes that take place in living organisms. Detailed knowledge of the biological processes or the metabolism is necessary in order to extract the most relevant features that need to be represented in the mathematical formulation. Knowledge about these processes is difficult to extract under laboratory conditions or during in-situ experiments.

Some of these mathematical descriptions or models for plankton evolution are based upon coupled nonlinear differential equations. Depending on the number of differential equations, these models can be divided roughly into two classes. The first class of models consists of a relatively large number of coupled differential equations. One of the purposes of designing such a model, is that each differential equation models one component that corresponds to a measurable quantity. Models in this class were created to reproduce the results from measurement campaigns. The authors of these models call them simple, which they probably are if one takes into account the multitude of factors in an ecosystem. But even if from a biological point of view these models might be simple, their mathematical properties are difficult to study due to the large number of parameters. The degree of complexity of such models renders them too complex from a dynamical systems point of view. Generally they cannot be studied with analytical non-numerical methods.

The other class of models consists of systems of two or three ordinary differential equations. Examples for this class are the models by (Steele & Henderson, 1981; Truscott & Brindley, 1994; Edwards & Brindley, 1996). They can more likely be solved with analytical methods. These authors were more interested to capture the qualitative behavior of the species in the model, rather than trying to fit model output to specific data. The results from simple models can be used by modelers dealing with large models as an indication about which aspects of the model formulation are the most crucial in determining the output. Important contributions in this sense were the works of (Steele & Henderson, 1992), (Evans & Parslow, 1985) and (Edwards & Brindley, 1996), that are often considered when formulating larger models.

The model used in this theses is based on the nutrient-phytoplanktonzooplankton (N, P, Z) model of (Steele & Henderson, 1981) and (Fasham, 1993). This model was further developed in the works by (Edwards & Brindley, 1996) and (Oschlies & Garçon, 1999; Martin et al., 2002). In this work we adopted the mathematical formulation of the equations given by (Pasquero et al., 2005). The model consists of three differential equations, describing the time evolution of the average concentration of nutrients N, phytoplankton P and zooplankton Z. The specific terms of the equations valid in the surface mixed layer are:

$$\frac{dN}{dt} = F_N = \Phi_N - \beta \frac{N}{k_N + N}P + \mu_N \left((1 - \gamma) \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z + \mu_P P + \mu_Z Z^2 \right)$$
(2.4)

$$\frac{dP}{dt} = F_P = \beta \frac{N}{k_N + N} P - \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_P P \qquad (2.5)$$

$$\frac{dZ}{dt} = F_Z = \gamma \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_Z Z^2.$$
(2.6)

The terms of the coupled differential equations are paraphrasing the following processes in the phytoplankton population:

$\frac{dN}{dt}$	=	nutrient upwelling - nutrient consumption $+$	
ac		+ recycling of biological material	
$\frac{dP}{dt}$	=	nutrient uptake - predation by zooplankton -	
uv		- phytoplankton mortality	
$\frac{dZ}{dt}$	=	uptake of phytoplankton - zooplankton mortality.	
uv			(2.7)

The terms on the right hand side of the nutrient equation represent the vertical nutrient supply due to upwelling of nutrients from the deep of the ocean, the conversion of nutrient into organic matter through the activity of phytoplankton and the recycling of organic matter present in the mixed layer respectively. The only significant reduction of nutrient in the system is due to the consumption by phytoplankton. The effect of nutrient sinking is not as significant as it typically occurs in nutrient depleted regions, and the associated removal effect is relatively small.

The phytoplankton dynamics is primarily regulated by primary production. Plankton is a grazer, it feed on organic macronutrients like nitrate, phosphate and silicic acid and on micronutrients like iron. Its growth is limited by the availability of nutrients. In the model this process is described by a Holling type II functional response. This type of functional response describes the food consumption rate of a randomly searching organism seeking nutrient. The Holling type II functional form implies that there exists a *maximum uptake rate* of nutrients that is determined by the time the organism needs to ingest the consumed nutrients. The length of time required to process each item sets an upper limit to the rate at which food can be consumed.

The maximum uptake rate is independent of the abundance of the grazers. This is a plausible assumption in the case of phytoplankton grazing on nutrients. There is no competition for nutrients in the sense that the phytoplankton does not actively search for nutrients but incorporates befalling nutrients. The grazers do not interfere with each other, so the primary production is only limited by the depletion of nutrients. The first term on the right hand side of Eq. [2.5],

$$PP = \beta \frac{N}{k_N + N}P \tag{2.8}$$

....

is defined as the primary production PP. The parameter β represents the maximum uptake rate of nutrient by phytoplankton and k_N is the half saturation concentration.

Phytoplankton dynamics is further regulated by grazing through zooplankton through a Holling type III response. It describes the phytoplankton mortality as a function a function of the so-called volume search rate of zooplankton. Zooplankton feeds on phytoplankton and other bacteria and has a wide range of feeding behavior that includes passive filter feeding and active predation. An active search for prey requires energy. It is plausible that in a period of food shortages a forager must change its search strategy in order not to spend more energy than can replenished by food. One method of doing this is either to move to a more favorable location or reduce the volume search rate. A simple way to model this active feeding behavior is to assume that the volume search rate is a function of the current food abundance. For example by assuming that the grazing rate of zooplankton is proportional to a power of the density of its prey. This functional response of the grazing rate is referred to as a Holling type III functional response. The first term in Eq. [2.6] is a Holling type III functional response that describes the growth rate of the zooplankton concentration Z in the presence of a phytoplankton concentration P. In this model it is proportional to the square of the phytoplankton concentration. For a detailed discussion of the Holling type II and III functional forms see (Gurney & Nisbet, 1998).

In this model zooplankton has an assimilation efficiency γ , a factor that describes which fraction of the total grazed biomass that is converted to zooplankton biomass. The remaining part of biomass is excreted and recycled together with the biomass of dead phytoplankton and zooplankton. The mortality term of phytoplankton, the last term in Eq. [2.5] is linear with a mortality rate μ_P . The mortality term of zooplankton, the last term in Eq. [2.6] is quadratical in Z. A quadratic mortality term parameterizes the effects of higher trophic levels. These are predators such as fish and krill. A nonlinear zooplankton mortality term has the effect of reducing oscillations introduced by the nonlinear grazing (Steele & Henderson, 1992; Edwards & Brindley, 1996; Edwards & Yool, 2000)

The regeneration efficiency μ_N is smaller than 1. A fraction of it is lost due to detritus sinking to deeper water, described by the factor $1 - \mu_N$, as not all biological substance is immediately available as nutrient. The nutrient source in the model, Φ_N , represents a vertical nutrient supply due to to upwelling. In this simple model the nutrient N represents a general nutrient concentration (as nitrate, phosphate or silicate) or micronutrient (e.g., iron) limiting phytoplankton growth. As no vertical structure is rendered, this nutrient input into the mixed layer is parameterized. This is usually represented by a *restoring flux* (RF) term, that controls the relaxation of the concentration of nutrients at the surface towards a value. The restoring flux (RF) term depends on the constant and large nutrient content in the deep reservoir, N_0 :

$$\Phi_N = S(\vec{x}, t)(N_0 - N) \tag{2.9}$$

The relaxation time 1/S is a function of space and time in our model. The parameter S measures the rate at which nutrient relaxes to the value N_0 . We imagine a restoring nutrient flux from the eutrophic layer with the concentration N_0 to the surface layer, where phytoplankton consumes and reduces nutrient concentrations N. This form of the nutrient flux can also be interpreted as the final difference approximation to a term that acts between two layers with vertical advective term that acts between the mixed layer and the deeper layers. This is the standard formulation used for chemostat models when the reservoir of nutrients has infinite capacity (Kot, 2001).

Other types of nutrient input processes, such as wind-driven dust deposition, are better represented by a nutrient flux that does not depend on the nutrient content in the mixed layer,

$$\Phi_N = \Phi_0(\vec{r}, t) \tag{2.10}$$

This type of nutrient flux can also exhibit spatiotemporal variability. In the model used for the area studied in this thesis, a region with a higher value of the nutrient relaxation rate S is situated above the island. This region represents an upwelling region, where water from the euphotic layer is transported to the mixed layer through vertical currents. Therefore the nutrient relaxation rate S is generally larger for an upwelling zone than for a region with only horizontal transport, where vertical transport of nutrients is mainly due to diffusion.

The values of the parameters of the biological model are shown in Table 2.1. Initially the ecosystem parameters were taken where possible from literature, mainly from (Fasham, 1995). According to (Oschlies & Garçon, 1999) the values chosen were determined by an optimization performed by the authors in a simulation for the North Atlantic. In the optimization loop, the biological model was coupled to advection model of the North Atlantic which resulted in too high values of chlorophyll during the spring bloom. Further tuning of the biological parameters was done by running one dimensional biological model and comparing the results with time series from the stations BATS and OW, and the station NABE close to the French Joint Global Ocean Flux Study (JGOFS). The obtained values are typical of midlatitude subeuphotic concentrations (Fasham, 1993; Oschlies & Garçon, 1999).



Figure 2.2: The temporal evolution of the three components system for the parameters in Table 2.1. N is the nutrient concentration, P is the phytoplankton concentration, Z is the zooplankton concentration and PP the primary production

With the choice of parameter values in Table 2.1 in spatially homogeneous conditions and for a constant nutrient input, the NPZ model has only one nontrivial stable fixed point Oschlies & Garçon (1999). In this thesis the biological model was coupled to a flowfield that has the main temporal and spatial scales adapted to the Canary Islands area. This velocity field is defined using units of time $T_c = 30$ days and length r = 25 km. The parameters of the biological system that are not non-dimensional must be transformed to these units of the flow field. The old and the new transformed parameter values used in the simulations of the coupled system are given in Table 5.2 in chapter 5. The time evolution of the biological system in units of time T_c and length r is plotted in Fig. 2.2. The values of the fix point solution are $N^* = 0.185$, $P^* = 0.355$ and $Z^* = 0.444 \text{ mmol N m}^{-3}$. There are not time dependent asymptotic solutions, regardless of the specific functional form

parameter	symbol	value
maximum uptake rate	β	0.66 day^{-1}
prey capture rate	η	$1.0 \text{ (mmol N m}^{-3})^{-2} \text{ day}^{-1}$
assimilation efficiency	γ	0.75
maximum grazing rate	a	$2.0 day^{-1}$
nutrient relaxation time	S	0.00648 day^{-1}
half-saturation food density	k_N	$0.5 \text{ mmol N m}^{-3}$
regeneration efficiency	μ_N	0.2
phytoplankton mortality	μ_P	0.03 day^{-1}
zooplankton mortality	μ_Z	$0.2 \text{ (mmol N m}^{-3})^{-1} \text{ day}^{-1}$
nutrient concentration btml	N_0	$8.0 \pmod{\text{N} \text{m}^{-3}}^{-1}$

Table 2.1: List of parameters used in the biological model

chosen for the nutrient supply, i.e. Eq. [2.9] or [2.10]. However, different choices of parameter values lead to different homogeneous solutions, and, in some cases, to the appearance of limit cycles (Edwards & Brindley, 1996).

2.3 The numerical algorithm

Advection-reaction-diffusion equations for active tracers are rather difficult to solve, numerically and analytically. The difficulties lie in the complex structure of the equations that describe the coupling between the hydrodynamic and the biological system. An analytical solution of this class of system of differential equations can only be derived for some specific one dimensional systems.

The set of three coupled differential equations describes the time evolution of the three components of the biological model, the concentrations of nutrients (N), phytoplankton (P) and zooplankton (Z). The model equations contain the advection and reaction terms for each component of the biological model. A diffusion process with a diffusion coefficient D acting on the concentrations of the fields is also included in the equation. The computations in this thesis were performed on an Eulerian grid whith the velocity-field $\mathbf{v}(\mathbf{x}, \mathbf{y}, \mathbf{t})$ derived from an analytical function. The equations of our model are min the Lagrangian form:

$$\frac{\partial N}{\partial t} + \mathbf{v} \cdot \nabla N = F_N + D\nabla^2 N,$$

$$\frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P = F_P + D\nabla^2 P,$$

$$\frac{\partial Z}{\partial t} + \mathbf{v} \cdot \nabla Z = F_Z + D\nabla^2 Z. \tag{2.11}$$

Here F_N , F_P , and F_Z represent the biological interactions in Eqs. [2.4], [2.5] and [2.6] discussed in section 2.2. The terms on the left hand side of the Eqs. [2.11] are the total derivatives of the three concentration fields advected by the flow on the Eulerian grid. These are the advection terms that account for the physical forcing. They consist of the sum of the partial derivatives of each concentration field over time and the dot product of the velocity vector \mathbf{v} and the gradient vector of the concentration fields symbolized by ∇ . The right side hand of the equations contains the reaction and diffusion terms. The first terms on the right hand side of the Eqs. [2.11] are the source terms F_N , F_P , and F_Z . They describe the effect of the biological activity in each gridpoint. Their specific form was discussed in section 2.2. The second term on the right side of the equation describes the effect of diffusion and diffusivity on the concentration fields. Diffusion is a physical phenomenon that has to be included in a realistic model. In the ocean, small scale turbulence induced by extrenal forcing mixes the water body enhancing the diffusion due to concentration gradients. The combined effect of diffusion and small scale turbulence on the concentration of a scalar field is referred to as *diffusivity*. Dispersion is also a term used to describe these effects. The diffusivity D describes how strong the concentrations of each species in neighbouring parcels interact with each other. For several reasons this term requires special attention. Determining the value for the effective diffusivity D is a delicate task.

As no model is capable of representing the currents at all scales in the ocean, a modeler has to find a way to parameterize the effects of small scale turbulence that cannot be explicitly resolved by the model. Simulating hydrodynamic flows coupled to biological systems requiring vast computational resources. Therefore many authors opt for a rather simple representation of dispersion in their models. The simplest and crudest means to represent net diffusive effects of turbulence is to use an *effective diffusivity*. In contrast to molecular diffusion, in oceanographic models effective diffusivity is the representation of the cumulative effect of dispersion by all the currents of a flow as a *Brownian motion*. The scale of the forcing leading to the currents is neglected, the quantity describes an average effect over all scales that are not resolved by the model. As the total number of flow structures that produce dispersion increases with the scale, the *estimates* of the effective diffusivity also increases with scale (Okubo, 1971). The values are estimated from data obtained in a wide range of tracer release experiments at different sites. When the effective diffusivity is plotted against the size of the tracer patch, one finds a linear dependence. This empirical relationship allows modelers to calculate an effective diffusivity for scales that they do not want to simulate explicitly by extrapolation. In this way effects in the change of turbulence behavior at certain scales may be taken into account by choosing a specific turbulence closure scheme, or the effect of turbulence can be treated as effective diffusivity at all scales. When coupled to a reactive tracer, such as phytoplankton, *effective diffusion* can strongly influence spatial structures (Martin, 2003). as the minimum resolution scale of our computation is 500 m, the value for the effective diffusivity D is taken to be $D \approx 10 \text{ m}^2/\text{s}$ following (Okubo, 1971) in the model used in this thesis.

A stringent requirement for the concentration fields described by Eq. [2.11] is that they shall be and must stay positive definite. Also sharp concentration gradients have to be preserved by numerical integration. To fulfill these requirements the temporal and spatial scales of the numerical integration algorithms must be adapted to each other. We shall return to this subject later in this section.

In the past, comparisons between different numerical advection schemes have shown that the nutrient supply in the euphotic layer is significantly affected by the numerics used (Oschlies & Garçon, 1999; Lévy et al., 2001). Primary production sensitivity to the advection schemes is comparable to uncertainties in the estimation of biological parameters (Lévy et al., 2001), and thus a major contributor to the errors in the results of biogeochemical models. Numerical errors often appear as non-physical diffusion and nonmonotonicity related to the appearance of under- and over-shoots in the presence of sharp gradients.

One can think of several methods to solve the coupled differential equations in Eq. [2.11]. The Eulerian integration of the equations is problematic due to the difficulty of finding correct spatial gradients in the concentration fields. We adopted a semi-Lagrangian approach to solve the coupled differential equations in Eq. [2.11] by numerical integration. This method is an extension of that introduced by (Abraham, 1998). Instead of trying to solve the advection-reaction-diffusion in Eq. [2.11] on an Eulerian grid, one rather computes the reactions in a large number of independent fluid parcels. The parcels motion is described by Eq. 2.2 whose explicit form is given in sections 3.3 and 4.3.1. Each fluid parcel represents a water volume, usually of the size of the grid spacing of the Eulerian grid on which the equations are solved. The structures in the velocity field on sub-grid scale are assumed to be correctly described by the diffusivity term. This assumption is of course a crude approximation and constitutes a limitation of the method. To describe the effects of inhomogeneities at small scales, it is necessary to include a sub-grid scale parameterization such as a turbulence closure scheme for the turbulence and a specific representation of the reactive components of the
system. The diffusion term on the right hand side of the equations Eq. [2.11] serves this purpose.

It is mentioned above that the reactions or biological activity are considered to take place in each fluid parcel separately. Each point on the grid is such a moving parcel that travels on a trajectory during each timestep of the integration. We chose to calculate the concentration fields in each gridpoint after each timestep. To obtain the starting concentration for the biological system in each parcel before each timestep of advection, we first integrate the trajectory of each water volume backward in time for a timestep. The concentration at this starting point of each trajectory is obtained by a suitable interpolation algorithm. This can be bilinear interpolation as used in this thesis, or for example bispline. The concentrations are interpolated from the concentrations on the grid computed in the previous timestep. The biological system is integrated forward in time using this starting value. The timestep of the integration of the advection and the biology are identical. There is no exchange between neighboring parcels during integration except for the effective diffusivity that is applied in a subsequent step. This allows sharp gradients in the concentration field to appear in case the biological system gives rise to such gradients. Alternately one can simply integrate the trajectories of the water parcels starting from a grid forward for a timestep. If the concentration of the species is required on a Eulerian grid, it can be interpolated on a regular field. Diffusion can then be implemented after each interpolation. To obtain good estimates, generally a large number of fluid parcels is integrated forward in time.

As the flow-field is derived from a streamfunction, the velocity field used in this thesis is non-divergent. The flow-field is therefore area preserving, meaning that only the shape of the parcels with a concentration can change during the advection. Stretching and folding of the parcels should not have a significant effect, as the timesteps of advection are small and should remain a sub-gridscale processes. Furthermore it is implied that the biological components do not swim freely but are advected by the flow within a parcel. This assumption is appropriate for plankton at mesoscale, since the size of the advected parcels is of the size of a couple of hundred meters. On these scales both phytoplankton and zooplankton are passively advected.

This numerical method can be implemented without diffusion in principle. The interpolation step induces a numerical diffusion of the order $D_n \propto dx^2/dt$. An explicit diffusion is implemented to allow mixing between neighboring parcels in certain time intervals. As mentioned previously it is important to make sure that the implemented diffusion, the effective diffusion D, has a realistic value. An important aspect of the numerical implementation is that the *numerical errors* induced by the interpolation between the points on the Eulerian grid must be of a smaller magnitude than the effective diffusion D. The numerical errors appear in form of a numerical diffusion D_n also referred to as numerical viscosity. If not taken care of, the numerical diffusion causes inaccuracy and propagates. In the worst case it eventually leads to numerical instability. The stability condition for the Eulerian diffusion step can be derived by the von Neuman stability analysis. When dt_d denotes the time step for the diffusion part, it reads $D \frac{dt_d}{dx^2} < 1$

The physical interpretation of the stability condition is that the maximum allowed time step of the integration is, up to a numerical factor D, the diffusion time across the grid-width. D is the estimated diffusion coefficient according to (Okubo, 1971). To satisfy the conditions $D > D_n$ and $D\frac{dt_d}{dx^2} < 1$, the diffusion timestep dt_d in our algorithm was set to be ten smaller than the advection time dt. Using the time units of T = 30 days and L = 25km for space, the dimensionless numerical values of the parameters used are D = 0.041472, dx = 0.01, dt = 0.01 and $dt_d = 0.001$.

A detailed discussion of the semi-Lagrangian methods can be found in (Strain, 1999, 2000). A comparison between the semi-Lagrangian method and a purely Eulerian method for solving an advection-reaction-diffusion equation is presented in (Pasquero et al., 2004). The authors conclude that when solving the system by integration of the equations on an Eulerian grid, diffusion cannot be set to zero, as the Eq. [2.11] contains the gradient of the velocity. The numerical error induced by this method leads to a smoothing of the concentration gradients that is more pronounced than in the case when the semi-Lagrangian method is used. Comparing the power spectra of the distribution of an advected field with the two methods reveals that the distributions are similar at large scales, but that the spectrum in the Eulerian case decays more rapidly at small scales. If one considers the higher efficiency of the numerics, the semi-Lagrangian method is superior to the Eulerian integration. In the standard implementation of an Eulerian method, according to (Oschlies & Garcon, 1999) most of the integration time is spent integrating advected and diffusive fluxes. In the semi-Lagrangian scheme the heaviest load is the interpolation step (Pasquero et al., 2004). The calculations presented here were performed using a bilinear interpolation scheme. Other authors choose more advanced but also time-consuming schemes such as the bispline. An analysis of the effects of the choice of a specific interpolation scheme was beyond the scope of this work.

Chapter 3

Studies of transport across an island wake

1

3.1 Abstract

Transport from nutrient-rich coastal upwellings is a key factor influencing biological activity in surrounding waters and even in the open ocean. The rich upwelling in the North-Western African coast is known to interact strongly with the wake of the Canary islands, giving rise to filaments and other mesoscale structures of increased productivity. Motivated by this scenario, we introduce a simplified two-dimensional kinematic flow describing the wake of an island in a stream, and study the conditions under which there is a net transport of substances across the wake. For small vorticity values in the wake, it acts as a barrier, but there is a transition when increasing vorticity so that for values appropriate to the Canary area, it entrains fluid and enhances cross-wake transport.

¹Chapter 3 has been published with modifications in (Sandulescu et al., 2006). The main differences compared with the published version are: the data in Figs. 3.2, 3.3, 3.5, 3.9, 3.11, 3.12, 3.13 was replotted, two additional subplots were added in Fig. 3.13 and the caption of this figure was modified accordingly. The text was not modified except for the exchange of the symbol N^c in the original text by the symbol R for the ratio of the particles transported across the wake.

3.2 Introduction

Chaotic transport in hydrodynamic flows (Aref, 2003; Ottino, 1989; Wiggins, 1992) is a subject generating a great amount of interest both in its fundamental aspects and in its applications to industrial, laboratory, and environmental flows. A class of problems of particular relevance in the context of ocean modeling is the one of transport across jets (Bower, 1991; Samelson, 1992a; Meyers, 1994; Rogerson et al., 1999; Cencini et al., 1999). One of the outcomes of these studies is that ocean jets can behave, depending on parameter regimes, both as barriers to the transport of the particles and as mixing enhancers, increasing the interchange of water masses across them (Bower et al., 1985). Typically there is an increased fluid transport when enhancing the time dependence of the jet, associated to an increased chaotic behavior of the fluid trajectories.

In this Paper we consider a related issue, namely that of fluid transport across a wake. The motivation arises from situations occurring in front of the Canary upwelling zone in the Northwest African coast (Arístegui et al., 1997; Barton et al., 1998, 2004; Pelegrí et al., 2005) (See Fig. 3.1). There is a strong mesoscale activity in the wake of the Canary Islands, originated from the current impinging on them from the North, and running southwards or southwestwards more or less parallel to the African coast. Wind stress in the lee region of the islands plays also a role in the generation of mesoscale eddies (Arístegui et al., 1997; Barton et al., 1998). At the same time there is intense upwelling of depth nutrient-rich waters at the African coast which is produced by winds parallel to the coast via the Ekman mechanism. These two systems interact giving rise to filaments of great biological productivity and complex dynamics. Our aim in this Paper is to explore a very simple kinematic mechanism for the formation of such filaments: entrainment by the wake. We will also determine whether the wake will act as a barrier, i.e. it will stop the flux of nutrient-rich water towards the ocean interior, or rather cross-wake transport will be increased by the presence of eddies. In the second case, which is the one realized in our model for parameter values appropriate for the Canary zone, the mechanism may be important to enhance biological productivity of ocean regions relatively far from the coastal upwelling. In any case, we stress that in this work we focus on the transport of particles into and across the wake and not on the long-range transport that could drive them far apart from it.

In general, one can identify three possible mechanisms contributing to the horizontal transport from a coastal upwelling across a wake: The first one is the direct effect of Ekman pumping that transports the upwelled waters in the direction opposite to the coast. In the second one, coastal water



Figure 3.1: The Canary Islands region, with the Canary current running southwestwards parallel to the African coast, where there is an intense upwelling, and impinging on the islands.

parcels become entrained by the wake, which stretches and deforms them into filamental features until some parts reach the ocean interior. Third, coastal waters may become captured inside eddies, which can travel long distances.

All of the three mechanisms require the wake to be permeable to fluid trajectories, and chaotic advection behind the island to be strong enough to allow transverse transport across the main current. There have been studies of chaotic transport in flows mmodelingisland wakes (Miller et al., 2002), but the emphasis was not in transverse transport. Transport of coastal waters inside eddies and filaments has been observed in the Canary area (Arístegui et al., 1997; Barton et al., 1998, 2004; Pelegrí et al., 2005). Nevertheless few attention has been devoted to the relative importance and interplay between the first two mechanisms. With numerical solutions of the Navier-Stokes equations in two dimensions (Duan & Wiggins, 1997; Shariff et al., 1990), it has been shown, for the wake behind a cylinder, that an important increase of cross-wake transport occurs in the Reynolds number range 100-200. The phenomenon has been studied in detail (Duan & Wiggins, 1997), and has been associated to topological changes in the structure of the wake, which allows lobes of fluid to be stretched into filaments that cross the wake. The similarity of this mechanism to what it is seen in the Canary area motivates our study, in which we try to identify an analogous mechanism in a geophysical setting. In particular, we will show a transition from a situation with a barrier that does not allow particles to cross the wake, and another one without barrier, where there is a net transport of matter across it. In the real ocean, phenomena such as eddy detachment and additional filamentation produced by hyperbolic regions in the neighborhood, can collaborate with the wake-crossing mechanism reported in this work to produce longrange transport. But these effects are absent in our model, and as already mentioned, we focus on the possibility of crossing the wake, i.e., in the fact that the particles visit the side of the wake opposite to the place from which they are released.

With this aim we use in this Paper a kinematic approach to analyze the interplay between the mechanisms of Ekman transport and entraining by the wake. We focus on horizontal transport on upper ocean layers by using a two-dimensional flow, and set up a model streamfunction having the qualitative features of the wake behind an island by modifying the streamfunction introduced in (Jung et al., 1993; Ziemniak et al., 1994) to model the wake behind a cylinder. Parameters are chosen in such a way that the relevant geometric features (sizes, time scales, speeds, ...) are comparable with the real situation in the Canary Islands zone. We do not expect this to be an accurate model of the real ocean dynamics, but since the spatial and tem-

poral scales are taken from observations, we expect it to capture the correct kinematics of the transport and the relative importance of the mechanisms involved.

In the following, we first discuss the properties of the velocity field used, and then characterize the amount of transport in several parameter regimes. To quantify it we define in the system an area outside the wake providing a continuous source of particles, and count how many of them are able to cross the wake for different parameter values. In our interpretation of the model as a representation of the Canary zone, the particle source area is intended to represent the upwelling water close to the African coast.

The Paper is organized as follows. In the next section we present the kinematic flow differentiating the three situations that we want to study: periodic flow, non-periodic flow and periodic flow with turbulent diffusion of the particles. Then in Section 4 we briefly comment on the dynamics of the particles. Section 5 contains the results of our work and in Section 6 we write down our conclusions.

3.3 An analytical model for the flow in the wake of an island

Full hydrodynamic simulations of flows in two or three spatial dimensions involve solving Navier-Stokes equations or approximations to it. In geophysical contexts, simplified turbulence closing schemes should be used to simulate the small unresolved scales. A simple alternative from which considerable insight has been gained in the past (Bower, 1991; Samelson, 1992a; Meyers, 1994; Rogerson et al., 1999; Cencini et al., 1999) is to consider, in two-dimensional incompressible situations, a model streamfunction $\Psi(x, y, t)$ giving a flow qualitatively similar to the one under study. The velocity components in xand y-direction and the equations of motion of fluid elements are:

$$\dot{x} = v_x(x, y, t) = \frac{\partial}{\partial y} \Psi(x, y, t),$$

$$\dot{y} = v_y(x, y, t) = -\frac{\partial}{\partial x} \Psi(x, y, t).$$
 (3.1)

We are interested in the transport perpendicular to the vortex street in the wake of an island. To keep the geometry of the island as simple as possible, we assume it to have a circular shape. Of course, this is a crude approximation to the Canary islands archipelago. However, observations report on the existence of vortex streets in the south of the islands which qualitatively can be understood as emerging from a single large obstacle. As in (Arístegui et al., 1997), the island of Gran Canaria will be chosen here as the representative obstacle of the whole archipelago. Our streamfunction is based on the one introduced in (Jung et al., 1993) and (Ziemniak et al., 1994), but we add to it the effect of Ekman pumping from the coast originated by the effect of the northern winds on the African coast. In addition, we will also eventually consider vortex trajectories more complex than in (Jung et al., 1993) and (Ziemniak et al., 1994). The kinematic model by Jung et al. was originally developed to describe the flow behind a cylinder of radius rlocated in the middle of a channel of width W. Satisfactory comparison was made with numerical solutions of the Navier-Stokes equation in the range of Reynolds numbers such that the velocity field is periodic in time (von Karman vortex street flow), i.e., for Re of order 100. It is remarkable that numerical simulations in (Arístegui et al., 1997) show that already at Re \approx 100, many orders of magnitude smaller than the true Reynolds number in the turbulent ocean, the flow around Gran Canaria given by a barotropic quasigeostrophic model reproduces some of the observed large scale features. This gives confidence to the hypothesis that the streamfunction in (Jung et al., 1993) and (Ziemniak et al., 1994), developed for flows in that order of Reynolds numbers, is a good starting point to model the large scale features of the island wake.

There are, however, many unrealistic features in it. Among them, the most noticeable is that the true geophysical flow is not time periodic. Another one is that it lacks of any of the small scale structures characteristic to real turbulent flows. To minimize these shortcomings, in this work we will present results for the transport across the wake of an obstacle for three different situations: In the first case we will use a streamfunction periodic in time, which is the direct extension of the model in (Jung et al., 1993) but including an Ekman term. In a second case the motion of the vortices, which in the original model is rectilinear, will have a stochastic component, giving rise to a non-periodic flow. In the third case we will add a random velocity component to the particle motion in the periodic streamfunction, as a way to investigate the impact on transport of small-scale turbulent diffusion. These three situations are described in the next subsections.

3.3.1 Periodic flow

The spatial coordinates are chosen such that the mean flow runs along the horizontal x direction, from left to right, put the center of the cylinder at the origin of coordinates, and measure lengths in units of the cylinder radius, so that r = 1. Under these conditions the streamfunction, based in (Jung et al.,

1993), will be written as

$$\Psi(x, y, t) = f(x, y)g(x, y, t). \tag{3.2}$$

The first factor f(x, y) ensures that the trajectories do not penetrate into the cylinder,

$$f(x,y) = 1 - e^{-a\left(\sqrt{x^2 + y^2} - 1\right)^2}.$$
(3.3)

There is a frictional boundary layer of width $a^{-1/2}$ on which the tangential velocity component tends linearly to zero, while the radial velocity component decreases quadratically. The cylinder surface can be considered as the union of an infinite number of parabolic fixed points.

The second factor g(x, y, t) models the background flow, the vortices in the wake, and the Ekman term:

$$g(x, y, t) = -wh_1(t)g_1(x, y, t) + wh_2(t)g_2(x, y, t) + u_0s(x, y)y + u_E(x - 1)\Theta(x - 1).$$
(3.4)

The first two terms describe the simultaneous presence of two vortices in the wake. They are of opposite sign but their maximal vortex strengths are equal and denoted by w. They are of Gaussian shape:

$$g_i(x, y, t) = e^{-\kappa_0 \left[\left(x - x_i(t) \right)^2 + \alpha \left(y - y_i(t) \right)^2 \right]}, i = 1, 2$$
(3.5)

 $\kappa_0^{-1/2}$ is the characteristic linear size of the vortices (the radius), and α gives the characteristic ratio between the elongation of the vortices in the x and y direction. The vortex centers move along the x direction according to

$$\begin{aligned} x_1(t) &= 1 + L\left(\frac{t}{T_c} \mod 1\right) , \quad y_1(t) = y_0 \\ x_2(t) &= x_1(t - T_c/2) , \qquad y_2(t) = -y_0 , \end{aligned}$$
 (3.6)

and their amplitudes are modulated by

$$h_1(t) = \left| \sin\left(\pi \frac{t}{T_c}\right) \right|$$

$$h_2(t) = h_1(t - T_c/2)$$
(3.7)

Thus, vortices are created behind the cylinder with a dephasing of half a period. Each of them moves a distance L along the x direction during a time T_c , then fades out, and the process restarts.

The third term in g(x, y, t) describes the background flow, a current of speed u_0 in the positive horizontal direction. The factor s(x, y) introduces

the shielding of this background flow behind the cylinder, allowing it to be replaced by the vortex structures. Its precise form is

$$s(x,y) = 1 - e^{-(x-1)^2/\alpha^2 - y^2}.$$
(3.8)

This shielded region is of size 1, i.e. of the size of the cylinder or island.

The last term in q(x, y, t) is absent in the original streamfunction of Jung et al. (Jung et al., 1993; Ziemniak et al., 1994). It models an additional velocity of constant strength u_E in the y direction acting only when the x coordinate of a particle is larger than 1, i.e. just behind the island (Θ is the Heavyside or step function, i.e. $\Theta(u) = 1$ if u > 0 and $\Theta(u) = 0$ if u < 0). This corresponds to a stream crossing the vortex street towards the negative y direction just past the cylinder. This term was introduced in order to take into account the existence of the Ekman-drift in the region of the Canary Islands which points towards the ocean interior. Plots of the streamlines of the flow without and with the Ekman-drift are shown in Figs. 3.2 and 3.3, respectively. The rectangle in the upper part is the area where a large number N of particles, initially equidistant, is repeatedly introduced at regular time intervals Δ . Their trajectories are followed by integrating the equations of motion (3.1) and from them the cross-wake transport is estimated (see below). This configuration aims at representing the transport of water parcels, rich in nutrients, from an upwelling region in the African coast towards the ocean interior.

3.3.2 Non-periodic flow

Real oceanic flows are never perfectly periodic. It is well known that structures that are perfect barriers to transport (Kolmogorov-Arnold-Moser (KAM) tori) in a periodic flow become leaky when the time-dependence is not exactly periodic (Wiggins, 1992), so that there is the possibility that the model defined in the previous subsection would underestimate transport. In addition, in the above presented periodic flow case, the trajectories of the vortices are rectilinear and regular which does not happen in the real case of Canary vortices. As a way to relax both limitations, we add some randomness to the vortex trajectories. Instead of moving along straight horizontal lines, $y_1(t) = y_0$, $y_2(t) = -y_0$, the vertical coordinates of the vortices move according to $y_1(t) = y_0 + \gamma \xi(t)$, and $y_2(t) = -y_1(t)$, where $\xi(t)$ is a normalized Gaussian white noise ($\langle \xi(t) \rangle = 0$, $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$) and γ the noise strength. Using this approach the periodicity of the streamfunction is broken, and some of the characteristic features of periodic flows, such as the existence of strict barriers to transport, will not be present in



Figure 3.2: The streamlines of the flow without Ekman flow, $u_E = 0$, at vortex strength w = 200. Other parameters as described in Sect. 3.3. The box where tracers are starting is drawn just above the cylinder with coordinates 0 < x < 1, and 2.1 < y < 2.5. The snapshots are at t = 0 (top-left), $t = T_c/4$ (top-right), $t = 2T_c/4$ (bottom-left), and $t = 3T_c/4$ (bottom-right).

this case. Again, particle trajectories starting in the upper rectangle are determined from equations (3.1).

3.3.3 Periodic flow with turbulent diffusion of the particles

For the preceding two cases, periodic and non-periodic flows, the trajectories of tracers are computed by integrating equations (3.1) with the given streamfunction. This streamfunction contains only large scale features and completely misses all the small scale turbulence that is characteristic to the real ocean.

A convenient way to include unresolved small scales in Lagrangian computations is to add to the velocity field experienced by the Lagrangian particle a fluctuating term representing small-scale turbulence (Griffa, 1996; Mariano et al., 2002). In our case,

$$\dot{\mathbf{x}}(t) = \mathbf{v}(\mathbf{x}, t) + \sqrt{2K}\eta(t). \tag{3.9}$$



Figure 3.3: Streamlines of the flow with Ekman flow $u_E = 2$ and other parameters, and time sequence of the snapshots, as in Figure 3.2.

where $\mathbf{x} = (x, y)$ and $\mathbf{v}(\mathbf{x}, t)$ is the velocity field given by Eq.(3.1) for the periodic flow case. This gives additional diffusion to particle trajectories. Usually the two-dimensional vector $\eta(t)$ is taken to be a Gaussian Markov process with a memory time of the order of some days (Buffoni et al., 1997; Falco et al., 2000). Here, to explore the impact of irregular unresolved motions in the opposite extreme to the deterministic situation considered in the previous sections, we use for $\eta(t)$ a two-dimensional Gaussian white noise of zero mean and correlations $\langle \eta(t) \cdot \eta(t') \rangle = \delta(t - t')$. For the strength Kwe take $K \approx 10 \ m^2 s^{-1}$, which is the effective eddy diffusivity estimated by (Okubo, 1971) as acting at the spatial scales of about 10 km, which are of the order of the spatial structures that begin to be missed from our streamfunction. In this case, some typical features of the periodic flow are lost, and even smooth dynamical systems structures become fuzzier. In particular, transport may occur even across perfect Lagrangian barriers.

3.4 Parameter estimation for the Canary zone

In this section we enumerate the relevant geophysical properties of the upper ocean levels of the Canary zone in order to be represented in the model. We extract the relevant information from references (Arístegui et al., 1997; Barton et al., 1998, 2004; Pelegrí et al., 2005). Although there are seasonal variations in most of the parameters, representative constant values are used here.

- A unique island is in the model. This mimics the Gran Canaria island, which seems to have most influence on the zonal mesoscale activity (Arístegui et al., 1997). Its linear size is of the order of 54 km, from which we take the radius of the model cylinder to be $r = 25 \ km$. This will be taken in the next Sections as the unit of length so that r = 1 there.
- Typically in this area the mean velocity is $0.05 \ m/s$ and in some periods of the year reaches $0.2 \ m/s$. We take a background flow velocity of $u_0 = 0.18 \ m/s$. In numerical experiments (with a large eddy viscosity) (Arístegui et al., 1997) a von Karman vortex street appears when the background flow is larger than $0.1 \ m/s$.
- Different sizes are observed for the Canary eddies, ranging from 50 to 100 kilometers, and depending on the distance to the island that generates them. In any case the mean radius of the eddies $\kappa_0^{-1/2}$ is comparable to that of the island that generates it. Thus we take $\kappa_0^{-1/2} = r$.
- Eddies are usually elliptic. Its eccentricity diminishes with the distance to the island. In our model we take $\alpha = 1$, that would represent circular vortices. But due to the part of the streamfunction representing shielding behind the cylinder, vortices are stretched and have some ellipticity.
- The rotation period of a buoy in an eddy (Pelegrí et al., 2005) ranges from 3 to 6 days (although increasing with time). This gives a linear velocity at their periphery (distance r from the center) of about 0.6 m/s. By equating this speed with typical values of derivatives of the streamfunction at the vortex periphery we estimate the vortex strength $w \approx 55 \times 10^3 \ m^2/s$.

- The shedding of eddies, as already commented, is not perfectly periodic. Nevertheless we take a typical interval between eddy shedding events of 15 days. Thus $T_c = 30$ days.
- Some measurements of eddy velocities indicate that they move towards the southwest at a velocity of 5-6 kilometers per day. The typical displacement during a time T_c is thus $L = 150 \ km = 6r$.
- Lifetime of the eddies is of several weeks, with some measurements reporting lifetimes of several months. Typically they remain close to the island for about one week. In the model the lifetime is the same as the period T_c , which is within the order of the magnitude of observed permanence in the wake.
- $a^{-1/2}$, the width of the cylinder boundary layer, is difficult to estimate since it is ill-defined at geophysical scales. Fortunately its value only affects motion close to the cylinder and its effect is unimportant in most of the velocity field. We take $a^{-1/2} = r$.
- The Ekman flow is originated by the wind stress $\tau = \rho_{air}c_dv^2$, where $\rho_{air} = 1.222 \ kg/m^3$ is the air density, $c_d \approx 0.0013$ is the drag coefficient between water and air, and v is the wind velocity, typically in the range [2.7, 8.7] m/s. Thus the wind stress is in a range [0.012, 0.12] N/m^2 . For a particular intermediate value of $v = 5 \ m/s$ we have $\tau = 0.040 \ N/m^2$.

The value of the Ekman speed is given by:

$$u_E = \frac{\tau}{\rho_0 f h},\tag{3.10}$$

being $\rho_0 \approx 1024 \ kg/m^3$ the sea water density, $f = 10^{-4} \ s^{-1}$ the Coriolis parameter at the Canary latitude, and h the depth of the Ekman layer. It ranges between 15 m and 100 m. We take the intermediate value $h = 50 \ m$, which can be justified from the expression $h = \sqrt{\frac{2A_v}{f}}$ for a vertical turbulent viscosity $A_v \approx 0.1 \ m^2/s$. With these parameter values, and the values for the wind stress, the Ekman velocity u_E is in the range [0.0023, 0.02] m/s.

In the following, in addition to measure lengths in units of r, we measure time in units of T_c . With this, the non-dimensional values of the parameters to be used in the model read: $r = T_c = a = \alpha = 1$, $\kappa_0 = 1$, L = 6, $u_0 = 18.66$, $u_E \in [0.2, 2]$, w = 200. In the non-periodic case we use $\gamma = 0.5$ and $y_0 = 0.5$ (i.e. half the island radius) for the parameters of the vortex trajectories.

3.5 Particle dynamics in the wake

The dynamics given by Equations (3.1) can be interpreted as the equation of motion of a one-degree of freedom Hamiltonian system with time-dependent Hamiltonian $\Psi(x, y, t)$. In our case we have an open system, meaning that the particles start in an incoming asymptotic region, pass a region where the dynamics is time dependent, and then leave the system through an outgoing asymptotic region. While in the time-dependent region of the system, particles are trapped by the vortices and whirled around for a while. Since the velocity field is time dependent, particles can be handed from one vortex to the following one and can remain in the region close to the cylinder for a relatively long time, even though the vortices leave this region quite rapidly. In fact there are periodic and localized trajectories organizing these long scattering orbits. They constitute the backbone of the so-called *chaotic saddle*, the unstable set of trajectories never leaving the wake region (Jung et al., 1993). This structure, and particularly its stable and unstable manifolds (the lines along which particles ending at the saddle approach it, and particles close to the saddle leave it, respectively) organize important trajectory characteristics in the wake. More in detail, the stable or contracting manifold of the chaotic saddle is the set of spatial points x such that particles starting from x approach the chaotic saddle as time advances. Similarly, the unstable or stretching manifold is the set of points such that their backward-in-time evolution approaches the chaotic saddle. Stable manifolds cannot intersect with themselves and with other stable manifolds, and the same holds for the unstable manifolds. Moreover, particle trajectories cannot cross these manifolds. However, stable and unstable manifolds can intersect each other. All these properties make them important templates organizing the particle trajectories in the system. Typically, vortex boundaries are areas of tangencies between stable and unstable manifolds. In the particular case of open flows, like the one studied in this work, the unstable manifold of the chaotic saddle is the set of points along which particles leave it, and, therefore, it is the set that is traced by a number of particles when they are launched in the system and take an long time to abandon it.

In Fig. 3.4 we show the stable and unstable manifolds of the chaotic saddle for two representative parameter sets. A clear change in the shape of the unstable manifold is seen when increasing the vortex strength w. The chaotic saddle itself, however, is only approached by particles starting on its stable manifold. In our configuration, and for the parameter values we use, these structures are closely packed very near the cylinder surface (see Fig. 3.4), in contrast with other situations studied in the literature (Jung et al., 1993; Ziemniak et al., 1994). As a consequence, for the initial conditions to be



Figure 3.4: Stable (in gray) and unstable (in black) manifolds of the chaotic saddle in the wake of the cylinder, for the case of the periodic flow and $u_E = 2$. Left: Snapshot taken at time $7T_c$ for vortex strength w = 10. Right: Snapshot taken at time $7T_c$ for vortex strength w = 200. In the inset we show a zoom of the manifolds in the area close to the cylinder. Other parameters as described in Sect. 3. The chaotic saddle itself is closely packed immediately behind the cylinder surface. The unstable manifold has been plotted by releasing a large number of particles left of the cylinder and very close to it, letting the flow to transport them for a long time ($7T_c$ as already indicated) so that only the ones lasting at this time in the wake region are still there and plotted. The stable manifold is plotted in the same way but releasing the particles right of the cylinder and running the flow backwards in time.

used in Sect. 3.6, tracked fluid particles will not intersect such manifold and they will not follow strongly chaotic recirculating orbits, but rather they will be advected downstream relatively fast. In addition they will leave the wake region along paths that are not perfectly aligned with the ssaddlesunstable manifold. Despite of this, we will see that the change of topology observed in Fig. 3.4 has global consequences in particle dynamics and that for realistic parameter values, corresponding to the right panel of Fig. 3.4, particles can cross the wake, and experience some stretching and dispersion.

3.6 Quantifying transport across the wake of an island

In this Section we report the numerical results obtained for the three different flows introduced in Section 2. Our objective is to quantify transport across a vortex street in the presence of a continuous source of particles representing the water parcels upwelling at the African coast. We model this source by placing test particles in the rectangle 0 < x < 1 and 2.1 < y < 2.5, i.e., above and at some distance of the cylinder (see Figs. 3.2 and 3.3). We place 200 new particles in the rectangle at regular intervals of time $\Delta = 0.01$ (in units of the flow period T_c), i.e. 20,000 new particles per period, and integrate their evolution under the flow. Particles are initially placed along four horizontal lines inside the rectangle, but this has no influence on the results described below.

Our open system is considered to be the region displayed in Figs. 3.2 and 3.3. Trajectories leaving this region are no longer integrated. The idea is that it will be impossible (for the periodic flow case) or practically impossible (for the non-periodic and turbulent cases) for the particles to return back to the region after they leave it. We count at every interval of time Δ how many particles have crossed the wake. There is some ambiguity in defining the transverse extent of the wake. Fortunately, as will become clear from the results presented later, the dynamics is such that particles either do not approach the central region behind the cylinder while they remain inside our region or rather they perform a rather large transverse excursion. Thus, any reasonable definition of 'crossing the wake' will give essentially the same results. In this Section we will count particles crossing the central line y = 0as 'having crossed the wake', and at the end of the Section we will show that the same results are obtained if counting them when crossing y = -1 (see Fig. 3.13). Particles crossing the chosen line several times are counted only once.

Because of the presence of the Ekman term u_E , all trajectories will eventually reach arbitrarily negative y coordinates if observed sufficiently far downstream. Clearly, this can not be considered to be a wake crossing, and we restrict our computation to the region shown in Figs. 3.2 and 3.3, where the vortices remain localized and thus it is the only part of the flow in which nontrivial dynamics occurs. Given the simple structure of the flow, even when wake crossing occurs, the particles can not go very far and typically they will not leave the proximity of the wake region. In a more realistic ocean setting, additional mechanisms can occur after wake crossing is anyway the



Figure 3.5: The ratio of particles crossing the wake, R, versus vortex strength w in the periodic flow. The different curves correspond to different values of the Ekman pumping, u_E , as indicated in the legend.

first step needed for such long-range transport to occur.

We fix all but two of the parameters of the model indicated in the previous section, namely the strength of the vortices, w, and the Ekman pumping u_E , which are varied in a realistic range. The measure of transport across the wake is performed by counting the number of particles crossing the line y = 0during each time interval Δ . A short transient after the launching of the first particles in the rectangle this quantity becomes a periodic function of time in the periodic flow case, and approximately periodic under the other two flows. To focus on average transverse transport, a quantity called R is computed as the ratio between the number of particles crossing y = 0 during 6 flow periods (after discarding an initial transient of 3 periods) and the total number of particles launched during that time (120,000 particles).

In Fig. 3.5 we plot R versus w for different values of the Ekman pumping strength u_E in the periodic flow case. The most relevant result is the absence of transport for small values of w, identifying the existence of a barrier that does not permit the entrance of particles in the wake. This barrier disappears when w crosses a critical value w_c which depends on the Ekman pumping strength u_E , $w_c = w_c(u_E)$. At a fixed value of u_E the proportion of crossing particles increases with increasing w as expected, above the critical threshold w_c . In Fig. 3.6 we plot the critical value w_c as a function of u_E . The value of w_c diminishes from approximately $w_c = 50$ for $u_E = 0.2$ to $w_c = 20$ for $u_E = 2$. Similarly for increasing u_E the ratio of particles crossing is



Figure 3.6: The critical values of the vortex strength, w_c , versus the velocity of the Ekman pumping u_E in the periodic flow.

significantly larger for a fixed w. Most importantly, for the typical realistic value w = 200 (see Sect. 3) one obtains a rather large proportion of crossing particles, R, independently of the value of u_E , so that one can expect that behind the Canary islands a net transport of particles from the coast to the opposite side of the island wake occurs.

Now we illustrate the transport mechanism by looking at particle distributions for the two different situations just identified. In Fig. 3.7 we show a snapshot of the particle positions, all of them launched at the horizontal lines in the marked rectangle at successive times. In the left panel we plot the case w = 10 in which the launching site is on the exterior of the barrier impeding transport across the wake. In the right panel (w = 200) we observe particles spreading through the wake of the island. Obviously a barrier no longer exists between the launching site and the lower parts of the wake. Transport occurs along a filament entrained into the wake that stretches the particle lines and later disperses them. The similarity of the tracers distribution with real features observed in the Canary area is remarkable (see for example Fig. 24 of (Barton et al., 1998), or Fig. 1 of (Arístegui et al., 2004)). This occurs despite the fact that, as advanced before, the trajectory structure in Fig. 3.7 is rather different from the saddle manifolds in Fig. 3.4. The difference arise because the launching site for the tracer particles is rather far from the manifolds.

Thus we can conclude that the formation of vortices in the wake of the Canary Islands together with Ekman pumping make up a possible mecha-



Figure 3.7: Plot of the spatial distribution of the tracers in the wake of the island for the case of the periodic flow and $u_E = 2$. Left: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 10. Right: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 200.

nism for the formation and entrainment of nutrient-rich filaments into the Canary wake, and eventually for transport of nutrients from the African coast to areas in the Atlantic beyond the islands. The effectiveness of this enrichment mechanism will depend however on how fast are the upwelled nutrients consumed by the biological populations near the coast (Pelegrí et al., 2005). Note that since the size of the island (and so the width of the wake) is 50 km, the time the particles would need to cross the wake if driven only by the Ekman flow (u_E is in the range [0.0023, 0.02] m/s) is between 28 – 250 days. Given that the mean flow $u_0 = 0.18 m/s$ transports particles out of the observation region shown in the figures (10 cylinder radii) in about 16 days, we see that no particles are able to cross the wake region behind the cylinder under the sole effect of the Ekman flow.

We now discuss the results for the other two flows considered. In Fig. 3.9 we show R versus w for different u_E in the non-periodic flow case, that is, with the random y component for the trajectories of vortex centers. The non-periodicity of the flow has been introduced to overcome strict barriers to transport that are not realistic. The results show that the effect of non-periodicity is not strong. We still observe a value w_c below which transport is extremely low, though it is non-zero now. Some particles can enter into the wake at low vortex strength $w < w_c$ due to the non-periodic nature of the flow, but their number is rather small. For higher vortex strength $w > w_c$ we observe an increasing net transport with increasing w. For a fixed u_E the critical value w_c is lower than the corresponding threshold in the periodic case. Fig. 3.8 shows distributions of tracers, again for w = 10 and w = 200. They share the qualitative features with the periodic case, Fig. 3.7, although



Figure 3.8: Plot of the spatial distribution of the tracers in the wake of the island for the case of the non-periodic flow and $u_E = 2$. Left: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 10. Right: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 200.



Figure 3.9: Proportion of particles crossing the wake, R, versus vortex strength w for the non-periodic flow. The different curves correspond to different values of the Ekman pumping, u_E , as indicated in the legend.

now there is much more particle dispersion after the filament enters the wake.

The same plots for the case of particles driven by the periodic flow with turbulent diffusion are shown in Figs. 3.11 and 3.10. For small values of the Ekman pumping u_E there is again a critical vortex strength w_c such that for $w < w_c$ only minimal transverse transport is observed. In the realistic value, $u_E = 2$, a non-negligible net transport of particles is observed already for low vortex strength w, in contrast to the previous cases where below the critical



Figure 3.10: Plot of the spatial distribution of tracers in the wake of the island for the case of the periodic flow with turbulent diffusion and $u_E = 2$. Left: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 10. Right: Snapshot of the distribution of the tracers at time $0.39T_c$ for vortex strength w = 200.



Figure 3.11: Proportion of particles crossing the wake, R, versus vortex strength w for the periodic flow with turbulent diffusion of the particles. The different curves correspond to different values of the Ekman pumping, u_E , as indicated in the legend.

value w_c transport was very low. Nevertheless there is still a sharp increase in effective transport when increasing w. Thus a remnant of the critical value w_c is still visible. In Fig. 3.10 the distribution of tracers is plotted for w = 10and w = 200, as in the previous cases. As expected from the introduction of turbulent diffusion particles become randomly dispersed, but always around



Figure 3.12: Comparison of transport across the wake for the three kinds of flows. The values of u_E and the type of flow that originated the data are indicated in the plot.

average paths similar to the previous cases.

The comparison among the three cases (periodic, non-periodic and turbulent) is shown in Fig. 3.12. Here we fix $u_E = 0$, $u_E = 1$ and $u_E = 2$, and plot R vs w for the different types of flows in one graph.

The smallest transverse transport, and the largest critical w_c , is always attained in the periodic flow case (which is the only case for which below w_c transverse transport is exactly zero). The smallest effective value of w_c is found for the case of particles with turbulent diffusion, and transport is also higher in this case for the smallest values of w and all u_E . At the other end of the considered range of w, i.e. going towards realistic values w > 150, the measured transport R is largest for the non-periodic flow case. The addition of the turbulent particle diffusion slightly increases transport at large w with respect to the purely periodic case, but the difference between these two cases is not large. This indicates that turbulent diffusion, at least as modelled here, has no strong influence on transverse transport in the realistic limit of large w, while non-periodic vortex movement is more significant.

Finally we return to the question about the distance over which transport across the wake occurs by comparing R for different positions of the line which the tracers have to pass in order to be counted as particles that have actually crossed the wake. In Fig. 3.13 we compare the proportions of particles crossing the lines y = 0 and y = -1 for the periodic flow case and $u_E = 1$. Similar results are obtained for other values of u_E and for the other



Figure 3.13: R versus w for $u_E = 1$ for the three flows, and two situations: the proportion of particles that cross the line y = 0, and the line y = -1. No remarkable differences can be observed.

two flows. The ratio of particles crossing the line y = 0 is slightly higher, but both remain very similar over the entire range of w, meaning that the measured transport does not depend significatively on the choice of the position of the line the tracers must cross in the wake. This was in fact quite obvious from the shape of the particle distributions (Figs. 3.7,3.8 and 3.10).

3.7 Conclusions

The biological activity around the Canary Islands and in the open ocean depends crucially on the availability of nutrients. An important source of these nutrients is provided by the upwelling near the African coast. In a simple scenario, we have shown that these nutrients can be transported over long distances perpendicular to the coast due to the formation of filaments that are entrained into the Canary wake by the eddies present there. The intensity of this horizontal transport depends strongly on the vorticity content of the vortices, characterized by w, and the strength of the Ekman pumping u_E . Other parameters do not affect transverse transport so strongly, except the mean flow velocity u_0 that has the obvious effect of transporting the particles faster or slower downstream the wake, and thus decreasing or increasing, respectively, their chances to cross it. The mesoscale structure mediating the crossing – a meandering filament – is very similar to real structures observed in the Canary region.

Since our approach is kinematic, and parameters are directly obtained from observations, our conclusions do not depend on the particular mechanism producing the wake eddies, being it flow separation, wind stress curl on the lee of the islands (Arístegui et al., 1997; Barton et al., 1998), etc. The simplicity of our approach has allowed us to identify different factors which enhance or diminish transport across the wake. We have found that cross-wake transport occurs always at large enough w. Periodic flow contains transport barriers which block the transport from outside the wake if the vortex strength is below a critical threshold value. Such a sharp threshold is replaced by a crossover to low transport in the non-periodic case, or when a simple model of turbulent diffusion is considered. These last mechanisms enhance transport, but in rather different ways: While turbulent diffusion influences transport for small vortex strength, mainly below and close to the critical transport threshold, non-periodicity of the flow enhances transverse transport at high vortex strength. Our model of turbulent diffusion acts effectively only at small scales, while non-periodicity changes the flow on larger scales yielding a stronger overall effect.

Here we have considered only transport of passive tracers. To study the impact of the phenomena discussed here on the biological productivity off from the African coast, coupling to models of plankton dynamics should be performed. This remains an interesting task for the future. **Probably, vertical upwelling produced inside the cyclonic eddies would have to be taken into account**.

Chapter 4

Biological activity in the wake of an island

2

4.1 Abstract

Hydrodynamic forcing plays an important role in shaping the dynamics of marine organisms, in particular of plankton. In this work we study the planktonic biological activity in the wake of an island which is close to an upwelling region. Our research is based on numerical analysis of a kinematic flow mimicking the hydrodynamics in the wake, coupled to a three-component plankton model. Depending on model parameters different phenomena are described: a) The lack of transport of nutrients and plankton across the wake, so that the influence of upwelling on primary production on the other side of the wake is blocked. b) For sufficiently high vorticity, the role of the wake in facilitating this transport and leading to an enhancement of primary production. Finally c) we show that under certain conditions the interplay between wake structures and biological growth leads to plankton blooms inside mesoscale hydrodynamic vortices that act as incubators of primary production.

 $^{^{2}}$ A modified version of this chapter has been submitted for publication (Sandulescu et al., 2007a). The main differences between the submitted manuscript and the present chapter are that the submitted version contains an appendix section where the numerical algorithm is described. Additionally, several individual words have been rephrased.

4.2 Introduction

Understanding the influence of hydrodynamic motions on the growth, productivity and distribution of marine organisms, especially in the context of plankton dynamics, is a major challenge recently addressed from a variety of perspectives (Mann & Lazier, 1991; Denman & Gargett, 1995; Abraham, 1998; Peters & Marrasé, 2000; Károlyi et al., 2000; López et al., 2001a,b; Martin et al., 2002; Martin, 2003). Vertical transport processes of nutrients are recognized as key factors controlling plankton productivity (Denman & Gargett, 1995). In particular, upwelling areas in the world's oceans are of fundamental importance for the growth of phytoplankton which is the base of oceanic food webs. They are characterized by nutrient rich waters coming to the surface from depths of over 50 meters. Nutrient enrichment enhances phytoplankton growth close to the upwelling regions, giving rise to an increase in zooplankton and fish populations in the area. More recently, the importance of horizontal fluid motion has also been pointed out (Abraham, 1998; López et al., 2001b; Hernández-García et al., 2002; Hernández-García et al., 2003; Martin, 2003). Mesoscale stirring redistributes and mixes plankton and nutrients laterally, giving rise also to enhanced productivity (Martin et al., 2002), or to bloom initiation (Reigada et al., 2003), and affects species competition and coexistence (Károlyi et al., 2000; Bracco et al., 2000). Satellite images illustrate the interaction between horizontal mesoscale motions and plankton dynamics.

Vertical upwelling and strong mesoscale activity occur simultaneously in several places of the globe. A stronger impact and a high complexity of the physical-biological interactions are expected there. Some of these areas are the Benguela zone, the Humboldt Current, or the Canary islands.

Though the phenomena we discuss are rather general, we illustrate them by using the Canary islands, which are close to the northwestern African coast, as a specific example. There, upwelling occurs at the African coast because of Ekman pumping induced by the dominant winds, and in addition, the Canary islands constitute an obstacle for the main ocean current in the area, flowing from Northeast to Southwest, originating a strong mesoscale hydrodynamic activity in their wake. The interaction between the vortices in the wake and the Ekman flow transporting nutrient-rich waters from the coastal upwelling seems to be at the heart of the observed enhancement of biological production in the open Atlantic ocean close to the Canary region. Motivated by this situation, the aim of this paper is to study, in a more general framework, the role of wake vorticity in redistributing upwelled nutrients and influencing phytoplankton growth.

To this end we combine the kinematic model flow introduced in (San-

dulescu et al., 2006) with a simple model of a Nutrient-Phytoplankton-Zooplankton (NPZ) trophic chain, and study the impact of the flow characteristics on the biological dynamics, particularly on the primary production (PP). We will use mainly parameter values of relevance in modeling the Canary wake, but we expect our results to have broader application. Only horizontal transport is explicitly taken into account in the flow, the upwelling is modelled as a source term in the nutrient equation. We address questions such as (i) whether the island wake is a barrier for the upwelled nutrients, or (ii) if rather the generated stirring mixes nutrients into poorer waters so that primary production is enhanced, or (iii) what is the impact of the presence of vortices and other wake structures on biological activity. Our main results are, on the one hand, that for a range of parameters which is realistic in the Canary area, primary production is enhanced in the part of the wake opposite to the upwelling zone. That is, the wake is not a barrier confining the region of high nutrients and plankton growth. On the other hand, there is a strong dependence of the productivity, and of the role of the vortices, on the inflow of biological components entering the wake due to transport by the main current. In some situations the vortices in the wake act as an *incubator* whose sole presence is enough to greatly enhance biological productivity in poor waters entering the region.

In the next section we present our general modeling framework, presenting the velocity field, the plankton model, and the boundary (environmental) conditions. In section 4.4 we present our results, organized on two subsections that contain our studies of concentrations of nutrients, plankton, and primary production in relation to hydrodynamic and inflow conditions, and another one in which we analyze the plankton content of vortices. Section 5.5 summarizes our conclusions.

4.3 Modeling framework

Figure 4.1 shows our twodimensional model domain. The main current flows from left to right, passing by the circular obstacle, which models the presence of an island, and giving rise to mesoscale vortices in its wake. Vertical hydrodynamic motion is not explicitly considered, but its effect on nutrient upwelling is modelled by a source of nutrients (shown as a small box in the upper zone of the domain). The associated Ekman flow points towards the interior of the domain. Our focus of study will be the lower part of the wake, the region A_s , marked with a box in the lower part of figure 4.1. Our primary objective is to determine if nutrient input from the upwelling region, which is on the opposite part of the wake, may enhance the biological activ-



Figure 4.1: The computational domain with a snapshot of the velocity field. Spatial coordinates, x and y, are in units of the island radius.

ity in this region, and to elucidate the role, as barriers or as transporters, of the wake and of the vortices present in it. In addressing this goal, we realize the importance of the contents of the water transported towards our domain by the main flow. In the context of the Canary islands situation, that will guide our selection of parameter values, the upper part of figure 4.1 represents the African coast, with the coastal upwelling. The obstacle is the Canary archipelago, more particularly the Gran Canaria island which plays an important role for the emergence of the vortices in the area. The main current is the Canary current, flowing from northeast to southwest. In this context our analysis may be of relevance to discuss enrichment of the open ocean beyond the Canary wake by input of coastal waters. More generally, it illustrates the interplay between transport, stirring, and biological dynamics.

4.3.1 The velocity field

We briefly introduce the velocity field used in this study. It is essentially the horizontal incompressible flow, derived from a time periodic streamfunction of period T_c , proposed by (Jung et al., 1993) to model kinematically the vortex street behind a cylinder at moderate Reynolds numbers, but modified to include a velocity component pointing towards the domain interior that mimics the Ekman flow associated with the upwelling (Sandulescu et al., 2006). Its technical description can be found in (Sandulescu et al., 2006). We next describe it qualitatively.

There is a maximum of two vortices simultaneously in the system. They are of opposite vorticity sign but their maximal vortex strength denoted by w is equal. They are created behind the circular obstacle with a phase difference of half a period, $T_c/2$. Each of the vortices travels a distance along the x direction for a time T_c and finally disappears. Then the process repeats periodically again. Since real oceanic flows are never perfectly periodic we add some randomness to the vortex trajectories.

The main background flow moves in the positive horizontal direction with a speed u_0 , and the Ekman drift, which is intended to model the flow from the coast towards the ocean interior, is introduced by considering an additional velocity u_E in the y direction acting in the region with x coordinate larger than the island radius r (see figure 4.1), i.e. just behind the island $(x \ge r)$. The circular obstacle, which is considered as a model island, has a radius r.

To adapt this general setup of the velocity field to a realistic and more specific situation we choose parameter values which are guided by the values in the Canary wake (Sandulescu et al., 2006): r = 25 km, $u_0 = 0.18$ m/s, $u_E = 0.02$ m/s, and $T_c = 30$ days. We consider two situations for the vortex strength in the wake. Previous results (Sandulescu et al., 2006) indicate that the wake entrains water from one side of the island towards the other in form of filaments for high values of the vortex strength which are realistic in the Canary area ($w = w_H \approx 55 \times 10^3$ m²/s), but that it is impermeable to transport when the vortices are weak (say $w = w_L \approx w_H/20 = 2.75 \times 10^3$ m²/s). We will analyze the plankton dynamics under these two vortex strengths, w_L and w_H , and also at intermediate ones. Despite the smallness of u_E , it is larger than the minimum needed to observe a transition from no transport to transport across the wake when increasing w (Sandulescu et al., 2006).

4.3.2 The NPZ model

Our description of the plankton population dynamics is based on a model developed by (Oschlies & Garçon, 1999). It describes the interaction of a three level trophic chain in the mixed layer of the ocean, consisting of nutrients N, phytoplankton P and zooplankton Z, whose concentrations evolve in time with the following NPZ dynamics:

$$\frac{dN}{dt} = F_N \equiv \Phi_N - \beta \frac{N}{k_N + N} P + \mu_N \left((1 - \gamma) \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z + \mu_P P + \mu_Z Z^2 \right),$$

$$\frac{dP}{dt} = F_P \equiv \beta \frac{N}{k_N + N} P - \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_P P,$$

$$\frac{dZ}{dt} = F_Z \equiv \gamma \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_Z Z^2.$$
(4.1)

The dynamics of the nutrients includes three different processes. There is a nutrient supply given by $\Phi_N = S(x,y)(N_0 - N)$ due to vertical mixing. S gives the inverse of the time scale for the nutrients to relax to the nutrient concentration N_0 below the mixed layer. Therefore S is the parameter accounting for the vertical nutrient supply due to upwelling. We take $S(x,y) = S_l = 0.00648 \ day^{-1}$ outside of the upwelling region and $S(x,y) = S_h = 100, S_l = 0.648 \ day^{-1}$ in the nutrient-rich upwelling area identified in figure 4.1. The nutrients are consumed by the phytoplankton according to a Holling type II functional response. The last three terms inside the parenthesis of the nutrient equation denote the recycling of a part of all dead organic matter. The phytoplankton grows upon the consumption of the nutrients, but its concentration is decreased due to grazing by zooplankton and to natural mortality. The grazing enters as a growth term for the zooplankton concentration with an efficiency factor γ . Zooplankton mortality is assumed to be quadratic. Additional details can be consulted in (Oschlies & Garçon, 1999) and (Pasquero et al., 2005). The parameters used are taken from (Pasquero et al., 2004) and presented in the Table 4.1.

The primary production, the rate at which new organic matter is produced, is given by the growth term in the phytoplankton dynamics

$$PP = \beta \frac{N}{k_N + N} P. \tag{4.2}$$

The dynamics of this food chain model is studied in detail in (Edwards & Brindley, 1996) and (Pasquero et al., 2004). Depending on the parameters of the model, it exhibits stationary or oscillatory behavior in the long-term limit. The chosen parameter values lead to a steady state. Using the values from Table 4.1 of the and fixing the vertical mixing to the lower value $S = S_l = 0.00648 \text{ day}^{-1}$ we obtain a steady state that will be called the *ambient* state: $N_{amb} = 0.185$, $P_{amb} = 0.355$ and $Z_{amb} = 0.444$ mmol N m⁻³. Thus, in the nutrient poor region occupying most of the domain the ambient primary production in steady state is $PP_{amb} = 0.0633$ mmol N m⁻³ day⁻¹. In the upwelling region, $S = S_h = 0.648 \text{ day}^{-1}$ and the steady state that would be reached under this nutrient input would be $N_{up} = 7.539$, $P_{up} = 0.603$ and $Z_{up} = 1.154$ mmol N m⁻³, and the primary production associated with these values would be $PP_{up} = 0.373$ mmol N m⁻³, i.e. nearly 6 times PP_{amb} .

parameter	value
β	0.66 day^{-1}
η	$1.0 \text{ (mmol N m}^{-3})^{-2} \text{ day}^{-1}$
γ	0.75
α	$2.0 \mathrm{day}^{-1}$
S_l	0.00648 day^{-1} (nutrient poor)
S_h	0.648 day^{-1} (nutrient rich)
k_N	$0.5 \text{ mmol N m}^{-3}$
μ_N	0.2
μ_P	0.03 day^{-1}
μ_Z	$0.2 \ (\text{mmol N m}^{-3})^{-1} \ \text{day}^{-1}$
N_0	$8.0 \text{ mmol N m}^{-3}$

Table 4.1: List of parameters used in the biological model

4.3.3 Complete model and input conditions

The coupling of the biological and the hydrodynamic model yields an advection-reaction system. We add also an eddy diffusion process acting on plankton and nutrients concentrations with diffusion coefficient D to incorporate the small scale turbulence, which is not explicitly taken into account by the large scale velocity field used. Following (Okubo, 1971) prescriptions, we take $D \approx 10m^2/s$, corresponding to spatial scales of about 10 km at which flow details begin to be absent from our large scale flow model. Thus our complete model is given by the partial differential equations:

$$\frac{\partial N}{\partial t} + \mathbf{v} \cdot \nabla N = F_N + D\nabla^2 N,$$

$$\frac{\partial P}{\partial t} + \mathbf{v} \cdot \nabla P = F_P + D\nabla^2 P,$$

$$\frac{\partial Z}{\partial t} + \mathbf{v} \cdot \nabla Z = F_Z + D\nabla^2 Z,$$
(4.3)

with the biological interactions F_N , F_P , and F_Z from Eq. (4.1), and the velocity field $\mathbf{v}(x, y, t)$ described in subsection 4.3.1. This system is numerically solved by means of a semi-Lagrangian algorithm on a grid. Additional details of the integration algorithm are reported in the section 2.3.

Since we are studying an open flow, inflow conditions into the left part of the domain should be specified. It turns out that the influence of inflow concentrations is rather important and we present here two cases that exemplify the two main behaviors we have identified: In the first one fluid parcels enter the computational domain with the ambient concentrations N_{amb} , P_{amb} , and Z_{amb} . This corresponds to the steady state for $S = S_l$, and represents the situation in which the exterior of the computational domain has the same properties as the part of the domain without upwelling. This input condition will allow us to focus on the interaction between the upwelling water and the main part of the domain containing the wake. In our second situation fluid particles transported by the main flow enter the domain from the left with a content of nutrients and plankton close to vanishing, corresponding to a biologically very poor open ocean outside the considered domain. To be specific, we take $N_L = 0.01 N_{amb}$, $P_L = 0.01 P_{amb}$ and $Z_L = 0.01 Z_{amb}$. Introducing the ratio f between input and ambient concentrations, $(N, P, Z)_{in} = f \times (N_{amb}, P_{amb}, Z_{amb})$, the low input situation corresponds to f = 0.01. Primary production in the inflow water is very low: $PP_L = 8.6 \times 10^{-6} \text{ mmol N m}^{-3} \text{ day}^{-1}$. This is more than 7000 times smaller than PP_{amb} . Since those concentrations are very low, we take into account that fluctuations may be important by adding to each of the concentrations (N_L, P_L, Z_L) of each fluid parcel entering the system an independent random amount of about $\sim 5\%$ of the inflow concentration. In this second situation there is mixing between three types of water: the 'ambient', the 'upwelled', and the 'inflow' ones. It turns out that the interaction between inflow and wake will be the responsible for the interesting behavior described below.

4.4 Results

In this section we first describe the outcome of the different scenarios considered in terms of primary production and plankton distributions, and then address in more detail the relation between vortex structures and plankton patches. The brackets <> shall denote spatiotemporal averages.

4.4.1 Primary production and plankton dynamics

We stress that one of the main observations in (Sandulescu et al., 2006) is that, for the flow parameters used here, there is a qualitative change in the transport behavior at vortex strength $w_c \approx w_H/10$: For weaker vortices, a plume of passive tracers released from the location of our upwelling area develops in the direction of the main flow with a slight transverse displacement due to the Ekman flow u_E but remaining far from our study region A_s . The wake is impermeable to transport. For $w > w_c$, however, the plume becomes a filament that is entrained by the vortices, so that it crosses the wake and reaches A_s . Note hat $w_L < w_c < w_H$.



Figure 4.2: The time evolution of the ratio between the primary production, PP, spatially averaged in A_s , and the ambient one PP_{amb} for the two inflow cases and two values of the vortex strength. The two upper lines are for ambient input concentrations, and the two lower for low inflow. Dashed-dotted lines $w = w_L$, solid lines $w = w_H$.

The observed behavior of our biological model when ambient concentrations are used at the inflow reflects rather directly this transport behavior: The two upper lines in figure 4.2 show the time evolution of the productivity averaged over the region A_s . The dashed-dotted line with a nearly constant value $PP/PP_{amb} \approx 1$ is obtained for $w = w_L < w_c$. The upwelling plume fertilizes the upper part of the computational domain, where higher concentrations of plankton are observed, but the lower part of the wake is unaffected by this and keeps its low ambient productivity value PP_{amb} nearly constant. When $w = w_H > w_c$ (upper solid line) productivity becomes enhanced with respect to its ambient value. It undergoes roughly periodic oscillations reflecting the periodic motion of the nutrient filament entrained by the vortices. The central column in figure 4.3 displays the phytoplankton spatial distribution at different time instants. A filament of high phytoplankton concentration appears in the system, sitting basically on top of the high nutrient filament (not shown) emerging from the upwelling and being entrained by the vortices. Zooplankton and primary production are also distributed in a similar way.

Figure 4.4 shows the average primary production (averaged over A_s and then averaged in time) as a function of the vortex strength w in the range



Figure 4.3: The Okubo-Weiss parameter (left column), the concentration of phytoplankton for ambient inflow (middle), and concentration of phytoplankton for the low inflow situation (right). Phytoplankton concentration is expressed in units of P_{amb} . Snapshots taken during one flow period, at $t/T_c = 14, 14.25, 14.5, 14.75$, from top to bottom.

 $[0.025w_H, w_H]$. As anticipated, a transition from essentially no enrichment by the upwelling to an increasing primary production occurs around $w = w_c \approx 0.1w_H$, confirming a direct influence of the physical transport process on the biological dynamics.

The dynamics in the low concentration inflow case is very different. For all values of w considered, the average primary production in A_s is smaller than the ambient one. This can be understood from the fact that fluid elements enter the domain with very low nutrient and plankton concentrations.

To understand additional features in this situation we have performed numerical analysis of the NPZ dynamics without hydrodynamic terms. Figure 4.5 shows the time needed for P and PP to reach their maximum values as a function of the initial conditions. With a mean flow of speed $u_0 = 0.18 m/s$, fluid elements directly advected by it (i.e. the ones in the region outside the wake) spend only 16 days inside the domain of horizontal extension 10r = 250km of figure 4.1. We see in Fig. 4.5 that for a fraction f of input to ambient concentrations such that f < 0.4 the maximum in P occurs later, so that we cannot expect considerable growth in A_s outside the vortices for the value of f = 0.01 used in in this work. But this observation is also puzzling, since the primary production reported in figure 4.2 for the low inflow case is not as small as the above argument would indicate: It is reduced just between 40% and 85% with respect to the ambient values.

Figure 4.3 (right column) clarifies the mechanisms involved. The spatial plankton distribution is rather different from the previous case. It is clearly related to the vortices and associated structures. The plankton concentration is very low outside. As in the case of ambient inflow, a plume with high nutrient concentration is present in the system due to upwelling and has a shape similar to the one in the ambient inflow case, which resembles the phytoplankton distribution of the central column of figure 4.3, but here it seems to have no effect in inducing phytoplankton growth. The time scale for plankton growth starting from small values is larger than the travel time through the computational area, thus these effects are observable only further downstream. Therefore, in the study area displayed in Figs. 4.1 and 4.3 the influence of the upwelling nutrient filament is masked by a more prominent mechanism, described below. In fact, in this low-inflow case, we note that the phenomenology observed in the area remains qualitatively unchanged if the upwelling is removed, although quantitative changes occur. This indicates that the dominant mechanism in the low inflow situation is not the mixing of upwelling and ambient waters, as in the ambient inflow situation, but the interaction of the inflow with the wake.

Figure 4.3 shows that phytoplankton growth in the vortices occurs after phytoplankton is transported into their interior by filaments emerging from


Figure 4.4: The spatiotemporal average, $\langle PP \rangle$, of the primary production in A_s in terms of its ambient value, PP_{amb} , as a function of the vortex strength w normalized by w_H . We plot the two inflow cases: ambient inflow concentrations (\bullet) and low inflow concentrations (\diamond).

the boundary of the circular obstacle. This complex structure – boundary of the obstacle, filaments emerging from it and rolling up around vortices - is well known from dynamical systems studies of this kind of flow (Jung et al., 1993; Ziemniak et al., 1994; Károlyi et al., 2000; Tél et al., 2005; Sandulescu et al., 2006), and is related to the so-called unstable manifold of the chaotic saddle, the main dynamical structure in the wakes occurring in time-dependent two dimensional open flows. Loosely speaking, it is the location of the fluid elements that take a long time to leave the proximity of the island, because of the complex recirculation emerging just behind the obstacle as well as the reduced velocities occurring near its boundary. A detailed analysis of these structures and their implications for the residence times will be published elsewhere. Particle release calculations allow us to realize that, although most of the incoming particles follow the mean flow and leave the system in the 16 days lapse estimated before, a fraction of them are captured by the wake structures with residence times of about 50 to 20 days for w_H and w_L , respectively. These long residence times allow the plankton concentration to build up in the filaments emerging from the obstacle, which gives rise to a plankton bloom later downstream, when the filaments are stretched and rolled up by the vortices.

Thus, recirculating structures in the island wake act as incubators that



Figure 4.5: Time needed for P (squares) and PP (circles) to reach their maximum values, as a function of initial concentrations (expressed as the ratio to ambient ones f) under the NPZ dynamics without flow. Time is in units of $T_c = 30$ days.

make fluid elements more productive before releasing them into the main current. It turns out that the peak values of the phytoplankton bloom in this low inflow case are larger than the ones under ambient inflow. This somehow paradoxical observation is explained by the fact that zooplankton values are relatively high under equilibrium ambient conditions, so that grazing control of the phytoplankton population is rather effective. By contrast, in the low inflow case zooplankton and thus grazing control is essentially absent. Zooplankton concentration begins to build up only when phytoplankton concentration has already reached larger values. Therefore it is responsible for the end of the bloom further downstream, but high phytoplankton values are attained before that.

Figure 4.2 (two lower curves) shows the time evolution of the primary production under the low inflow conditions. Even for $w = w_L$, for which the wake blocks nutrient fertilization of A_s from the upwelling, primary production shows an oscillating behavior, reflecting the oscillations of the wake structure which is the responsible for the plankton growth. Figure 4.4 shows the increase in primary production by increasing the vortex strength w at a value of $w = w_c \sim 0.1w$. In the range of w considered there is an increase in primary production by a factor of about 2.17, larger than the factor 1.2 of increase attained under ambient inflow.



Figure 4.6: The spatiotemporal average of the primary production in A_s in terms of its ambient value $\langle PP \rangle / PP_{amb}$ as a function of the inflow concentrations at vortex strength $w = w_H$ (solid line) and $w = w_L$ (dashed line). f is the fraction of the ambient concentrations in the fluid entering the system.

So far we have described two very distinct inflow situations and studied the impact of vortices by varying the vortex strength. We now fix the vortex strength w to the high, w_H , and low, w_L , values and describe the primary production mean behavior for intermediate inflow cases in Fig. 4.6 (solid line is for w_H and dashed line for w_L). The inflow concentrations are now varied in terms of the factor f giving the fraction of the ambient concentrations present in the inflow: $(N, P, Z) = f \times (N_{amb}, P_{amb}, Z_{amb})$. f = 0.01 and f = 1 are the low and ambient input cases discussed above. We see, for both values of w, an increase of the average production in A_s with the biological content of the inflow, which is rather fast until $f \approx 0.2$.

Maximum values of P and PP are shown in Fig.4.7 for different initial conditions, with and without flow. A contrasting behavior is seen, remarking the fact that the flow is not simply redistributing biological material but also changing qualitatively the dynamics.

Although we have considered in this paper the effective diffusion value $D = 10m^2/s$, we present in Fig.4.8, for completeness, the effect of reducing D. We see that, in general, primary production is slightly reduced. For low values of w, in the low inflow case, which is the situation in which productivity is concentrated just in filaments emerging from the circular boundary, we see



Figure 4.7: Maxima of P (circles) and PP (squares), normalized to the corresponding ambient values, vs input fraction f. The data with solid-line are computed without flow, and the ones with dashed-line are in the presence of flow ($w = w_H$). In this last case the maxima are the ones occurring in the time-dependent values of the spatial averages (in A_s) of P and PP.

larger production for smaller diffusion. The reason for this is the lack of dilution of the emerging filaments.

4.4.2 Vortices and plankton distribution

It is well known that vortices are responsible for a large part of the transport and mixing phenomena at mesoscale on the ocean surface (Barton et al., 1998; Pelegrí et al., 2005; Martin et al., 2002). They influence biological dynamics, and most of the studies have focused on the effect of the relatively large vertical motions induced by their cores. Here we focus instead on horizontal processes. In this section we consider the case $w = w_H$, where strong vortices are present in the system, and characterize the plankton distributions relative to vortex positions for the two different inflow conditions, that highlight the two different primary production enhancement mechanisms discussed above. Similar results are expected for other values of w.

We make use of the Okubo-Weiss parameter (Okubo, 1970; Weiss, 1991) W (a precise definition is included in the Appendix 4.6) to identify in an objective way the interior and the exterior part of vortices. Flow regions with W < 0 are vorticity dominated, and can be identified as the inner



Figure 4.8: Average value of PP in A_S , computed as a function of w/w_H , for a small value of the diffusion coefficient, $D_s = 0.1D$, divided by $\langle PP \rangle$, the average production obtained for the value $D = 10m^2/s$ used in the rest of the paper. Filled circles correspond to the low inflow case, while squares are for the ambient inflow one.

part of vortices. Regions with W > 0 are strain dominated and outside vortices. The leftmost column of figure 4.3 displays the W values, showing clearly the position of the vortices. These positions can be correlated with the phytoplankton distributions displayed in the other columns.

Figure 4.9 shows the spatial average of phytoplankton concentrations and primary production inside each of the two vortices and outside them. These regions are identified with the help of the Okubo-Weiss parameter W. The primary production time series is qualitatively similar to the phytoplankton one, although slightly shifted towards earlier times. This is so because PPcontains the influence of the nutrient dynamics, whose temporal evolution anticipates the phytoplankton one. The zooplankton time series (not shown) are also qualitatively similar but shifted towards later times.

In the case of ambient inflow the interior of the vortices (dashed and continuous lines) contains the same quantity of plankton as the inflow, namely the ambient concentration. Only when additional nutrients from the upwelling zone are entrained we observe bursts localized in time. Thus most of the biological activity is in the outside area (which includes the upwelling zone). This quantifies what is seen in the middle column of figure 4.3: plank-



Figure 4.9: Upper panels: time evolution of the spatial average of the phytoplankton concentration P (normalized with P_{amb}) inside the two vortices: y < 0 (lower vortex, solid line) and y > 0 (upper vortex, dashed line), and out (outside, dot line). Left is for ambient inflow concentrations, and right for low inflow concentrations. Vortex strength is always $w = w_H$. Lower panels: primary production in the same locations and situations.

ton appears mainly in filaments that wind around the vortex periphery basically without entering them. The asymmetry observed between the content of the two vortices arises from the fact that, due to the different sense of rotation of the vortices more nutrients are transported towards the vicinity of the lower vortex than to the upper one (cf. figure 4.3).

The situation is rather different in the low inflow case. The range of the concentration oscillations is now larger, and the content of the two vortices oscillates in antiphase. The largest concentration values occur now inside vortex cores, leading to peak bloom values larger than before. Minima are also smaller so that averages in regions such as A_s give an overall smaller plankton content and primary production. Filamental structures close to the boundary of the island transport concentrations towards the vortices where

the species are trapped and transported downstream. During this motion their concentrations are homogenized by small scale turbulence (modeled by the diffusion term in Eq. 4.3) and the classical dynamics that the system of equations (4.1) exhibits in a homogeneous situation occurs: nutrient consumption by phytoplankton induces a large phytoplankton bloom which is stopped by the grazing by zooplankton, that also experiences growth, until all three components approach the final equilibrium value $(N_{amb}, P_{amb}, Z_{amb})$. This steady state for the vortex content occurs only further downstream.

4.5 Conclusions

We have presented numerical results on the biological dynamics in the wake of an island close to a coastal upwelling area. Parameter values were appropriate for the Canary Islands region but we expect our results to be of greater generality.

Two different scenarios have been identified and discussed. In the first one, occurring when the region outside the focus area has properties similar to it, we have identified an enrichment mechanism of one side of the wake by nutrients upwelled on the other side. It occurs at sufficiently high vortex strength of the vortices present in the wake. Vortices entrain water from one side of the island in the form of filaments that are transported across the wake. Filaments of this type are observed in satellite images of the Canary area (Barton et al., 1998; Pelegrí et al., 2005). When the vortex strength is low, transport from one side to the other of the wake is blocked. This scenario is a rather direct translation of the behavior of passive tracers under similar flow (Sandulescu et al., 2006).

The second scenario becomes evident when the waters surrounding the study area are biologically much poorer. Now fertilization by the upwelling is not relevant, but we have identified a mechanism for primary production increase in the wake: The large residence times of some of the fluid particles in particular structures of the island wake allow them to become enriched by the ambient nutrient sources. Filaments from the wake structures feed this enriched water into the vortices, and the nonequilibrium plankton dynamics there leads to strong plankton blooms confined inside the vortices. The biological significance of hydrodynamical structures in the wake of obstacles has been recognized before (Károlyi et al., 2000; Scheuring et al., 2000; Tél et al., 2005), but in these cases the relevance was associated with their complex geometric structure that allowed fine intertwining of filaments containing different species or substances. The mechanism presented here seems to be different and associated with large residence times in the wake, leading

to a kind of *incubatory* effect. We expect this mechanism to be at work in many types of island wakes, even if they are not associated with upwelling systems.

4.6 Appendix: The Okubo-Weiss parameter

The Okubo-Weiss parameter W (Okubo, 1970; Weiss, 1991) is a method used to distinguish areas in which the flow is dominated by vorticity from those areas where the flow is strain dominated. It is given by

$$W = s_n^2 + s_s^2 - \omega^2, (4.4)$$

where s_n , s_s are the normal and the shear components of strain, and ω is the relative vorticity of the flow defined as:

$$s_n = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}, \quad s_s = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \quad , \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}.$$
 (4.5)

We chose the critical threshold value to be $W_c = 0$. For areas where W is below W_c the flow is vortex dominated, otherwise we consider the flow to be strain dominated.

Chapter 5

The biological and hydrodynamic timescales

3

5.1 Abstract

We study the interplay of hydrodynamic mesoscale structures and the growth of plankton in the wake of an island, and its interaction with a coastal upwelling. Our focus is on a mechanism for the emergence of localized plankton blooms in vortices. Using a coupled system of a kinematic flow mimicking the mesoscale structures behind the island and a simple three component model for the marine ecosystem, we show that the long residence times of nutrients and plankton in the vicinity of the island and the confinement of plankton within vortices are key factors for the appearance of localized plankton blooms.

5.2 Introduction

The interplay between hydrodynamic motion and the distribution of marine organisms like phytoplankton and zooplankton is a major challenge recently addressed in numerous studies (Mann & Lazier, 1991; Denman & Gargett, 1995; Abraham, 1998; Peters & Marrasé, 2000; Károlyi et al., 2000; López et al., 2001a,b; Martin et al., 2002; Martin, 2003; Sandulescu et al., 2007a).

³A modified version of this chapter has been has been published with modifications in Sandulescu et al. (2007b). The main differences compared with the published manuscript are are that the published version contains an appendix section where the numerical algorithm is described. Additionally, several individual words have been rephrased.

The growth of phytoplankton in the world's oceans depends strongly on the availability of nutrients. Thus, one of the essential factors controlling the primary production is the vertical transport of nutrients. Coastal upwelling is one of the most important mechanisms of this type. It usually occurs when wind-driven currents, in combination with the Coriolis force, produces Ekman transport, by which surface waters are driven away from the coast and are replaced by nutrient-rich deep waters. Due to this nutrient enrichment, primary production in these areas is strongly boosted, giving rise also to an increase of zooplankton and fish populations.

On the other side, the interplay between plankton dynamics and horizontal transport, mixing and stirring has been investigated in several studies recently (Abraham, 1998; López et al., 2001b; Hernández-García et al., 2002; Hernández-García et al., 2003; Martin, 2003). Horizontal stirring by mesoscale structures like vortices and jets redistributes plankton and nutrients and may enhance primary production (Martin et al., 2002; Hernández-García & López, 2004). An example of the structures formed is in Fig. 5.1. Horizontal transport can also initiate phytoplankton blooms and affects competition and coexistence of different plankton species (Károlyi et al., 2000; Bracco et al., 2000).

Vertical upwelling in connection with strong mesoscale activity occurs in several places on Earth. One of these regions is the Atlantic ocean area close to the northwestern African coast, near the Canary archipelago. The main water current in this area flows from the Northeast towards the Canary islands, in which wake strong mesoscale hydrodynamic activity is observed (Arístegui et al., 1997). The interaction between the vortices emerging in the wake of the Canary islands and the Ekman flow seems to be essential for the observed enhancement of biological production in the open southern Atlantic ocean close to the Canary islands (Arístegui et al., 2004). The aim of this paper is to study the interplay between the redistribution of plankton by the vortices and the primary production. In particular we focus on the role of residence times of plankton particles in the wake of the island. Though we believe that our study is relevant for different areas in the world, we focus on the situation around the Canary archipelago to be specific.

In this work we consider the coupling of the kinematic flow introduced in (Sandulescu et al., 2006) to a simplified model of plankton dynamics with three trophic levels, and study the impact of the underlying hydrodynamic activity and the upwelling of nutrients on primary production in different areas of the wake. In (Sandulescu et al., 2007a) it has been reported that vortices, in particular, play an essential role in the enhancement of primary production. Our main objective in this paper is to analyze this mechanism in detail and show that the extended residence times of plankton within vortices



Figure 5.1: Satellite image of filamental structures of phytoplankton distributions in the Southern ocean in the neighborhood of Tasmania (image by NASA CZCS http://disc.gsfc.nasa.gov/oceancolor/scifocus/ classic_scenes/01_classics_tasmania.shtml)

are responsible for the observation of localized algal blooms in them.

The organization of the paper is as follows. In section 5.3 we present the general framework of our system, indicating the hydrodynamical and the biological model, as well as their coupling. Our main analysis is devoted to the mechanism of the appearance of a localized plankton bloom within a vortex (Sec. 5.4). We study the residence times of plankton within vortices and in the neighborhood of the island. Additionally we clarify the role of the chaotic saddle embedded in the flow in the wake of the island. Finally we summarize and discuss our results in Sec. 5.5.



Figure 5.2: The simplified island wake model setup.

5.3 General framework: velocity field, plankton model and boundary conditions

Our system consists of a hydrodynamic flow with an embedded obstacle and vortices in its wake. The model contains also an incoming jet perpendicular to the main flow that models an Ekman flow coming from the coast, and a nutrient-rich region at a distance from the obstacle simulating a coastal upwelling zone. A sketch of the model is shown in Fig. 5.2. With this simplified geometry we mimic the essential features of the hydrodynamic flow in the Canaries (note that the whole Canary archipelago is approximated by one cylindrical island). In particular, in the wake of the obstacle strong mesoscale activity is observed in the form of a periodic detachment of vortices, which then travel in the main flow direction.

We use the kinematic model first developed by (Jung et al., 1993), which we modified by the introduction of the Ekman flow (Sandulescu et al., 2006). This model is coupled to a simple population dynamics which features the interaction of nutrients N, phytoplankton P and zooplankton Z. The next two subsections are devoted to the introduction of the hydrodynamic as well as the biological model before discussing the results of coupling both models to study the feedback between hydrodynamics and phytoplankton growth. 5.3. General framework: velocity field, plankton model and boundary conditions

5.3.1 The hydrodynamic model

We now introduce the velocity field. Details can be found in (Sandulescu et al., 2006). The setup of our hydrodynamic model is based on a horizontal flow pattern. As Fig. 5.2 shows the main current runs from left to right along the horizontal x direction. The center of the cylinder is placed at the origin of the coordinate system. We consider a two-dimensional velocity field which can be computed analytically from a stream function Ψ . The velocity components in x- and y-direction and the equations of motion of fluid elements are:

$$\dot{x} = v_x(x, y, t) = \frac{\partial}{\partial y} \Psi(x, y, t),$$

$$\dot{y} = v_y(x, y, t) = -\frac{\partial}{\partial x} \Psi(x, y, t).$$
 (5.1)

The stream function is given by the product of two terms (Jung et al., 1993):

$$\Psi(x, y, t) = f(x, y)g(x, y, t).$$
(5.2)

The first factor f(x, y) ensures the correct boundary conditions at the cylinder, $f(x,y) = 1 - \exp\left[-a\left(\sqrt{x^2 + y^2} - 1\right)^2\right]$. This term introduces a frictional boundary layer of width $a^{-1/2}$ on which the tangential velocity component tends linearly to zero, while the radial velocity The second factor g(x, y, t) modcomponent decreases quadratically. els the background flow, the vortices in the wake, and the Ekman flow $g(x,y) = -wh_1(t)g_1(x,y,t) + wh_2(t)g_2(x,y,t) + u_0s(x,y)y + u_E\Theta(x-1)x.$ The vortices in the wake are of opposite sign but their maximal vortex strengths are equal and denoted by w. They are described by $g_i(x, y, t) = \exp\left(-\kappa_0 \left[(x - x_i(t))^2 + \alpha (y - y_i(t))^2 \right] \right), i = 1, 2.$ $\kappa_0^{-1/2}$ is the characteristic linear size of the vortices, and α gives the characteristic ratio between the elongation of the vortices in the x and y direction. The vortex centers move along the x direction according to $x_1(t) = 1 + L(t/T_c \mod 1)$ and $x_2(t) = x_1(t - T_c/2)$, and at values of $y_i(t)$ described below. The amplitudes of the vortices are $h_1(t) = \left| \sin \left(\pi \frac{t}{T_c} \right) \right|$ and $h_2(t) = h_1(t - T_c/2)$ respectively, so that each of them travels along the x direction for a time T_c and disappears. The background flow moves in the positive horizontal direction with a speed u_0 . The factor $s(x,y) = 1 - \exp\left[-(x-1)^2/\alpha^2 - y^2\right]$ describes the shielding of the background flow by the cylinder in a phenomenological manner, using the same elongation factor α as for the vortices.

The Ekman drift, which is intended to model the flow from the coast towards the ocean interior, is introduced by considering an additional velocity of constant strength u_E in the y direction acting only at x coordinates larger than 1, i.e. just behind the island. This corresponds to a stream crossing the vortex street towards negative y values beyond the cylinder.

Real oceanic flows are never perfectly periodic. Therefore we use a nonperiodic version of the kinematic flow just presented. Non-periodicity is achieved by adding some randomness to the vortex trajectories. Instead of moving along straight horizontal lines, $y_1(t) = y_0$, $y_2(t) = -y_0$ (y_0 constant), the vertical coordinates of the vortices move according to $y_1(t) = y_0 + \epsilon \xi(t)$, and $y_2(t) = -y_1(t)$, where $\xi(t)$ is a normalized Gaussian white noise ($\langle \xi(t) \rangle = 0$, $\langle \xi(t) \xi(t') \rangle = \delta_{tt'}$) and ϵ is the noise strength.

The parameters of the model are chosen in such a way that they represent properly the geophysical features of the Canary zone. These values are given in Table 5.1. To make the model dimensionless we measure all lengths in units of the island radius r = 25 km and all times in units of the period $T_c = 30$ days. For a discussion of all parameters we refer to (Sandulescu et al., 2006), where the adaptation of the model to the situation around the Canary islands is discussed in detail.

parameter	value	dimensionless value
r	25 km	1
u_0	$0.18 \mathrm{~m/s}$	18.66
$\kappa_0^{-1/2}$	$25 \mathrm{km}$	1
α	1	1
w	$\approx 55 \times 10^3 \ m^2/s$	200
T_c	30 days	1
L	6r = 150 km	6
$a^{-1/2}$	$25 \mathrm{km}$	1
u_E	$0.02 \mathrm{~m/s}$	2
y_0	r/2=12.5 km	0.5
ϵ	$68.5 \text{ km } \text{day}^{1/2}$	0.5

Table 5.1: List of parameters used in the hydrodynamical model

5.3.2 The biological model

One can find in the literature a large variety of different models used to analyse the dynamics of marine ecosystems. Their complexity ranges from simple ones with only a few interacting components (Steele & Henderson, 1981, 1992) to large comprehensive ones (Baretta et al., 1997). We use a system which is based on a three component model developed by (Steele & Henderson, 1992) and later modified by (Edwards & Brindley, 1996) and (Oschlies & Garçon, 1999).

The model describes the interaction of three species in a trophic chain, namely nutrients N, phytoplankton P and zooplankton Z, whose concentrations evolve in time according to the following dynamics:

$$\frac{dN}{dt} = F_N = \Phi_N - \beta \frac{N}{k_N + N}P + \mu_N \left((1 - \gamma) \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z + \mu_P P + \mu_Z Z^2 \right) \\
\frac{dP}{dt} = F_P = \beta \frac{N}{k_N + N} P - \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_P P \\
\frac{dZ}{dt} = F_Z = \gamma \frac{\alpha \eta P^2}{\alpha + \eta P^2} Z - \mu_Z Z^2$$
(5.3)

Let us now briefly discuss the meaning of the different terms (cf. (Oschlies & Garçon, 1999) and (Pasquero et al., 2004) for details): the dynamics of the nutrients is determined by nutrient supply due to vertical mixing, recycling by bacteria and consumption by phytoplankton. Vertical mixing which brings nutrients from lower layers of the ocean into the mixed layer is parameterized in the biological model, since the hydrodynamical part considers only horizontal transport of nutrients. For the vertical mixing we assume N_0 as a constant nutrient concentration below the mixed layer. Thus the mixing term reads:

$$\Phi_N = S(x, y)(N_0 - N), \tag{5.4}$$

where the function S determines the strength of the upwelling and will be discussed in more detail below. The nutrients are consumed by phytoplankton with a saturation characteristics described by a Holling type II functional response. The recycling by bacteria is modelled by the last three terms in the bracket. A part of all dead organic matter as well as the exudation of zooplankton is degraded by bacteria, though the dynamics of the bacteria themselves is not included in the model. The phytoplankton grows upon the uptake of nutrients, but its concentration is diminished by zooplankton (grazing term) and due to natural mortality. Grazing, modelled by a Holling type III function, enters also as a growth term for the zooplankton dynamics multiplied by a factor γ taking into account that only a part of the food

parameter	value	dimensionless value
β	$0.66 \mathrm{day}^{-1}$	19.8
η	$1.0 \pmod{\text{N}}{\text{m}}^{-3}^{-2} \text{day}^{-1}$	0.12288
γ	0.75	0.75
a	$2.0 day^{-1}$	60
S_l	0.00648 day^{-1} (nutrient poor)	0.1944
S_h	0.648 day^{-1} (nutrient rich)	19.44
k_N	$0.5 \text{ mmol N m}^{-3}$	7.8125
μ_N	0.2	0.2
μ_P	0.03 day^{-1}	0.9
μ_Z	$0.2 \text{ (mmol N m}^{-3})^{-1} \text{ day}^{-1}$	0.384
N_0	$8.0 \text{ mmol N m}^{-3}$	125

Table 5.2: List of parameters used in the biological model

is converted into biomass of the zooplankton, while the other part $(1 - \gamma)$ goes to recycling. The natural mortality of zooplankton is assumed to be quadratic because this term does not only model natural mortality but also the existence of higher predators which are not explicitly considered (Edwards & Bees, 2001). The parameters used are taken from (Pasquero et al., 2004) as presented in Table 5.2. To obtain dimensionless quantities convenient for the numerics, space is measured in units of r, time in units of T_c and mass in units of 10^{12} mmol N.

The primary production is defined as the growth term in the phytoplankton dynamics:

$$PP = \beta \frac{N}{k_N + N}P \tag{5.5}$$

The function S, measuring the strength of vertical mixing in this model is a crucial quantity for the coupling between the hydrodynamical and the biological model, because it quantifies the *local* nutrient supply. As shown in Fig. 5.2 we assume that there exists an upwelling zone which is located in a small rectangular region on one side of the island. According to this assumption, we assign two different values to the parameter S(x, y). In the upwelling zone there is a strong vertical mixing leading to nutrient rich waters in the mixed layer. There we assume $S(x, y) = S_h = 0.648 \text{ day}^{-1}$, while in all the surrounding waters upwelling is much lower so that we assign $S(x, y) = S_l = 0.00648 \text{ day}^{-1}$ which is a hundred times lower.

The dynamics of this model is different depending on the choice of parameters. The long-term behavior can be either stationary with constant concentrations of N, P and Z or oscillatory. We refer for more details to (Edwards & Brindley, 1996) and (Pasquero et al., 2004). We use a parameter set where the system possesses a stable steady-state. Using the parameter values from Table 5.2 and setting the vertical mixing $S = S_l = 0.00648 \text{ day}^{-1}$ we obtain as a steady state $N_{amb} = 0.185$, $P_{amb} = 0.355$ and $Z = Z_{amb} = 0.444$ mmol N m⁻³. In this nutrient poor region the ambient primary production is $PP_{amb} = 0.0633$ mmol N m⁻³ day⁻¹.

5.3.3 The coupled model

The coupling of the biology and the hydrodynamics yields a system of advection-reaction-diffusion equations. Thus the complete model is given by the following system of partial differential equations:

$$\frac{\partial N}{\partial t} + \vec{v} \cdot \nabla N = F_N + D\nabla^2 N,
\frac{\partial P}{\partial t} + \vec{v} \cdot \nabla P = F_P + D\nabla^2 P,
\frac{\partial Z}{\partial t} + \vec{v} \cdot \nabla Z = F_Z + D\nabla^2 Z,$$
(5.6)

with the biological interactions F_N , F_P , and F_Z from Eq. (5.3), and the velocity field $\vec{v}(x, y, t)$ from Eqs. (5.1) and (5.2). The diffusion terms take into account the small scale turbulence with eddy diffusivity D. We take $D \approx 10m^2/s$, as corresponding to the Okubo estimation of eddy diffusivity at scales of about 10 km (Okubo, 1971), the scales which begin to be missed in our large scale streamfunction. This advection-reaction-diffusion system is solved numerically by means of the method explained in Appendix A. As we are studying an open flow, the inflow conditions into the left part of the computational domain have to be specified to complete the model definition. Depending on the choice of the inflow concentrations we observe different behavior. A detailed analysis can be found in (Sandulescu et al., 2007a), here we only recall the main results which are the basis of the analysis we present here. We have studied two essentially different inflow conditions:

1. In the first one fluid parcels enter the computational domain with the ambient concentrations N_{amb} , P_{amb} , and Z_{amb} corresponding to the steady-state for $S = S_l$. In this case the exterior of the computational domain has the same properties as the part of the domain without upwelling.

2. In the second one fluid parcels transported by the main flow enter the domain from the left with a very small content of nutrients and plankton, corresponding to a biologically very poor open ocean outside the considered domain. In particular we take $N_L = 0.01 \times N_{amb}$, $P_L = 0.01 \times P_{amb}$ and $Z_L = 0.01 \times Z_{amb}$, leading to very low primary production in the inflow water $PP_L = 8.6 \times 10^{-6}$ mmol N m⁻³ day⁻¹. Since those concentrations are very low, we take into account that fluctuations may be important by adding to each of the concentrations (N_L, P_L, Z_L) some Gaussian noise term (cf. (Sandulescu et al., 2007a) for details).

The two inflow concentrations yield different behavior as shown in Fig. 5.3. In the first case we observe high biological activity connected with a high primary production in the area outside vortices. Namely, this is the area of the nutrient plumes advected from the upwelling region (Fig. 5.3) left column). By contrast, in the second inflow case (Fig. 5.3 right column) we obtain a high phytoplankton concentration within the vortices. It turns out that here the vortices act as incubators for primary production leading to localized plankton blooms. In the first inflow case the behavior is easy to understand since due to higher nutrient concentrations in the upwelling region and its neighborhood a high growth of phytoplankton is expected. The response on the upwelling of nutrients in the second case is less obvious. Therefore the main objective of this paper is to find out the mechanism of the localized plankton blooms. Such a bloom is shown in Fig. 5.1 as a red (due to high phytoplankton content) vortex which is detached from the island traveling into the open ocean. Therefore, in the rest of this work we consider only the situation that the concentrations at inflow are at their low values $N_L, P_L, Z_L.$

5.4 The mechanism of emergence of localized plankton blooms

After specifying the complete model and its dynamics we now investigate the behavior of the coupled biological and hydrodynamical system from different perspectives to clarify the mechanism of localized enhancement of phytoplankton and primary production connected to vortices. Firstly we study the biological model alone to understand the interplay between the three biological components N, P and Z leading to a sharp increase of phytoplankton for some time interval. This study yields a certain biological time

scale for the growth of plankton which we compare in a second step to the hydrodynamical time scale obtained from the investigations of residence times in vortices. Thirdly we discuss the role of the chaotic saddle embedded in the flow for the emergence of localized enhanced plankton growth.

5.4.1 Plankton growth

To study the enhancement of primary production and the emergence of localized algal blooms we have first to analyze the dynamics of the biological model. There is no commonly accepted definition of an algal bloom. Usually a large increase in the phytoplankton concentrations is considered as a bloom. In most cases such blooms are observed once or twice a year due to seasonal forcing. In our case the phytoplankton bloom is not related to an external forcing and appears only on a rather short time scale.

Models for singular blooms without external seasonal forcing rely on two different mechanisms: On the one hand there appears a sharp increase in phytoplankton as a result of an enrichment with nutrients (Edwards & Brindley, 1996; Huppert et al., 2002). On the other hand blooms can occur due to certain triggering perturbations in the plankton concentrations. The latter mechanism is related to a so-called excitable dynamics (Truscott & Brindley, 1994). Our model system belongs to the first system class.

Since the long-term behavior of our model is stationary for the pparameterization used, the emergence of a sharp increase in phytoplankton is a transient phenomenon and its time scale is important for the mechanism of localized enhancement of the primary production. The time evolution of the three components and the primary production PP of the model system towards the steady state concentrations C_{amb} (C = N, P, Z, or PP) is shown in the upper part of Fig. 5.4. With starting concentrations $0.01 \times$ steadystate concentrations of N_{amb} , P_{amb} and Z_{amb} , first the nutrient concentration increases and, after a time lag, primary production and phytoplankton concentration follow with a large increase. This growth is approximately exponential when the nutrients reach their maximum. Finally, with a larger time lag the concentration of predators (zooplankton) increases as well and the bloom ends due to two factors: nutrient depletion and increased grazing by zooplankton. For comparison in the lower panel of Fig. 5.4 the time evolution of the system with starting concentrations $0.3 \times$ steady-state concentrations of N_{amb} , P_{amb} and Z_{amb} is plotted. With higher starting concentrations the overshooting in nutrient and phytoplankton concentrations at the beginning of the time evolution is less pronounced (because there are more predators already present) and the concentrations converge faster towards the steadystate.



Figure 5.3: The concentration of phytoplankton (normalized with the steady state concentration, P_{amb}) for inflow at ambient concentrations (left) and for low inflow concentrations (right). Snapshots taken at $t/T_c = 4.1, 4.35, 4.6, 4.85$.



Figure 5.4: The time evolution of the concentrations (C) of the species and the primary productivity. Starting concentrations are $0.01 \times$ steady state concentrations of N, P and Z (top) and $0.3 \times$ steady state concentrations of N, P and Z (bottom).

From these simulations we can estimate the time scale for the biological growth: To reach the maximum of the bloom, only 15 to 25 days are necessary depending on the initial condition. The time scale for the whole relaxation process is about $2T_c$, i.e. about 60 days. To understand the interplay between the biological growth and the hydrodynamic mesoscale structures we have to compare this biological time scale with the hydrodynamic one.

5.4.2 The residence time of fluid parcels in the wake

As pointed out in (Sandulescu et al., 2007a) the hydrodynamic mesoscale structures are important for the enhancement of primary production in the wake of the island. To gain more insight into the interplay of hydrodynamics and plankton growth we now quantify the time scales for the relevant hydrodynamic processes. To this end we study the various structures in the hydrodynamic flow which have a significant influence on the residence times of nutrients and plankton in the wake of the island. Firstly, far away from the island (top and bottom of Fig. 5.2) the flow is strain dominated and particles like nutrients and plankton are advected with the background flow of speed u_0 . Thus the residence time of particles released away from the island (with y > 2 and y < -2, x = 0) is about 16 days.

Secondly we note the existence of the eddies. They are characterized by a dominance of vorticity compared to strain. Thus particles are trapped in the vortex once entrained to it. The particles will rotate in the vortex for



Figure 5.5: The trajectory of a fluid parcel released in the flow at the coordinates (-1.15, 0) at time t = 0. Its subsequent positions are plotted with a dimensionless time step $\Delta t = 0.001$.

some time, but since this confinement is not perfect and vortices exist only for some time they leave the vortex and move away with the background flow out of the computational area (cf. Fig. 5.5).

Thirdly we consider two other geometrical objects which are also relevant for the residence time of particles in the vortex street: the chaotic saddle and the cylinder boundary. As shown in (Jung et al., 1993; Duan & Wiggins, 1997) there exists a chaotic saddle which is embedded in the flow beyond the island. At least for short time scales, this invariant set determines the residence times of particles. Particles released in the neighborhood of the chaotic saddle will approach it along its stable manifold and will leave it along its unstable manifold. The unstable manifolds at two different times (the manifolds and the saddle are time-dependent) are presented in Fig. 5.6. As compared with the configuration in (Jung et al., 1993), the manifolds are tightly packed close to the cylinder, because of the parameters used here. To obtain an estimate for the residence time on the chaotic saddle we use a method suggested by (Jung et al., 1993). We sprinkle a large number N of tracer particles (N = 62500) in the area $[0, 2] \times [-1, 1]$ and integrate their trajectories forward in time. If the dynamics in the region is mainly hyperbolic, the number of particles remaining in the area of the chaotic saddle decreases as $N \sim \exp(-\kappa t)$ with κ the escape rate or $1/\kappa$ the mean residence time on the saddle. Figure 5.7 shows the residence times obtained with this method, and Fig. 5.8 shows the decay of the number of particles in the region as a function time. We note that the expected exponential decay occurs only for very short time scales. By fitting this initial time decay, the corresponding escape rate is $\kappa = 3.1/T_c$, and therefore the residence time of tracers in the hyperbolic part of the saddle is $\tau \sim 10$ days. For larger times the particle number in the region decays as a power law. The reason for this



Figure 5.6: The unstable manifold of the chaotic saddle. Snapshots taken at $t = 1.5 T_c$ (left plot), and $2 T_c$ (right plot).

power-law behavior is the non-hyperbolic dynamics near the boundary of the cylinder. As already shown by (Jung et al., 1993) particles stay for a long time in the vicinity of the cylinder giving rise to another long-term statistics of the residence times of the tracers. Thus the number of particles decays as $N \sim t^{-\gamma}$ with $\gamma = 0.96$. The residence times in the vicinity of the island can be estimated as $\tau \sim 85$ days, measured from the decay to a fraction e of the initial number: $N(\tau) = N(0)/e$.

Overall we obtain a residence time statistics which reflects a combination of the three components in the flow: the cylinder, the chaotic saddle and the vortices. This leads to residence times of tracers in the wake as long as 90 days (cf. Fig. 5.7). Note that in Fig. 5.7 one can see that tracers having the longer residence times are either located close to the cylinder or on the chaotic saddle. The residence times in the vortices are determined by their travel time which is about 50 days.



Figure 5.7: Residence time (coded in color in units of T_c) of fluid elements inside the area as a function of its initial positions at time $t/T_c = 0.25$.

5.4.3 The interplay of biological and hydrodynamical residence times

To understand the emergence of localized enhancement of primary production we have to analyze the interplay of the different time scales relevant for coupled biological and hydrodynamical processes. Biological evolution needs about 30-60 days to reach the steady-state when entering the computational domain with very low concentrations of nutrients and plankton. Due to the exponential growth in the beginning of the growth phase, we obtain a plankton bloom after about 25 days. Outside the vortex street the travel time of tracers through the computational domain is only about 16 days due to the background flow of $u_0 = 0.18$ m/s. Therefore we cannot expect a considerable growth of plankton outside the vortex street, since the residence time of plankton and nutrients is too short.

Let us now analyze the situation within the mesoscale structures of the flow in the wake of the island. As the residence time close to the island



Figure 5.8: The number N of tracers inside the area $[0, 2] \times [-1, 1]$ as a function of time in units of the period of the flow T_c . Vortex strength w = 200. Left panel shows the very early decay, with an exponential fit. Right panel plots the overall decay, with a power law fit at large times.

is about 85 days the concentrations of nutrients and plankton have already reached the steady-state which is also indicated by the green color in the right column of Fig. 5.3. Some of the particles in the vicinity of the island come close to the stable manifold of the chaotic saddle visible as the filaments which detach from the cylinder. These filaments are stretched and folded along the unstable manifold of the chaotic saddle, being diffusively diluted during the process by mixing with the poor surrounding waters. Thus, very thin filaments of low plankton and nutrient concentration are produced which are first rolled around the vortices and then entrained by them. Inside the vortex the concentrations become homogeneised to a low value. These very low concentrations of plankton experience the bloom cycle described in Section 5.4.1 during the time they are trapped and confined by the vortex. Since the travel time for the vortices is about 50 days, plankton in them has time to grow. Therefore we observe a localized plankton bloom when the vortex has travelled a distance of ~ 100 km which corresponds to a residence of the plankton in the vortices of $\sim 15-20$ days. After 40-50 days and ~ 200 km we obtain steady-state concentrations and the former filamental structure within the vortex is smeared out by our diffusion term which mimics small scale turbulence.



Figure 5.9: Finite Size Lyapunov Exponent distributions. We plot the field $\lambda_+ - |\lambda_-|$ at time $t/T_c = 0.25$. Stable and unstable manifolds in the flow are approximated by the most positive and most negative filaments in the distributions.

5.4.4 The emergence of filamental structures due to strong mixing

In the previous subsection we have stated that the transport of nutrients and plankton from the vicinity into the interior of a vortex happens by filaments which are entrained by the vortex. To explain this stretching mechanism we now study the mixing process around the vortices in more detail using a method to visualize exponential divergence of the trajectories of initially nearby particles.

The usual tool to analyse exponential divergence in dynamical systems theory is the computation of Lyapunov exponents. In order to adjust this concept to local processes, we compute finite size Lyapunov exponents (FSLE) which are based on the idea that one measures the time necessary to obtain a final prescribed distance δ_f starting from an initial distance δ_0 (Artale et al., 1997; D'Ovidio et al., 2004). For a two-dimensional flow we obtain two Lyapunov exponents λ_+ and λ_- (see appendix B).

Maxima in the spatial distribution of λ_+ , the positive or expanding FSLE, approximate the underlying stable manifold of the chaotic flow (Joseph & Legras, 2002; D'Ovidio et al., 2004), the direction along which parcels approach the saddle. The contracting FSLE, λ_- , detects the underlying unstable manifold of the chaotic flow, the direction along which parcels are stretched out of the saddle. For details on how to calculate these scalar fields see Appendix B.

The FSLEs were calculated choosing the initial separation δ_0 equal to the gridsize and the final separation δ_f equal to the radius of the island and the vortices, since this is the scale of the motion in the wake. As both stable and unstable manifolds cannot be crossed by fluid parcels they are barriers (Artale et al., 1997; D'Ovidio et al., 2004). The scalar field $\lambda_{+} - |\lambda_{-}|$ is plotted in Fig. 5.9. FSLEs are Lagrangian measures, which are computed from trajectories that remain in the flow for a long time, in our case for up to $3T_c$. Therefore even though they are plotted as a snapshot, the visualized structures reflect the stretching and folding of the fluid parcels during this long time. The stable and unstable manifolds are intertwined around the vortex cores and at the island. Stable and unstable manifolds are crossing the wake allowing for transport across the vortex street. They intersect each other in hyperbolic points, regions of strong mixing. This stretchingcompressing mechanism leads to low nutrient and plankton concentrations transported into the interior of the vortex, and thus becoming the starting concentrations for the localized plankton bloom.

5.4.5 On the role of the upwelling region of nutrients

Finally we discuss the importance of the vertical mixing of nutrients in the upwelling zone for the emergence of a plankton bloom inside vortices. Comparing Fig. 5.3 left and right column it is obvious that in the case of an inflow with steady-state conditions (left column), the nutrient plume which appears in the neighborhood of the upwelling zone gives rise to a phytoplankton bloom (red filamental plume). Such a plume is almost absent under low inflow conditions (right column). Though the nutrient supply due to vertical mixing is identical for both inflow conditions, it seems to have a limited effect in the low inflow case. One argument has been already discussed above: The background flow transports the nutrients too fast so that the very small plankton concentrations can not grow to reach high values during the travel time through the computational area. The growth of phytoplankton is visible only further downstream. This leads to the conclusion that the plankton bloom inside the vortex is only slightly influenced by the extra nutrients entrained from the upwelling zone in the low inflow situation. To strengthen this statement we present in Fig. 5.10 the plankton dynamics when the upwelling is removed. We note that the concentration values for phytoplankton and zooplankton are slightly lower compared to the upwelling regime, but qualitatively there is no change observable. Thus localized phytoplankton blooms in vortices are possible in the wake of an island just due to the mechanism discussed in Subsec. 5.4.4 without any extra nutrient supply due to upwelling.

5.5 Conclusions

We have analyzed the interplay between hydrodynamic mesoscale structures and biological growth in the wake of an island. Parameter values for the kinematic hydrodynamic flow were chosen to match the observations for the Canary island region, but since the basic hydrodynamic features studied here are commonly observed in other areas too, we expect our results to be of general validity. Our study is focused on the emergence of a plankton bloom localized in a vortex in the wake of an island. In a previous paper (Sandulescu et al., 2007a) it has been pointed out that under certain conditions a vortex may act as an incubator for plankton growth and primary production. Here we have revealed the mechanism of such a plankton bloom. If the hydrodynamic flow far away from the island is dominated by a jet, then the hydrodynamic time scale is much faster than the biological one, so that considerable growth of plankton cannot be observed. By contrast, in the wake of an island we obtain a much slower time scale which becomes comparable to the biological one giving rise to an exponential growth of phytoplankton and thus to the emergence of a plankton bloom within a vortex. The essential factors for this phenomenon to happen are (i) the long residence times in the vicinity of the island leading to an enrichment of nutrients and plankton in the neighborhood of the island; (ii) the transport and subsequent entrainment of nutrients and plankton to the interior of the vortex due to filamental structures emerging with the chaotic saddle beyond the island, and (iii) the confinement of plankton in the vortex. Though the upwelling of nutrients in an upwelling zone enhances the emergence of localized plankton blooms, it is not a precondition for this phenomenon to occur. The extra nutrients supplied by vertical mixing in areas away from the vortex street are not a part of the mechanism explained here. Upwelling could be more effective if the vortices directly cross upwelling zones when traveling through the ocean. Similar situations have been considered in (Bracco et al., 2000). There it has been shown that under conditions where upwelling occurs only directly within vortices, a plankton bloom within a vortex can be initiated. Therefore the mechanism reported in (Bracco et al., 2000), which relies mostly on upwelling, is different from the mechanism reported here.



Figure 5.10: The concentration of phytoplankton in the absence of upwelling. Snapshots taken at $t/T_c = 4.1, 4.35, 4.6, 4.85$. The color coding is in the same range as in Fig. 5.3.

5.6 Appendix: Finite Size Lyapunov-Exponents

Stretching by advection in fluid flows is often described by means of Lyapunov exponents. They are defined as the average of the exponential rate of separation of initially infinitesimally separated parcels. For application with data sets from tracer experiments the infinite time limit poses a problem. To study non-asymptotic dispersion processes, Finite Size Lyapunov Exponents (FSLE) have been introduced (Artale et al., 1997; D'Ovidio et al., 2004). The FSLE technique allows us to characterize dispersion processes and to detect and visualize Lagrangian structures, such as barriers and vortices. The FSLE are computed by starting two fluid elements at time t close to the point **x** but at a small distance δ_0 , and let them to evolve until their separation exceeds δ_f . From the elapsed time, τ_+ , the FSLE is calculated as

$$\lambda_{+}(\mathbf{x}, t, \delta_{0}, \delta_{f}) = \frac{1}{\tau_{+}} \log \frac{\delta_{f}}{\delta_{0}}$$
(5.7)

The positive subindexes indicate that the tracers are advected forward in time. λ_+ is a scalar measure giving the stretching rate in the flow as it is the inverse of the separation time τ .

The same definition can be applied to tracers integrated in the negative direction in time. λ_{-} gives the contraction rate in the flow at the specified position:

$$\lambda_{-}(\mathbf{x}, t, \delta_0, \delta_f) = \frac{1}{\tau_{-}} \log \frac{\delta_f}{\delta_0}$$
(5.8)

Regions with high values of λ_+ and λ_- trace out approximately the stable and unstable manifolds of the chaotic saddle. These manifolds cannot be crossed by fluid parcel trajectories and thus greatly influence the transport in the area.

Chapter 6 Conclusions

Aquatic and terestrial eco-systems can be described on the basis of food webs in which species on higher levels feed on species that are on a lower level in the web. The competition between species for the available resources is the force driving the evolution of the changes in the fauna and flora over longer time. Those species that are best adapted to the environmental conditions will have more offspring and survive due to their competitive advantage. Processes in a ecosystem with interdependent species should therefore be studied in the context of environmental conditions. In this thesis, the effects of the external oceanic factors, such as transport of nutrients and hydrodynamic forcing, on a trophic food web with three levels were studied with generic models of the oceanic flow in the region of the Canary Islands.

The effects of the hydrodynamic forcing on plankton activity are in particular interesting in areas with intense mesoscale activity. Mesoscale structures such as eddies, vortices, fronts and oceanic jets are inducing inhomogeneities in the structures of the flow. These gradients in the velocity fields can be associated with transport in the vertical and the horizontal. Oceans are a stratified medium with homogeneous physical and biological properties in the different depth layers. The plankton species that populate the different depth layers are especially adapted to the prevailing physical conditions, such as nutrient concentration and light. Mesoscale structures may disrupt locally the vertical stratification and push the eco-systems in the layers out of the asymptotical equilibrium. Hydrodynamic forcing in the horizontal opens possibilities for interesting biological effects in areas with intense mesoscale activity. The Canary islands region is one of the relatively few regions in the world with strong mesoscale activity and upwelling of nutrients.

We have studied the fluid transport across the Canary wake. Wakes can have a dual nature regarding transport. Depending on the parameter regimes wakes can behave as barriers to transport or as can enhance transport. This dual behavior is of particular relevance for the biological activity in the Canary Islands area, as the main nutrient source in the area is located close to the African coast. From there the nutrients are transported in a direction transversal to the main flow, the Canary current. The wind forcing drives this current at a constant speed parallel to the African coast towards the Canary archipelago, from north-east towards south-west. Wind stress in the lee region of the islands influences the formation of mesoscale vortices. The vortices detach in the wake of the islands and travel with the main current parallel to the coast for a couple of hundreds of km and finally dissipate. Their rotating cores entrain water masses over large distances towards the open Atlantic. At the same time the wind induces the upwelling of nutrient rich water from deeper layers near the coast in an Ekman spiral. The two mechanisms, vortices and Ekman pumping, interact giving rise to filaments of great biological productivity and are responsible the transport of the nutrients across the Canary wake. Typically the fluid transport across a wake increases with the amplitude of the frontal instability. For the Canary region three possible scenarios can lead to nutrient rich waters being transported across the wake and far out into the ocean: (i) direct transport by the Ekman pumping across the wake, (ii) coastal waters become entrained by the wake and are stretched to filaments that are transported over long distances, or (iii) transport of nutrient rich waters in the interior of the vortices and are carried over long distances. The mechanisms require the wake to allow cross wake transport. This condition is fulfilled in time dependent flows, as particles are advected along chaotic trajectories. The study presented in this thesis is primarily concerned with determining the possibility of particles to visit a region on the opposite side of the wake. Transport in the upper layers of the ocean is studied with a two-dimensional flow-model from a streamfunction that captures the main features of the flow in the area, using spatial and temporal scales derived from observations. The upwelling of nutrient rich water is simulated by launching particles continuosly in a rectangular area in the flow. The scale of the transport across the wake is determined by counting the number of particles that cross a horizontal line on the opposite side of the wake per unit of time. The dependence of the transport across the wake is analyzed in the kinematic flow for three cases. In the first case for a time-periodic flow including an Ekman term. In a second case the motion of the vortices, which in the original model is rectilinear, has a stochastic component, giving rise to a non-periodic flow. In the third case a random velocity component is added to the particle motion in the periodic streamfunction, as a way to investigate the impact on transport of small-scale turbulent diffusion. The parameters that have a relevant impact on the transport across the wake are the Ekman flow velocity and the vortex strength. The mesoscale structures connected to the crossing are meandering filaments, similar to those observed in the Canary region. For all three flow characteristics we find that transport across the wake is possible for large enough vortex strength. The crossover from the no-transport to the transport situation in the wake is a sharp one in the periodic flow. In the non-periodic case and the turbulent case the transition is gradual. These mechanisms enhance the transport in different ways: the aperiodicity in the flow leads to changes in the flow on large scales enhancing transport at high vortex strength, while turbulent diffusion acts on small scales and contributes to lowering the critical vortex strength required for transport.

Phytoplankton and its primary production makes up the basis the oceanic food web. Horizontal and vertical transport of nutrients close to upwelling regions by mesoscale structures that redistribute and mix plankton and nutrients have a positive effect on the primary production. The aim of the study presented in this thesis is to study in a general framework the role of wake vorticity in redistributing upwelled nutrients and influencing phytoplankton growth in the Canary Islands region. The enhanced biological activity observed in the wake is based on the favorable conditions resulting from the interaction between the vortices in the wake and the Ekman flow transporting nutrient rich waters from the upwelling zone. The vortices in the wake transport nutrients across the wake mixing nutrients into poorer waters and enhance primary production there. There is a strong dependence of the productivity and of the role of the vortices on the inflow of biological components entering the wake with the main current. If the region outside the focus area is at ambient equilibrium, biological activity is enhanced on the opposite side of the wake at sufficiently high vortex strength, as filaments of nutrients are transported across the wake. If the water surrounding the study area are biologically much poorer, a different mechanism for the increase of the primary production in the wake is relevant: Fluid parcels remain longer times in particular structures of the island wake. Recirculating structures in the island wake act as *incubators* that make fluid elements more productive before releasing them into the main current. It turns out that the peak values of the phytoplankton bloom in this low inflow case are larger than the ones under ambient inflow. This somehow paradoxical observation is explained by the fact that zooplankton values are relatively high under equilibrium ambient conditions, so that grazing control of the phytoplankton population is rather effective. Filaments from these wake structures feed these waters into the vortices. As the vortices detach, a strong plankton bloom is observed in the confined water that remaines at non-equilibrium due to stretching by the vortex. The significance of the hydrodynamic stirring in the wake of obstacles has been previously recognized, but the mechanisms related to the enhancement of the primary production were associated with their complex geometric structure. The mechanism described here is related to the long residence time and redistribution in the island wake.

The correct interpretation of the numerical results presented in this thesis requires an in depth knowledge of the time scales of both biological and hydrodynamic processes in the wake. This interplay between the timescales of the redistribution of the nutrients by the hydrodynamic forcing and the primary production in the Canary Islands region is studied. The focus lies on the residence times of water parcels in the wake of the island, and in the mesoscale structures. The models used are the same as in the previous chapters, with parameters chosen in order to match the observation for the Canary Island region. The underlying structures in the time dependent flow, the chaotic saddle and the boundary layer have different effects on the residence times of fluid parcels. While hyperbolic effects from the chaotic saddle are dominant on the short timescales, non-hyperbolic effects from the island boundary layer are predominant on the long timescales, both determining the residence times of water parcels in the wake. Algal blooms are sharp increases in the phytoplankton concentration that are triggered by favorable environmental conditions. In the system considered here the time evolution of the bloom is an overshoot in the phytoplankton concentration during a transient after which the concentration relaxes towards an asymptotic value, the steady-state. The period until the peak concentrations and the asymptotic solution are reached is in general dependent on initial conditions. These are set by the concentrations in the Canary current. The studied area can be divided into areas with different hydrodynamic properties. The timescales of the biological processes in these regions are compared to the timescales of the residence times of water parcels in them. Far away from the island the hydrodynamic flow is dominated by the Canary current. The hydrodynamic timescale is much faster than the biological one and therefore biological one, so that considerable biological growth cannot be observed in that areas. Residence times in the wake close the island are in the same order as the biological time scales and the concentrations of nutrients and plankton can reach the steady-state values. Some of the particles in the vicinity of the island come close to the stable manifold of the chaotic saddle visible as the filaments which detach from the cylinder. Very thin filaments of low plankton and nutrient concentration are produced which are first rolled around the vortices and then entrained by them. These filaments are stretched and folded along the unstable manifold of the chaotic saddle, being diffusively diluted during the process by mixing with the poor surrounding waters. Inside the vortex the concentrations become homogeneised to a low value and experience the bloom cycle. The transport and the subsequent entrainment

of nutrients and plankton to the interior of the vortex are visualized by the method of finite-size-Lyapunov-exponents (FSLE), that detects the filamental structures in the fluid that emerge with the chaotic saddle beyond the island. The upwelling of nutrients in the upwelling zone enhances the emergence of localized plankton blooms but is not necessary for this phenomenon to occur. Similar studies with vortices showed that local blooms can also occur due to upwelling inside the vortices.

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Erklärung

Hiermit erkläre ich, daß ich die vorliegende Dissertation selbstständig verfasst habe und nur die angegebenen Hilfsmittel verwendet habe. Teile der Dissertation wurden bereits veröffentlicht bzw. sind zur Veröffentlichung eingereicht, wie an den entsprechenden Stellen angegeben. Die Dissertation hat weder in Teilen noch in ihrer Gesamtheit einer anderen wissenschaftlichen Hochschule zur Begutachtung in einem Promotionsverfahren vorgelegen.

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